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STOCHASTIC SENSITIVITY ANALYSIS OF NONLINEAR CRACK DETECTION USING QUADRATIC TEAGER-KAISER ENERGY

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Abstract: *This study investigates the influence of uncertainties in materials on nonlinear crack detection, particularly in cases where responses exhibit slight nonlinearity. A stochastic sensitivity analysis approach is conducted on a novel indicator for nonlinear crack detection, known as the quadratic Teager-Kaiser energy, and its performance is compared with the harmonic indicator derived from the frequency spectrum. The method considers each input parameter as a random variable individually, as well as all input parameters collectively as random variables, by incorporating the partial sensitivity factor of each input and the coefficient of variation of all inputs. The findings indicate that, among all material parameters discussed, quadratic Teager-Kaiser energy exhibits relatively greater sensitivity than frequency spectrum responses, with elastic modulus being the most influential parameter in both cases. These results suggest that considering the uncertainties in input parameters is crucial when utilizing harmonic indicators for crack detection, especially for slight cracks.*

Keywords: Sensitivity analysis, Crack detection, harmonic indicators, Quadratic Teager-Kaiser energy

1. Introduction

Fatigue cracks can occur in structural components subjected to cyclic loads, posing significant risks to structural integrity and safety. These cracks exhibit dynamic behaviour characterized by periodic opening and closure during tension and compression, referred to as “breathing” behaviour. This dynamic phenomenon can generate corresponding nonlinear components in the system responses, providing potential opportunities for non-destructive testing and nonlinear crack detection. Recent research has focused on the nonlinearity induced by the “breathing” behaviour of cracks, emphasizing the significance of sub- and super-harmonics in nonlinear crack detection (Andreaus et al., 2007; Broda et al., 2016). Inspired by the vibro-acoustic modulation technique, several nonlinear crack detection methods based on harmonics have been developed (Semperlotti et al., 2009; Kim et al., 2011). However, these harmonics are typically much weaker than the fundamental harmonic, particularly in the early stages of crack formation. In a previous study (Cao et al., 2022), a novel concept known as the energy modulation effect (EME) was introduced to enhance these harmonics. The associated indicator, quadratic Teager-Kaiser energy (Q-TKE), has been validated as a robust tool for nonlinear crack detection. Nevertheless, the EME also complicates the relationship between the corresponding responses and input parameters. In certain cases, uncertainties in the input parameters may lead to variations in the Q-TKE spectrum or the emergence of interference peaks, potentially resulting in misinterpretation of the results. Therefore, it is essential to further investigate the impact of these uncertainties on Q-TKE. Sensitivity analysis is a vital statistical technique widely employed in structural design optimization and uncertainty quantification (Borgonovo and Plischke, 2016; Kleijnen, 2010). Probabilistically, it can be categorized into deterministic and stochastic approaches (Novak et al., 1993). Compared to deterministic sensitivity analysis, stochastic sensitivity analysis treats input parameters as random variables with probability distributions, enabling the extraction of statistical information about the output. This study employs stochastic sensitivity analysis methods on Q-TKE and compares them to the indicator derived from the frequency domain to assess the impact of material uncertainties on nonlinear crack detection.

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2. Q-TKE for nonlinear crack detection

Under a harmonic excitation, the steady-state vibration response of a cracked beam can be expressed by the multiple harmonics.

$$f(t)=\sum A_n \cos\left(\omega_n t+\varphi_n\right) \tag{1}$$

where $f(t)$ is the response, A_n, ω_n and φ_n represent amplitude, angular frequency, and phase of the nth harmonic. The amplitude of the nth harmonic is much smaller than that of the fundamental harmonic due to the weak "breathing" behaviour of crack. To address this problem, the extended Teager-Kaiser energy (TKE) operator $\psi(*)$ is introduced to measure the energy of the response $E(t)$.

$$\psi(f_i(t), f_j(t)) = \frac{\mathrm{d}f_i(t)}{\mathrm{d}t}\frac{\mathrm{d}f_j(t)}{\mathrm{d}t} - f_i(t)\frac{\mathrm{d}^2 f_j(t)}{\mathrm{d}t^2} \tag{2}$$

where $f_i(t)$ and $f_j(t)$ are the ith and jth order harmonic components of the responses. By substituting Eq. (1) into Eq. (2), the TKE can be rewritten as

$$E(t) = \sum \psi(A_n \cos(\omega_n t + \varphi_n)) + \sum_{m\neq n} \psi(A_m \cos(\omega_m t + \varphi_m), A_n \cos(\omega_n t + \varphi_n)) \tag{3}$$

Each term of Eq. (3) can be expanded respectively as follow:

$$\sum \psi(A_n \cos(\omega_n t + \varphi_n)) = A_n^2 \omega_n^2 \tag{4}$$

$$\sum_{m\neq n} \psi(A_m \cos(\omega_m t + \varphi_m), A_n \cos(\omega_n t + \varphi_n)) = \frac{1}{2} A_m A_n (\alpha_{mn} + \beta_{mn}) \tag{5}$$

where α_{mn} and β_{mn} are the linear combinations of harmonic components with frequency $\omega_m \pm \omega_n$, and amplitudes $\omega_m \omega_n$ and ω_n^2 (Cao et al., 2022). Eq. (4) shows that the interaction of same harmonic components will only generate a constant, while Eq. (5) shows that the interaction of different harmonic components will produce a new harmonic component, whose amplitude and frequency are modulated by the corresponding mth and nth subcomponents. Remarkably, the amplitude of new harmonic will generally be enhanced by the lower-order harmonic, especially for the fundamental frequency because $A_1 \gg A_n$ ($n = 2,3,...$). In some cases, the basic EME remains unable to enhance the amplitude of higher-order harmonics to a detectable level. Consequently, the concept of the differential Q-TKE is further proposed for processing real discrete response samplings.

$$E_2[n] = (f_n^2 - f_{n-1} f_{n+1})^2 - (f_{n-1}^2 - f_{n-2} f_n)(f_{n+1}^2 - f_n f_{n+2}) \tag{6}$$

3. Stochastic sensitivity analysis

In stochastic sensitivity analysis, we select a certain input parameter X_i as the random variable with its probability distribution while other input parameters are kept at their mean values. For a chosen X_i, the coefficient of variation (CoV) of response R_i can be calculated, known as the partial coefficient of variation CoV_{R_i}. The ratio of partial CoV and CoV of X_i is defined as the partial sensitivity factor α_i^{CoV} to indicate the relative influence of individual input variable on the variability of system response.

$$\alpha_i^{CoV} = CoV_{R_i} / CoV_{X_i} \tag{7}$$

If K input parameters are considered as random variables, the following approximate formula may be satisfied:

$$CoV_R \approx \sqrt{\sum_{i=1}^{k} (\alpha_i^{CoV} CoV_{X_i})^2} \tag{8}$$

where CoV_R represents CoV of the response considering all input parameters as random variables. It can be learnt from Eq. (8) that the actual impact on output is dependent on the square of the sum of each partial coefficient of variation $CoV_{R_i}^2$.

4. Stochastic finite element modelling and results analysis

A steel cantilever beam with a “breathing” crack is simulated in ANSYS for stochastic sensitivity analysis. Fig. 1 illustrates the beam with zoomed-in views of the crack during compression and tension. The depth of crack is 10% of the beam's depth, with no specified width. The dynamic behavior of the crack is modelled using a contact pair, with Rayleigh damping applied at a ratio of 0.1. A transverse concentrated harmonic excitation is applied at the midline of the top surface, 30 mm from the free end. Simultaneously, velocity responses are recorded at the upper-edge midpoint of the free end. To induce relatively pronounced nonlinearity, the excitation frequency is selected to be close to the first-order natural frequency. The stochastic model considers the elastic modulus, Poisson’s ratio, and material density as random variables, Tab. 1. Latin Hypercube sampling is employed by the FReET software (Novak, 2023) to enhance sampling accuracy while maintaining a very small sample size through stratified sampling strategies. Consequently, samples I-VI are generated.

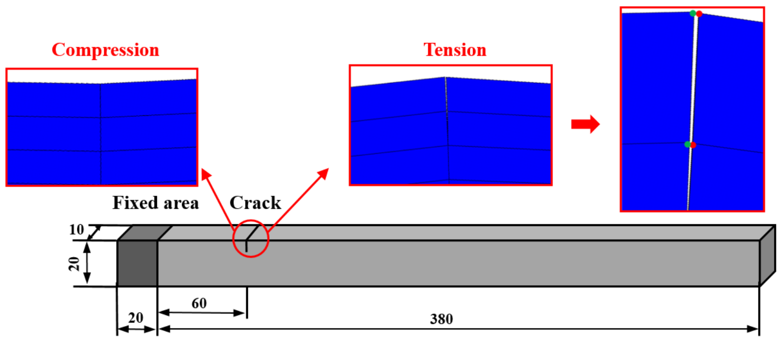

Fig. 1: Numerical model of the beam with zoomed-in views of the crack during compression and tension.

Tab. 1: Statistical information and Latin Hypercube samples of material parameters.

Parameter	Mean	Std	PDF	I	II	III	IV	V	VI
E ($\times 10^8$ kPa)	2.05	0.0683	Normal	1.9476	2.0034	2.0355	2.0645	2.0966	2.1524
μ	0.26	0.03	Normal	0.2536	0.3050	0.2150	0.2395	0.2805	0.2664
ρ (kg/m^3)	7850	33	Normal	7872.5	7827.5	7800.5	7899.5	7843.0	7857.0

The elastic modulus, Poisson’s ratio, and material density are treated as individual random variables in scenarios A, B, and C, while other parameters are kept at their mean values. In scenario D, all parameters are considered collectively as random variables. The first four harmonics are analyzed in both frequency and Q-TKE domains. As shown in Tab. 2 and Fig.2, the harmonics in Q-TKE domain exhibit significantly greater sensitivity to all parameters compared to those in the frequency domain. In both domains, the elastic modulus has the dominant influence on all harmonics, while the Poisson ratio and material density hold secondary effects. Notably, material density is almost negligible in the frequency domain (less than 10%). In the frequency domain, the approximate relationship in Eq. (8) is satisfied perfectly, with a maximum error lower than 5%. However, in the Q-TKE domain, there is a gap between the results calculated using partial CoV and obtained directly at the scenario D. Even so, the approximate relationship of the even harmonics remains within an acceptable range. A possible reason leading to this phenomenon can be that the modulation of Q-TKE complicates the nonlinear relationships between these harmonic and selected variables, leading to some orders no longer satisfying the pre-conditions for the approximate relationship.

Tab. 2: Stochastic sensitivity analysis of the first four harmonics in frequency and Q-TKE domains.

Parameters	Frequency				Q-TKE			
	1st	2nd	3rd	4th	1st	2nd	3rd	4th
α_A^{CoV}	11.516	15.506	10.381	20.597	185.329	60.873	92.299	27.787
α_B^{CoV}	1.913	11.092	0.657	10.137	104.935	9.472	65.456	14.597
α_C^{CoV}	0.394	0.255	0.963	1.658	139.215	7.235	38.629	15.503
CoV_{Cal}	0.049	0.080	0.044	0.096	1.069	0.261	0.502	0.147
CoV_{R_D}	0.049	0.083	0.044	0.092	0.461	0.258	0.322	0.102

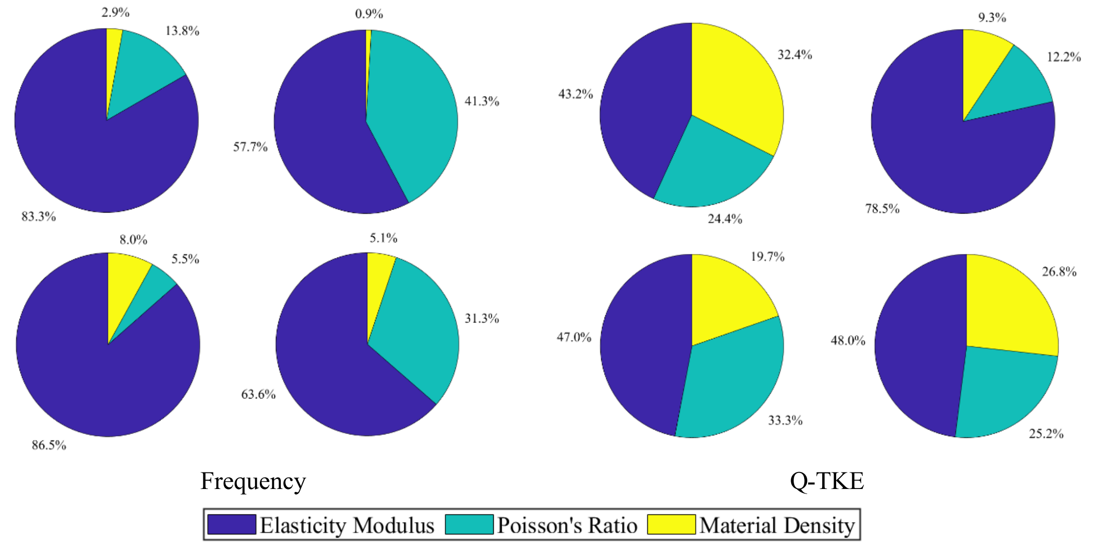

Fig. 2: Comparison of the sensitivity using CoV (influence in percentage) for the first four harmonics.

5. Conclusions

The study presents the impact of material uncertainties on nonlinear crack detection, focusing specifically on two important diagnostic indicators: frequency harmonics and Q-TKE harmonics. Through stochastic sensitivity analysis, it was demonstrated that Q-TKE harmonics show a greater sensitivity to all material parameters compared to frequency harmonics. Consequently, when identical levels of change occur in the material parameters, crack detection based on Q-TKE harmonics may exhibit increased instability. The analysis ranks the material parameters from most to least significant as follows: elastic modulus, Poisson's ratio, and material density. This ranking indicates that greater attention should be given to potential deviations in the elastic modulus when employing these methods for crack detection. Additionally, the parity of the Q-TKE harmonic order reveals significant differences in the sensitivity factors and the proposed approximation relationship. This suggests that relevant nonlinear crack detection methods could be further developed by considering the parity of Q-TKE harmonics in future research.

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