OVERVIEW OF TENSEGRITY – II : HIGH FREQUENCY SPHERES

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The paper continues the overview of tensegrity, part I of which deals with the fundamental classification of tensegrities based on their topologies. This part II focuses on special features, classification and construction of high frequency tensegrity spheres. They have a wide range of applications in the construction of tough large scale domes, in the field of cellular mechanics, etc. The design approach of double layer high frequency tensegrities using T-tripods as compresion members for interconnecting the inner and outer layers of tendons is outlined. The construction of complicated single and double bonding spherical tensegrities using a repetitive pattern of three-strut octahedron tensegrity in its flattened form is reviewed. Form-finding procedure to design a new tensegrity structure or improve the existing one by achieving the desired topology and level of prestress is discussed at the end. The types of tensegrities, their configurations and topologies studied in both parts of this overview paper can be helpful for their recognition in different technical fields and, consequently, can bring their broader applications.

Keywords : *double layer tensegrity, high frequency, T-tripod, form-finding*

1. Introduction

Sphere-like tensegrity structures are needed in a number of applications. High-frequency spheres represent geodesic shapes created on the basis of principal polyhedrons (see Fig. 1) but composed of a greater number of members. Many methods have been developed for disintegrating the basic polyhedral form into a larger number of components. Knowledge of geometrical constraints of high frequency spheres and resulting polyhedrons, is important to design tensegrity structure while maintaning its structural and fabrication limits. For applications where the stiffness of basic tensegrities is not sufficient, double-layer tensegrities may be applied. Methods for organizing tensegrity trusses into spheres are based on geodesic subdivisioning of an octahedron [1].

2. Formation of geodesic polyhedrons

Most of the geodesic polyhedrons are derived from five principal polyhedrons as depicted in Fig. 1.

A triangular grid is constructed by dividing the edges of each triangle of the principal polyhedron into equal number of parts and then join them by drawing lines between them as depicted in Fig. 2. The number of subdivisions for an edge is called the frequency of a particular subdivisioning and denoted by 'ν'. When the average length of the members

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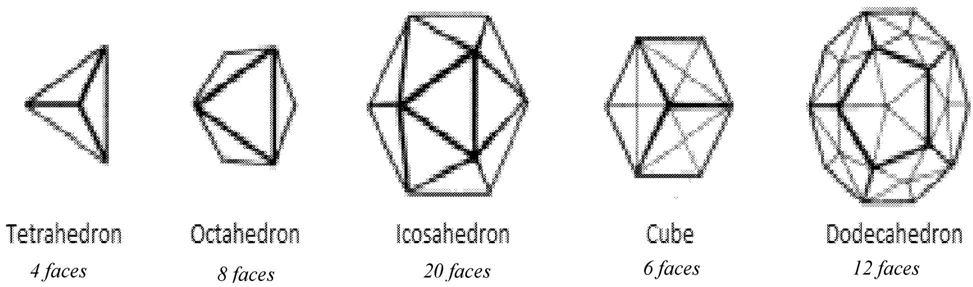

Fig.1: Principal (Platonic) polyhedrons (from [2])

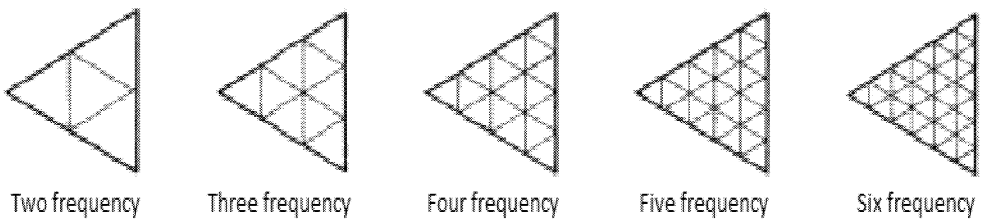

Fig.2: Division of triangular faces of polyhedron into various frequencies (from [2])

remains constant, more members imply that linear dimensions of the whole structure increase proportionally to the frequency [3]. The subdivisions with even frequencies only are used in tensegrity designing [2].

2.1. Diamond structures

As in the model for a T-tetrahedron, the tensegrities are considered to be a collection of tendon triangles interconnected via struts and tendons with their vertices lying on a sphere. The lengths of the struts as well as the lengths of the tendons forming tendon triangles are fixed. In addition to different number of tendons, another difference between $2\,\nu$ diamond T-tetrahedron (see Fig. 3(a)) and Zig-zag T-tetrahedron (see Fig. 3(b)) is in the position of tendon triangles. The T-icosahedron with diamond pattern is actually a special case of the two frequency diamond T-tetrahedron with all tendons having the same length.

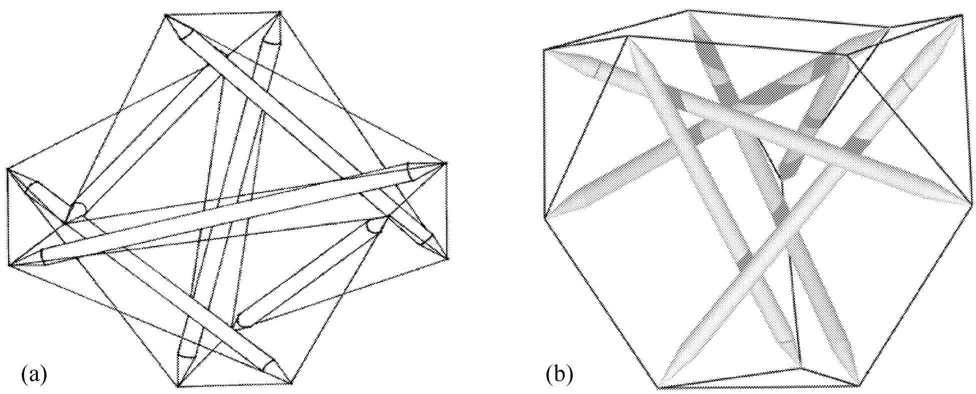

Fig.3: (a) $2\,\nu$ diamond T-tetrahedron and (b) Zig-zag T-Tetrahedron (from [4])

In Fig. 4, label a represents equilateral, whereas b represents isosceles triangles. The bold lines represent the geodesic breakdown lines used in the tensegrity design. It is called a 4ν structure because its geometry derives from the 4ν geodesic subdivision of the tetrahedron [4]. An example of 4ν diamond T-tetrahedron is illustrated in Fig. 5.

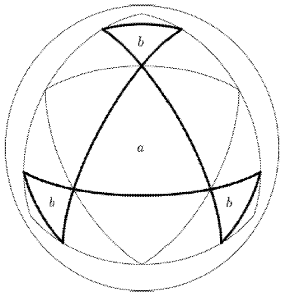

Fig.4: Face triangle of a four frequency tetrahedron projected onto a sphere (from [4])

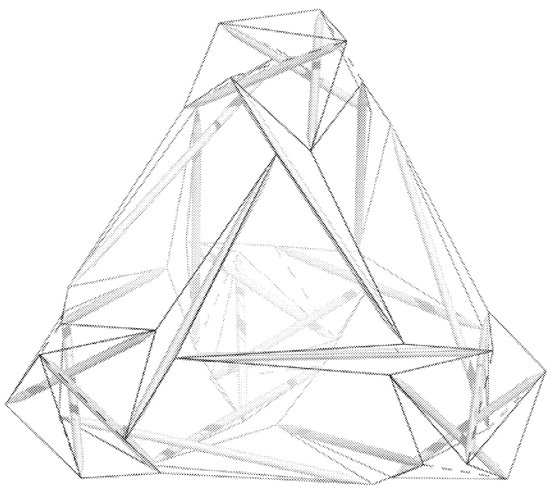

Fig.5: 4ν diamond T-tetrahedron (from [4])

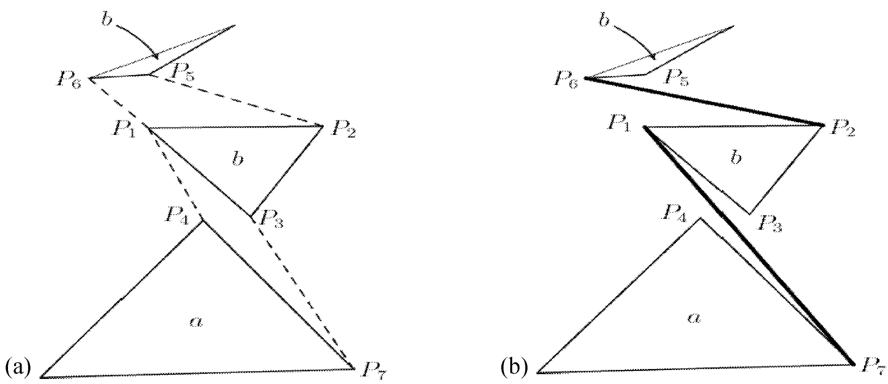

Fig.6: (a) Tendons arrangment (dotted lines) and (b) Struts arrangment (bold lines) of 4ν diamond T-tetrahedron (from [4])

Arrangment of struts and tendons connecting two types of tendon triangles, a (equilateral) and b (isosceles), is shown in Fig. 6. Adjacent tendon triangles are always connected by a pair of tendons (see Fig. 6(a)) and a strut (see Fig. 6(b)) [4].

2.2. Zig-zag structures

A zig-zag structure has a similar arrangement of struts and tendons as the corresponding diamond structure. In a diamond tensegrity structure two adjacent triangles are interconnected by two tendons (in addition to a strut), whereas in a zig-zag tensegrity they are connected by only one strut and one tendon (see Fig. 7). From the structural point of view each strut is traversed by a 'zig-zag' of three tendons [4]. An example of $4\,\nu$ zig-zag T-tetrahedron tensegrity is presented in Fig. 8.

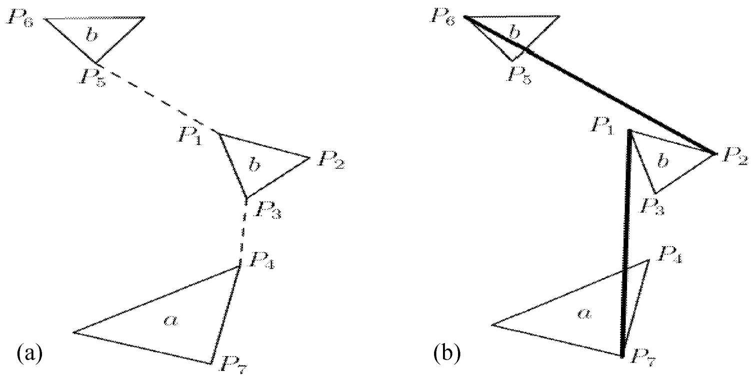

Fig.7: (a) Tendons arrangement and (b) Struts arrangement of $4\,\nu$ Zig-zag T-Tetrahedron (from [4])

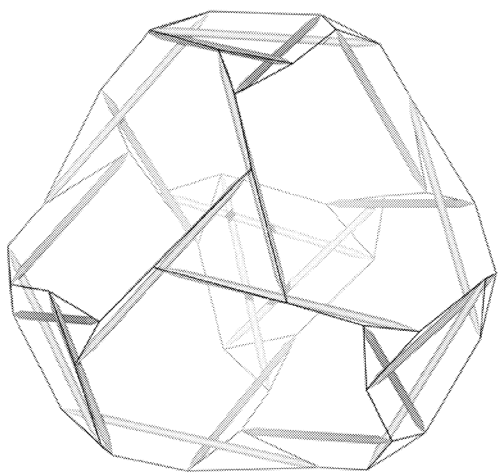

Fig.8: $4\,\nu$ Zig-zag T-tetrahedron (from [4])

3. Trusses or double-layer tensegrities

For most of the tensegrities discussed so far, the tensile members create a single continuous spherical layer, thus they are rather compliant and tend to vibrate significantly in many practical applications. Even the high frequency spherical tensegrities have little resistance to concentrated load. These drawbacks are overcome by developing space truss configuration for tensegrity structures which would be analogous to the space truss arrangements developed for the geodesic dome. Tensegrity space trusses are characterized by outer and inner layers of tendons interconnected by a collection of struts and tendons. Consequently, the structure becomes more rigid and has more resistance to concentrated loads [4, 8].

3.1. Tensegrity tripod

In tensegrity prism, removing the triangle of tendons corresponding to one of the ends and pressing the struts at the free end close to each other, resist and stay apart. The structure obtained is called tensegrity tripod or T-tripod as illustrated in Fig.9. When attached to other tensegrity structures, this composite compression member can keep them apart and enable creation of multilayer tensegrities [4].

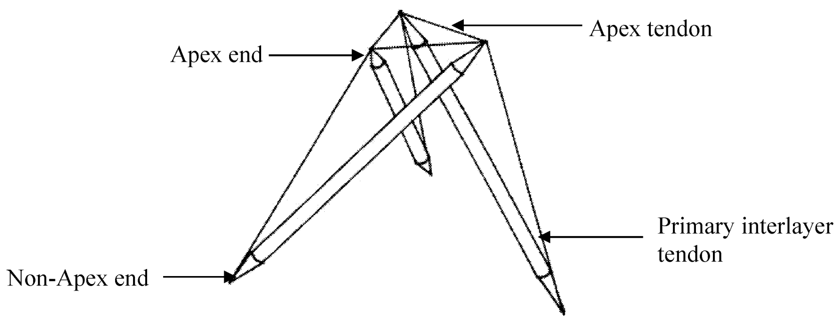

Fig.9: Tensegrity Tripod (T-tripod) (from [4])

T-tripod has 6 tendons, three of them called as apex or outer convergence tendons binding the three struts together at one end to form a triangle representing the apex of tripod and the remaining three tendons are called as primary interlayer tendons which join the apex end of one strut to the non-apex end of an adjacent strut in the T-tripod (see Fig. 9) [3]. Alternatively, another compression member called 'T-polypod' is designed from any polylateral prism by eliminating its tendons on one end and can be used instead of T-tripods [4].

When T-tripods are used as compression members to support a spherical single-layer tendon network, the T-tripods struts lie outside the layer of tendons (see Fig. 10). This eliminates the intractable interference problems occurring in larger structures when supported by simple two-hubbed struts as they lie in the same layer which they support [5].

3.2. Construction of double tensegrity sphere using T-tripod

3.2.1. Outer convergence and corresponding binding

Consider a sphere made of single-layer network supported by T-tripods as described in the previous section. This network consists of vertex-connected rings of tendons called as polylaterals (non-planar equivalent of a polygon) which are connected pairwise in vertexes (hubs). The condition is not more than one vertex is shared between adjacent polylaterals, and every vertex is shared by exactly two polylaterals (here, triangles created by edges

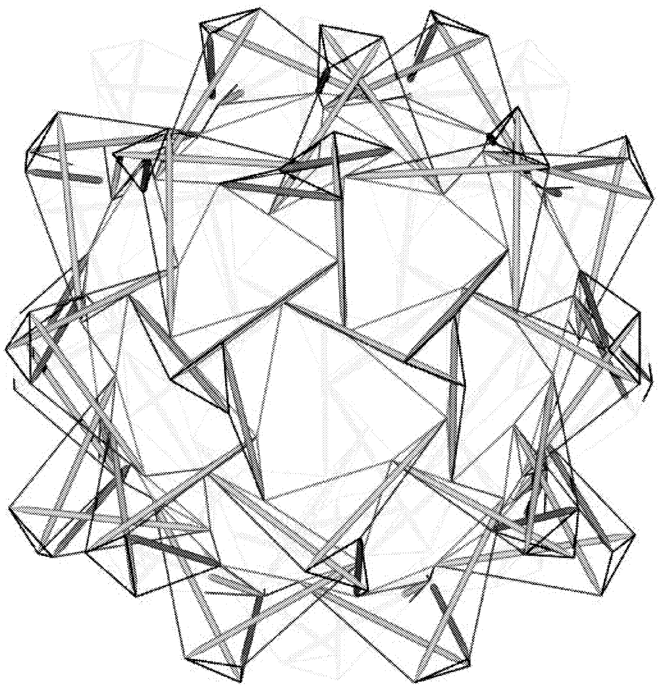

Fig.10: Spherical assembly of tripods embedded in the tensegrity network (from [4])

of neighbouring polylaterals are not considered as polylaterals). The smallest polylateral is a triangle of three hubs connected by three tendons [3, 7]. The apex tendons of the T-tripods lie on the outer surface called as outer convergence. Apart from continuous inner single-layer spherical network there is a discontinuous outer network formed by the tendon triangles of the apexes of the T-tripods which serve as compression members. These apex tendons (triangles) are called as outer convergence tendons [4]. To achieve more structural stability, the outer network is completed by binding together the T-tripod apexes using another set of tendons called outer binding tendons (see Fig. 11). In this way the outer convergence (layer of tendons) is created.

Although the lengths of outer network tendons are different, the outer and inner networks should be bound in such a way that both networks have the same topology. The distance between the three non-apex ends of struts of the t-tripod increases on untwisting the T-tripod (i.e. they move further apart from each other by elongating the primary interlayer tendons). Thus, the outer convergence tendons should be bound together in such a way that tension in the outer binding tendons untwists the T-tripods. Consequently, the outer network of tendons is pushed farer from the inner network by the virtue of expanding compression members i.e. untwisting of T-tripods [4].

3.2.2. Inner convergence and corresponding binding

Struts from several different T-tripods converge at inner network where they are connected together by tendons called as inner convergence tendons forming a polylateral and having a similar topology as outer binding tendons. The remaining tendons of the inner network creating triangles are called inner binding tendons. When the appropriate tendons

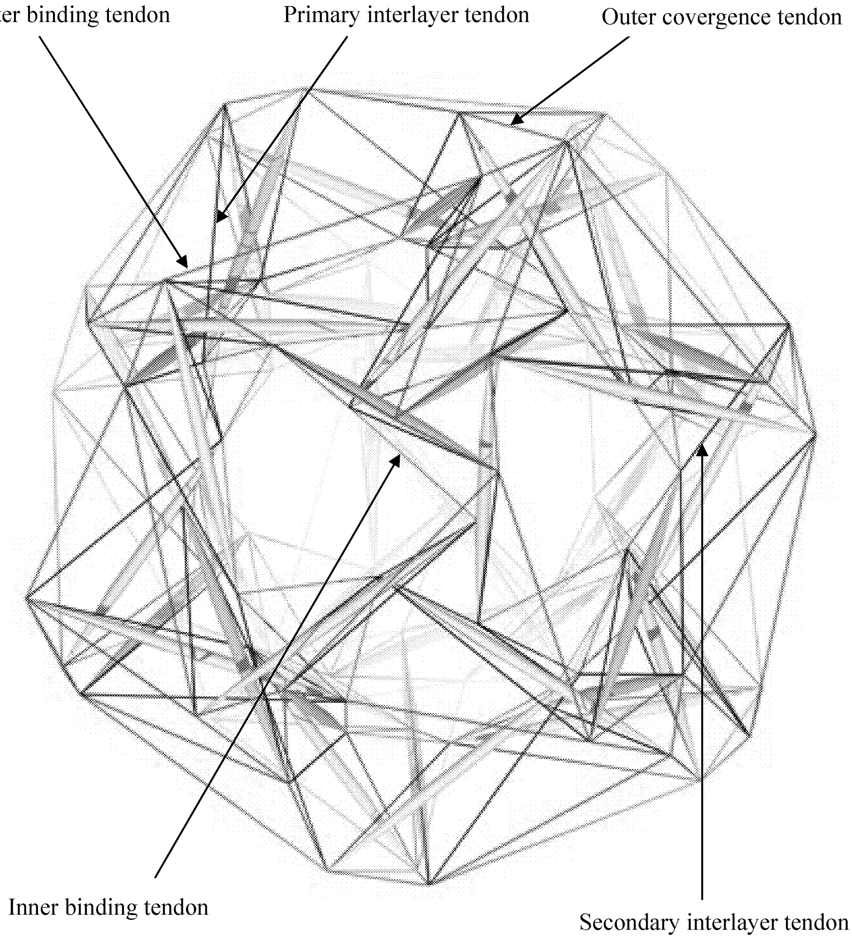

Fig.11: 4ν T-Octahedron double-layer tensegrity (from [4])

are added to connect the convergence polylateral to the opposite ends of the struts they are called secondary interlayer tendons and complete the truss network as illustrated in Fig. 11. The tensegrity truss generated using this method has two groups of geometrical shapes in each layer, triangles and polylaterals.

In the final design, a triangle or polylateral from one group will be completely surrounded by polylaterals from another group [4]. The combination of triangles and polylaterals yield a more rigid structure than general polylaterals only [6]. In this configuration each strut is secured by 12 tendons, which is the minimum number of tendons required to rigidly fix one system in its surrounding system [7]. In planar view, the truss section looks same viewed either from inside or outside the tensegrity sphere. This technique can be applied to any tensegrity network on the condition that it can be divided into alternating polylaterals [5]. In this way maximum stiffness can be achieved with minimum strut density, as all T-tripods act as compression members (between outer and inner convergence) and convergences (layers) consist of tendons only. The geodesic networks on the basis of this approach belong to the class of double layer tensegrities [4]. Most of the structures discussed till now have spherical symmetry. The drawback of this technique is that it does not result into a perfect spherical shape network as triangles eliminate the polyhedrons which have much more faceted appearence [5]. The final outcome is a rigid tensegrity space frame aimed at extremely large-scale applications for instance, for covering superstructure of a space station [4].

4. Spherical tensegrity

The spherical tensegrities have a wide scope of applications, especially in the field of architecture, cellular mechanics, etc. Further, a dome can be induced by truncating the tensegrity sphere. The simplest form of tensegrity is an octahedron consisting of three compression members i.e. the struts cross and do not touch each other as they pass at the centre. They are held together only at their terminals by the triangular tension net (tendons) as depicted in Fig. 12(a).

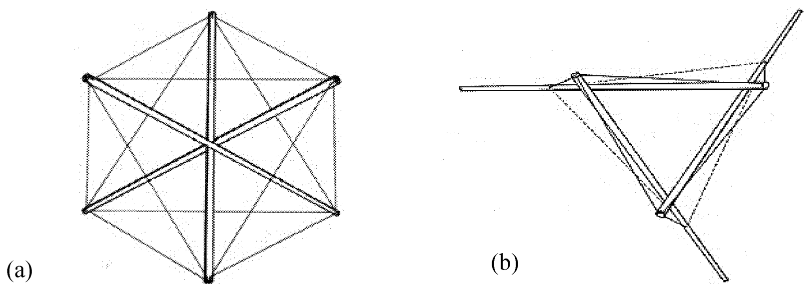

Fig.12: (a) Three-strut tensegrity octahedron
(b) Corresponding flattened form (from [7])

The same three islanded struts of the tensegrity octahedron may be mildly reorganized or asymmetrically transformed into flattened form by removing three tendons and lengthening or shortening the others (see Fig. 12 (b)). The struts may be of the same or different lengths and the vertexes can be shifted along the struts from their original positions at their ends. Even in flattened form the compression members still do not touch each other. This fundamental unit used repeatebly to construct the spherical tensegrity structures [7].

Formation of a tensegrity structure can be done in two ways. The first way is called single-bonding in which a single strut of basic three strut structure in flattened form is directly connected to the strut of another basic three strut structure in flattened form accomodating the tensegrity sub-unit hexagonal ring of tendons as depicted by dotted lines in Fig. 13(b).

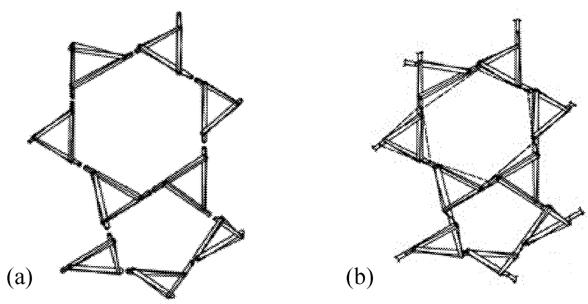

Fig.13: Complex of basic three-strut tensegrities, with axial alignment whose exterior terminals (a) before and (b) after joining in single bond as 90-strut tensegrity (from [7])

Another way known as double bonding, the basic structures in flattened form has direct connection between struts on a straight line (as that of single bonding) and also, are joined by two parallel struts with the help of tendons. Thus, basic three-strut tensegrities in flattened form may be joined to form a complex, 270-struts, isotropic triacontrahedron tensegrity geodesic spheres either in single-bonding with positive rotating triangles (see Fig. 14(a)) or double-bonding with negative rotating triangles (see Fig. 14(b)).

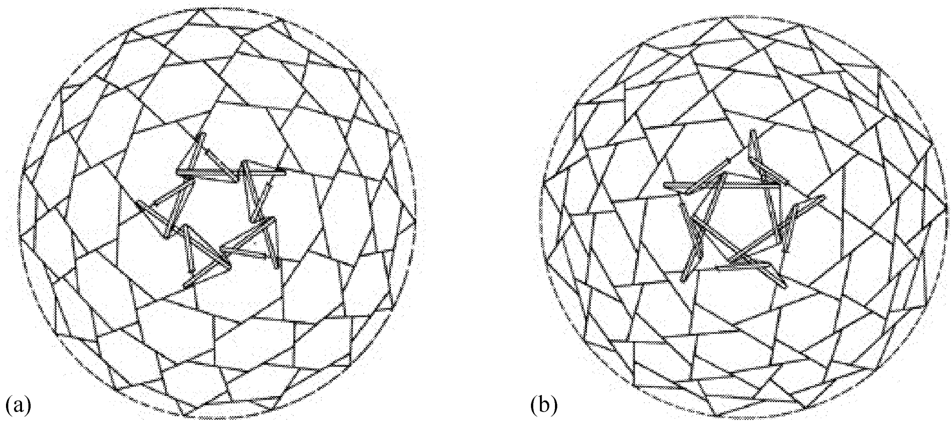

Fig.14: A 270-strut isotropic tensegrity geodesic sphere forming a complex six-frequency triacontahedron tensegrity (a) Single bonded clockwise rotating tendon triangles and (b) double bonded anticlockwise rotating tendon triangles (from [7])

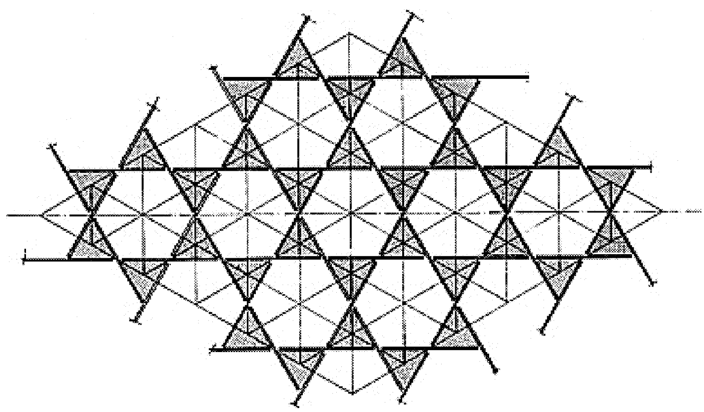

Fig.15: The basic 12 frequency matrix (from [7])

Within the tensegrity sphere compression generated in discontinuous struts is balanced by the tension applied by omni integrated tendons network. The spherical tensegrity consist of continuous tension network of single or double tendons between the ends of discontinuous compression network of islanded struts. The load distribution and uniform stress flow within the system can be better achieved with double tendon tension network. These tendons always yield in obtuse or acute 'V' shape at their point of contact with islanded struts [7].

In a spherical tensegrity system, the tension tendons are positioned closely to the struts and they do not yield away. The basic 12-frequency tensegrity matrix (see Fig. 15) employs collections of the basic three-strut units joined at dead centre between single-bonded and double-bonded discontinuities and the shaded triangles represent the sites for each of the three-strut units [7].

5. Form-finding technology

Any tensegrity structure is designed by keeping its parameters in mind mainly, certain amount of pre-stress, length of its constituents and basic morphology. A new topologic and geometric design aims at achieving a certain higher level of prestress and to keep it under the externally applied forces while maintaning its geometrical constraints [9]. As tensegrities belong to the class of statically indeterminate structures, their static analysis requires an initial form-finding procedure [10, 11]. Form-finding is technique based on mathematical programming view of tensegrity for finding a certain geometrical configuration which makes the system stable on its own (an intrinsic property of statically indeterminate structures with members bearing tension or compression, only). It involves computing of the shape of a tensegrity structure when being pre- stressed and sets in self-equilibrium.

Form-finding procedure is very useful for civil engineers, architects and the people dealing with mechanisms and structures. Although designed to achieve the same goal, it has been defined in contradictory ways by many authors [12, 13, 14]. Therefore, based on a comprehensive literature study, the authors have proposed the following definition: Form-finding is a method to design and generate the stable geometrical configuration of a tensegrity structure (using mathematical modelling) inspired by other geometrical forms and structures, under a given condition of pre-stress such that it will remain stable and maintain its shape under a certain range of external forces and impacts.

The truncated tetrahedron was analysed for its form-finding tensegrity structure with the same length for all cables and another length for all struts [9]. In Fig. 16, first image represents truncated icosahedron in polyhedron form and the second one represents its corresponding tensegrity structure created using form- finding procedure. The truncated icosahedron is combination of hexagonal and pentagonal polylaterals, whereas the corresponding form-finding tensegrity has structure of pentagons surrounded by triangles. In addition, the tensegrity structure consists of 30 compression elements (thick bars) and 120 tension elements (thin black bars) that do not provide smooth edges as that of the corresponding polyhedron. Thus it is evident, that the form-finding tensegrity structures may or may not be similar to the polyhedron from which it originates [10, 11, 14].

The form-finding methods in tensegrity architectures are broadly divided into kinematical methods and statical methods. In statical method, a relationship is set between equilibrium configurations of a structure with a given topology and the forces in members, which are then analysed using various methods, for instance force-density method, energy method, etc. In other words, it will search for equilibrium configurations that permit the existence of a state of prestress in the structure with certain required characteristics. On the other hand, kinematical method is characterised by increasing (decreasing) the length of the struts (tendons) and keeping the length of tendons (struts) constant until a maximum (minimum) of length of strut (tendon) is achieved to stabilise the system. This is a practical approach to build the tensegrity structure without requiring the tendons to be in pre-tension [10].

Further, the kinematical methods are classified into analytical approach, non-linear optimization and pseudo-dynamic iteration, whereas statical methods include analytical

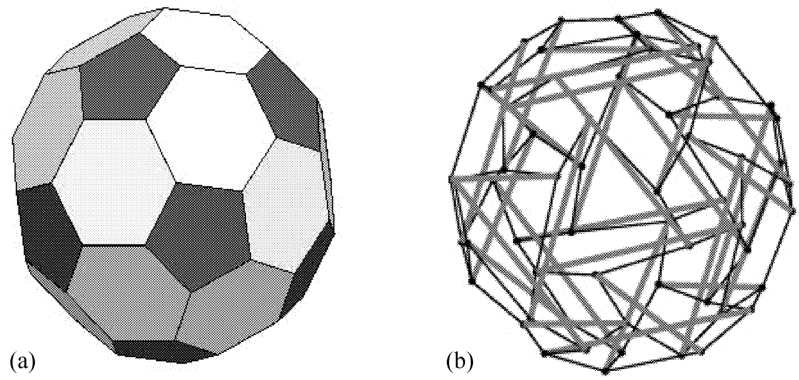

Fig.16: (a) Truncated Icosahedron in polyhedron form and (b) The corresponding tensegrity structure having rhombic configuration designed using form-finding technology (from [14])

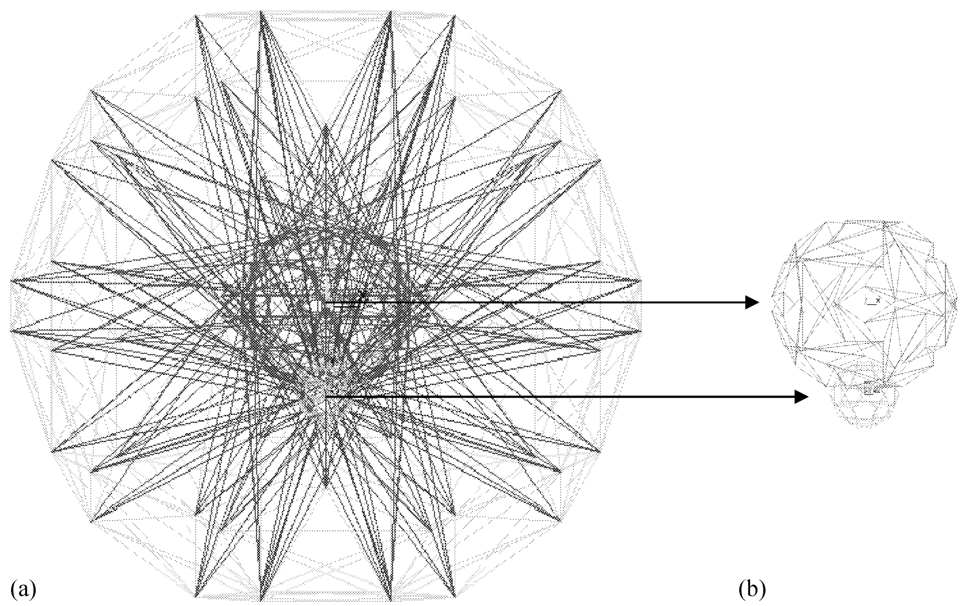

Fig.17: (a) Tensegrity principle based computational model of smooth muscle cell cytoskeleton; (b) Nucleus and centrosome designed using form-finding technique

methods, based on formulation of linear equations of equilibrium in terms of force densities, energy minimization and a search for equilibrium configurations of the struts of the structure connected by tendons whose lengths are to be determined using a reduced set of equilibrium equations. A lot of new methods were also introduced in recent years. These methods do not require the members to be in state of pre-stress [10]. All the form-finding methods presented so far (based on different approaches) have been summarised and tabulated by Jaun & Tur [13]. In most of the existing form-finding procedures; however, assumptions on the tension coeficients (force divided by length as a variable for each element), the element lengths or the symmetry of the whole structure must be imposed a priori [11]. Therefore, the evaluation of complex tensegrity structures with these procedures remains dificult. A form-finding procedure typically gives the value of critical parameters such as (i) a twisting angle

(ii) a cable-to-strut ratio (iii) a force-to-length ratio also known as the tension coeficient or the force density coeficient [14]. The advanced tensegrity principle based computational model of smooth muscle cell cytoskeleton is developed at Brno University of Technology with its nucleus and centrosome tensegrities designed using form-finding technology.

Following are the four methods to evolve new tensegrity structures [2]:

a) To postulate a new concept of tensegrity or to modify an existing one.
b) Solution for different relationships between struts and tendons which can be achieved in any of the following ways. First, interpreting the existing relationship between them in a different way. Second, manipulate the number of struts and tendons until a new pattern is achieved. Third, discover the relation from an entirely new figure. And finally, consider various ways in which the struts and tendons can be related to each other.
c) Discover or develop new polyhedral figures which serve as bases for tensegrity systems based on already established relationships between the struts and tendons.
d) Extend an existing idea of figures; for instance, develop tensegrity structure from two layer figure to three layer diamond structures, geodesic domes [4].

6. Conclusion

In part I, we have introduced the principal tensegrities and their classification based on topologies and weaving configurations, whereas in this part of the paper we have overviewed the fabrication of complex high frequency spheres and double layered tensegrity structures using form-finding technology. Several methods to create higher order tensegrity structures have been presented and illustrated with relevant examples applicable to various fields. The objective of this two-part overview paper is to introduce amazing tensegrity structures and their extraordinary features to the researchers and students from various fields, to encourage them to contribute in this area by creating their new applications.

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