# MAGNETO-THERMAL CONVECTION IN WALTERS' (MODEL B') ELASTICO-VISCOUS FLUID SATURATED BY A DARCY-BRINKMAN POROUS MEDIUM

G. C. Rana*, S. K. Kango**, Kalpna Chadha***

*The present paper investigates the thermal convection in Walters' (model B') fluid saturated by a porous medium in the presence of uniform vertical magnetic field. For the porous medium, Brinkman model is employed and Walters' (model B') fluid model is used to describe the rheological behavior of elastico-viscous fluid. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the magnetic field and viscoelasticity introduce oscillatory modes. For stationary convection, it is observed that the Walters' (model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid. The effects of Darcy number, magnetic field and medium permeability have been discussed analytically and numerically in detail. The case of overstability has also been discussed and a sufficient condition for the non-existence of overstability is derived.*

Keywords: *Darcy-Brinkman porous medium, magnetic field, thermal convection, Walters' (model B') fluid*

## 1. Introduction

The problem of thermal convection in porous medium has attracted considerable interest during the last few decades, because it has various applications in geophysics, soil sciences, ground water hydrology, astrophysics, food processing, oceanography, limnology and engineering etc.

Many researchers have investigated thermal convection problems by taking different types of fluids. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Bhatia and Steiner [2] have studied the thermal instability of a Maxwellian visco-elastic fluid in the presence of magnetic field while the thermal instability in a viscoelastic fluid in hydromagnetics has been considered by Sharma [3].

The medium has been considered to be non-porous in all the above studies. The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium are given by Vafai and Hadim [4], Ingham and Pop [5] and Nield and Bejan [6]. Lapwood [7] has studied the convective flow in a porous medium in hydromagnetics using

* G. C. Rana, Department of Mathematics, Sidharth Govt. College, Nadaun-177 033, Himachal Pradesh, India
** S. K. Kango, Department of Mathematics, Govt. College, Haripur-177 103, District Kullu, Himachal Pradesh, India
*** K. Chadha, Department of Mathematics, Govt. P. G. College, Hamirpur-177 005, Himachal Pradesh, India

linearized stability theory whereas Rayleigh instability of a thermal boundary layer in flow through porous medium has been considered by Wooding [8]. The gross effect when the fluid slowly percolates through the pores of the rock is represented by the well known Darcy's law.

There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walters' (model B') elastico-viscous fluid having relevance in chemical technology and industry. Walters' [9] reported that the mixture of polymethyl methacrylate and pyridine at containg 30.5g of polymer per litre with density 0.98 g per litre behaves very nearly as the Walters' (model B') elastico-viscous fluid. Walters' (model B') elastico-viscous fluid form the basis for the manufacture of many important polymers and useful products. Sharma and Rana [10] have studied the stability of Walters' (model B') superposed fluid in porous medium. Baris [11] has studied the steady three-dimensional flow of a Walter's B' fluid in a vertical channel whereas Sharma and Rana [12] have studied thermal instability of an incompressible Walters' (model B') elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium.

The effect of magnetic field on thermal instability of Walters' (model B') elastico-viscous fluid finds importance in geophysics, particularly, in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Sharma et al. [13] has studied the stability of stratified Walters' (Model B') fluid in porous medium in the presence of suspended particles and variable magnetic field whereas Sharma and Kango [14] studied the effect of suspended particles and variable magnetic field on the stability of two superposed fluids in porous medium and found that magnetic field completely stabilizes the system.

Recently, Shivakumara et al. [15] have studied the effect of thermal modulation on the onset of thermal convection in Walters' B viscoelastic fluid in a porous medium whereas Joneidi et al. [16] studied the homotopy analysis method to Walter's B fluid in a vertical channel with porous wall. Nadeem and Akbar [17] studied the peristaltic flow of Walter's B fluid in a uniform inclined tube whereas Kuznetsov and Nield [18] studied thermal instability in a porous medium layer saturated by a nanofluid in a Darcy-Brinkman porous medium. Kango et al. [19] studied the thermosolutal instability in Walters' (model B') fluid in the presence of Hall currents in porous medium in hydromagnetics while the convection in Walters' (model B') fluid in a Darcy-Brinkman porous medium is studied by Rana [20]. More recently, Rana and Jamwal [21] studied the effect of rotation on the onset of compressible viscoelastic fluid saturating a Darcy-Brinkman porous medium and found that Darcy number has stabilitizing effect on the system.

Keeping in mind the importance in various applications mentioned above, our interest, in the present paper is to study the thermal convection in Walters' (model B') elastico-viscous in the presence of uniform vertical magnetic field in a Darcy-Brinkman porous medium.

## 2. Mathematical model and perturbation equations

Here, we consider an infinite horizontal layer of an electrically conducting Walters' (model B') elastico-viscous fluid of depth $d$ in a porous medium bounded by the planes

$z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity $\varepsilon$ and permeability $k_1$, which is acted upon by a uniform vertical magnetic field $H(0,0,H)$ and variable gravity $g(0,0,-g)$ as shown in figure 1. This layer is heated from below such that a steady adverse temperature gradient $\beta = (|\mathrm{d}T/\mathrm{d}z|)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

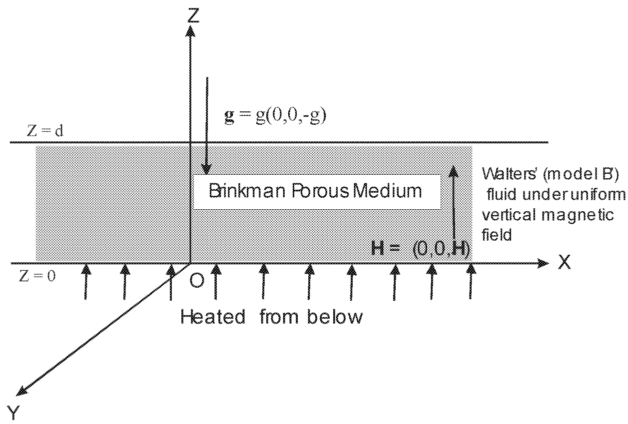


*Fig.1: Geometrical configuration of physical situation*

The hydromagnetic equations in porous medium (Chandrasekhar [1], Walters' [9], Kuznetsov and Nield [18] and Rana [19], Rana and Jamwal [21]) relevant to the problem are

$$\frac{1}{\varepsilon}\left[\frac{\partial\vec{q}}{\partial t} + \frac{1}{\varepsilon}(\vec{q}\nabla)\vec{q}\right] = -\frac{1}{\rho_0}\nabla p + \vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(\frac{\mu}{\rho_0} - \frac{\mu'}{\rho_0}\frac{\partial}{\partial t}\right)\vec{q} + \\ + \frac{\tilde{\mu}}{\rho_0}\nabla^2\vec{q} + \frac{\mu_\mathrm{e}}{4\pi\rho_0}(\nabla\times\vec{H})\times\vec{H} \ , \tag{1}$$

$$\nabla\vec{q} = 0 \ , \tag{2}$$

$$E\frac{\partial T}{\partial t} + (\vec{q}\nabla)T = \kappa\nabla^2 T \ , \tag{3}$$

$$\nabla\vec{H} = 0 \ , \tag{4}$$

$$\varepsilon\frac{\partial\vec{H}}{\partial t} = \nabla\times(\vec{q}\times\vec{H}) + \varepsilon\,\eta\,\nabla^2\vec{H} \ . \tag{5}$$

where

$$E = \varepsilon + (1-\varepsilon)\frac{\rho_\mathrm{s} C_\mathrm{s}}{\rho_0 C_\mathrm{f}} \ ,$$

$\rho_\mathrm{s}$, $C_\mathrm{s}$, $\rho_0$, $C_\mathrm{f}$ denote the density and heat capacity of solid (porous) matrix and fluid respectively.

The equation of state is

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right] \ , \tag{6}$$

where the suffix zero refers to values at the reference level $z = 0$. Here $\rho$, $\mu$, $\mu'$, $p$, $\varepsilon$, $T$, $\mu_\mathrm{e}$, $\alpha$, $\vec{q}(0,0,0)$ and $\vec{H} = (0,0,\vec{H})$ stand for density, viscosity, viscoelasticity, pressure, medium porosity, temperature, magnetic permeability, thermal coefficient of expansion, velocity of the fluid and magnetic field. Here $\tilde{\mu}$ and $\kappa$ are the effective viscosity and effective thermal diffusivity of the porous medium.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$q = (0,0,0)\ , \qquad q_{\mathrm{d}} = (0,0,0)\ , \qquad T = -\beta\, z + T_0\ , \qquad \rho = \rho_0\,(1 + \alpha\,\beta\, z)\ .$$

Let $q(u, v, w)$, $\theta$, $\delta p$ and $\delta\rho$ denote, respectively, perturbations in fluid velocity $q(0,0,0)$, temperature $T$, pressure $p$ and density $\rho$ while $\nu$ and $\nu'$ denote respectively the kinematic viscosity and kinematic viscoelasticity of the fluid.

The change in density $\delta\rho$ caused by perturbation $\theta$ temperature is given by

$$\delta\rho = -\alpha\,\rho_0\,\theta\ . \tag{7}$$

Then equations (1)–(5), on linearization, give

$$\frac{1}{\varepsilon}\frac{\partial q}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}\,\alpha\,\theta - \frac{1}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\vec{q} + \frac{\tilde{\mu}}{\rho_0}\nabla^2\vec{q} + \frac{\mu_{\mathrm{e}}}{4\pi\,\rho_0}(\nabla\times\vec{h})\times\vec{H}\ , \tag{8}$$

$$\nabla\vec{q} = 0\ , \tag{9}$$

$$E\,\frac{\partial\theta}{\partial t} = \beta\, w + \kappa\,\nabla^2\theta\ , \tag{10}$$

$$\nabla\vec{h} = 0\ , \tag{11}$$

$$\varepsilon\,\frac{\partial H}{\partial t} = (\vec{H}\,\nabla)\vec{q} + \varepsilon\,\eta\,\nabla^2\vec{h}\ . \tag{12}$$

The boundaries are taken to be free as well as perfect conductors heat and the adjoining medium is electrically non-conducting. The case of two free surfaces is a little artificial except in the case of stellar atmospheres. However this assumption allows us to obtain the analytical solution without affecting the essential features of the problem. The boundary conditions appropriate for the problem are

$$w = \frac{\partial^2 w}{\partial z^2} = 0\ , \quad \theta = 0\ , \quad \text{at} \quad z = 0 \quad \text{and} \quad z = d \tag{13}$$

and $\vec{h}$ is continuous with an external vacuum field.

Equations (8)–(12) give

$$\begin{aligned}\frac{1}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 w) = -\frac{1}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\nabla^2 w + g\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\alpha\,\theta\, + \\ + \frac{\tilde{\mu}}{\rho_0}\nabla^4 w + \frac{\mu_{\mathrm{e}}\,H}{4\pi\,\rho_0}\frac{\partial}{\partial z}(\nabla^2 h_{\mathrm{z}})\ ,\end{aligned} \tag{14}$$

$$\left(E\,\frac{\partial}{\partial t} - \kappa\,\nabla^2\right)\theta = \beta\, w\ , \tag{15}$$

$$\varepsilon\left(\frac{\partial}{\partial t} - \eta\,\nabla^2\right)h_{\mathrm{z}} = H\,\frac{\partial w}{\partial z}\ , \tag{16}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \; .$$

## 3. Dispersion relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities have $x$, $y$ and $t$ dependence of the form

$$[w, \theta, h_{\rm z}] = [W(z), \Theta(z), K(z)] \, \exp(i\, k_{\rm x}\, x + i\, k_{\rm y}\, y + n\, t) \; , \tag{17}$$

where $k_{\rm x}$ and $k_{\rm y}$ are the wave numbers in the $x$ and $y$ directions, $k = (k_{\rm x}^2 + k_{\rm y}^2)^{1/2}$ is the resultant wave number and $n$ is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (17), equations (14)–(16) in non-dimensional form, become

$$\left[1 + \left(\frac{P_{\rm l}}{\varepsilon} - F\right)\sigma - D_{\rm A}\,(D^2 - a^2)\right](D^2 - a^2)\, W + \frac{g\,\alpha\, a^2\, d^2\, P_{\rm l}\,\Theta}{\nu} - \\ - \frac{\mu_{\rm e}\, H\, d\, P_{\rm l}}{4\pi\, \rho_0\, \nu}\,(D^2 - a^2)\, D\, K = 0 \; , \tag{18}$$

$$(D^2 - a^2 - E\, P_{\rm r}\,\sigma)\,\Theta = -\left(\frac{\beta\, d^2}{\kappa}\right) W \; , \tag{19}$$

$$(D^2 - a^2 - Q_{\rm r}\,\sigma)\, K = -\left(\frac{H\, d}{\varepsilon\,\eta}\right) D\, W \; , \tag{20}$$

where we have put $a = k\,d$, $\sigma = n\,d^2/\nu$, $P_{\rm l} = k_1/d^2$ is the dimensionless medium permeability, $P_{\rm r} = \nu/\kappa$, is the thermal Prandtl number, $Q_{\rm r} = \nu/\eta$, is the magnetic Prandtl number, $F = \nu'/d^2$, is the dimensionless kinematic viscoelasticity, $D_{\rm A} = \tilde{\mu}\, K_1/(\mu\, d^2)$, is the Darcy Brinkman number and $D^* = d\,{\rm d}/{\rm d}z$ and the superscript $*$ is suppressed.

Eliminating $K$ and $\Theta$ between equations (18)–(20), we obtain

$$\left\{\left[1 + \left(\frac{P_{\rm l}}{\varepsilon} - F\right)\sigma - D_{\rm A}\,(D^2 - a^2)\right](D^2 - a^2)\,(D^2 - a^2 - Q_{\rm r}\,\sigma)\,(D^2 - a^2 - E\,P_{\rm r}\,\sigma)\right\} W = \\ = R\, a^2\, P_{\rm l}\,(D^2 - a^2 - Q_{\rm r}\,\sigma)\, W + \frac{Q\, P_{\rm l}}{\varepsilon}\,(D^2 - a^2)\,(D^2 - a^2 - E\,P_{\rm r}\,\sigma)\, D^2\, W \; , \tag{21}$$

where

$$R = \frac{g\,\alpha\,\beta\, d^4}{\nu\,\kappa}$$

is the thermal Rayleigh number, and

$$Q = \frac{\mu_{\rm e}\, H^2\, d^2}{4\pi\,\rho_0\,\nu\,\eta}$$

is the Chandrasekhar number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non conducting. The boundary conditions appropriate to the problem are (Chandrasekhar, [1])

$$W = D^2\, W = D\, Z = \Theta = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad 1 \tag{22}$$

and the components of $h$ are continuous. Since the components of the magnetic field are continuous and the tangential components are zero outside the fluid, we have

$$D\,K = 0\ , \tag{23}$$

on the boundaries. Using the boundary conditions (22) and (23), we can show that all the even order derivatives of $W$ must vanish for $z = 0$ and $z = 1$ and hence, the proper solution of equation (21) characterizing the lowest mode is

$$W = W_0\,\sin\pi z\ , \qquad W_0 \text{ is a constant.} \tag{24}$$

Using equation (24) in (21), we obtain the dispersion relation

$$R_1 = \frac{(1+x)\,(1+x+i\,E\,P_{\mathrm{r}}\,\sigma_1)}{x\,P}\left[1+\left(\frac{P}{\varepsilon}-\pi^2\,F\right)i\,\sigma_1 + D_{\mathrm{A}_1}\,(1+x)\right] + \\ + \frac{Q_1}{\varepsilon}\,\frac{(1+x)\,(1+x+i\,E\,P_{\mathrm{r}}\,\sigma_1)}{x\,(1+x+i\,Q_{\mathrm{r}}\,\sigma_1)}\ , \tag{25}$$

where

$$R_1 = \frac{R}{\pi^4}\ , \qquad D_{\mathrm{A}_1} = \frac{D_{\mathrm{A}}}{\pi^2}\ , \qquad x = \frac{a^2}{\pi^2}\ , \qquad i\,\sigma_1 = \frac{\sigma}{\pi^4}\ , \qquad P = \pi^2\,P_l\ , \qquad Q_1 = \frac{Q}{\pi^4}\ .$$

Equation (25) is required dispersion relation accounting for the effect of magnetic field, medium permeability and viscoelasticity on thermal convection in Walters' (model B') elastico-viscous fluid in a Dracy-Brinkman porous medium.

## 4. Stability of the system and Oscillatory modes

Here we examine the possibility of oscillatory modes, if any, in Walters' (model B') elastico-viscous fluid due to the presence of magnetic field, viscoelasticity and gravity field. Multiply equation (18) by $W^*$ the complex conjugate of $W$, integrating over the range of $z$ and making use of equations (18)–(20) with the help of boundary conditions (22) and (23), we obtain

$$\left[1+\left(\frac{P_l}{\varepsilon}-F\right)\sigma\right]I_1 + D_{\mathrm{A}}\,I_2 + \frac{\mu_{\mathrm{e}}\,\varepsilon\,\eta\,P_l}{4\pi\,\rho_0\,\nu}\,(I_3 + Q_{\mathrm{r}}\,\sigma^*\,I_4) - \\ - \frac{\alpha\,a^2\,g\,\kappa\,P_l}{\nu\,\beta}\,(I_5 + E\,P_{\mathrm{r}}\,\sigma^*\,I_6) = 0\ , \tag{26}$$

where

$$I_1 = \int_0^1 (|D\,W|^2 + a^2\,|W|^2)\,\mathrm{d}z\ ,$$

$$I_2 = \int_0^1 (|D^2\,W|^2 + a^4\,|W|^2 + 2\,a^2\,|D\,W|^2)\,\mathrm{d}z\ ,$$

$$I_3 = \int_0^1 (|D^2\,K|^2 + a^2\,|K|^2 + 2\,a^2\,|D\,K|^2)\,\mathrm{d}z\ ,$$

$$I_4 = \int_0^1 (|D\,K|^2 + a^2\,|K|^2)\,\mathrm{d}z\ ,$$

$$I_5 = \int_0^1 (|D\,\Theta|^2 + a^2\,|\Theta|^2)\,\mathrm{d}z\ ,$$

$$I_6 = \int_0^1 (|\Theta|^2)\,\mathrm{d}z$$

and $\sigma^*$ is the complex conjugate of $\sigma$. The integral part $I_1, \dots, I_6$ are all positive definite.

Putting $\sigma = \sigma_\mathrm{r} + \mathrm{i}\,\sigma_\mathrm{i}$ in equation (26) and equating the real and imaginary parts, we obtain

$$\left[\left(\frac{P_l}{\varepsilon} - F\right) I_1 + D_\mathrm{A}\,I_2 + \frac{\mu_\mathrm{e}\,\varepsilon\,\eta\,P_l}{4\pi\,\rho_0\,\nu}\,Q_\mathrm{r}\,I_4 - \frac{\alpha\,a^2\,g\,\kappa\,P_l}{\nu\,\beta}\,E\,P_\mathrm{r}\,I_6\right]\sigma_\mathrm{r} = = -I_1 - D_\mathrm{A}\,I_2 - \frac{\mu_\mathrm{e}\,\varepsilon\,\eta\,P_l}{4\pi\,\rho_0\,\nu}\,I_3 + \frac{\alpha\,a^2\,g\,\kappa\,P_l}{\nu\,\beta}\,I_5\ , \tag{27}$$

and

$$\left[\left(\frac{P_l}{\varepsilon} - F\right) I_1 + D_\mathrm{A}\,I_2 + \frac{\mu_\mathrm{e}\,\varepsilon\,\eta\,P_l}{4\pi\,\rho_0\,\nu}\,Q_\mathrm{r}\,I_4 - \frac{\alpha\,a^2\,g\,\kappa\,P_l}{\nu\,\beta}\,E\,P_\mathrm{r}\,I_6\right]\sigma_\mathrm{i} = 0\ . \tag{28}$$

It is evident from equation (27) that $\sigma_\mathrm{r}$ is positive or negative. Therefore, the system is stable or unstable. Equation (28) implies that $\sigma_\mathrm{i} = 0$ or $\sigma_\mathrm{i} \neq 0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes are introduced due to presence of magnetic field and viscoelasticity which were non-existent in their absence.

## 5. The stationary convection and discussion

For stationary convection, putting $\sigma = 0$ in equation (25), reduces it to

$$R_1 = \frac{1+x}{x}\left[\frac{1+x}{P} + \frac{(1+x)^2\,D_{\mathrm{A}_1}}{P} + \frac{Q}{\varepsilon}\right]\ , \tag{29}$$

which expresses the modified Rayleigh number $R_1$ as a function of the dimensionless wave number $x$ and the parameters $D_{\mathrm{A}_1}$, $P$, $Q_1$ and Walters' (model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter $F$ vanishes with $\sigma$.

To study the effects of Darcy number, magnetic field and medium permeability, we examine the behavior of $\partial R_1/\partial D_{\mathrm{A}_1}$, $\partial R_1/\partial Q_1$ and $\frac{\partial R_1}{\partial P}$ analytically and numerically.

From equation (29), we obtain

$$\frac{\partial R_1}{\partial D_{\mathrm{A}_1}} = \frac{(1+x)^3}{x\,P}\ , \tag{30}$$

which shows that Darcy number has stabilizing effect on the system which is an agreement with the result derived by Kuznetsov and Nield [18], Rana [20] and Rana and Jamwal [21]. Also, in fig. 2, $R_1$ increases with the increase in $D_{\mathrm{A}_1}$ as shown. Thus, Darcy number has stabilizing effect, which clearly verifies the analytical result (30).

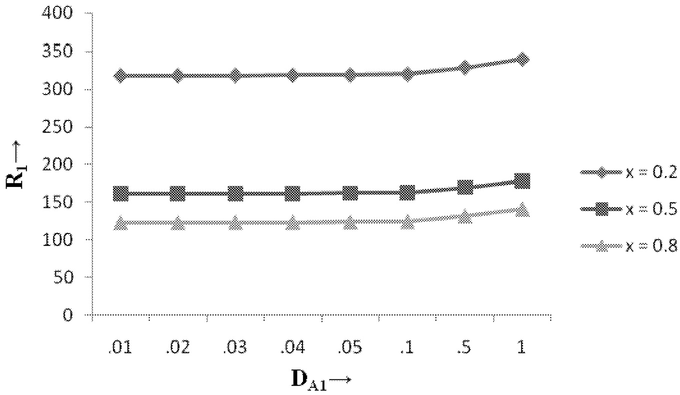


*Fig.2: Variation of Rayleigh number $R_1$ with Darcy number $D_{A_1}$ for $Q_1 = 15$, $\varepsilon = 0.3$, $P = 0.4$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$*

From equation (29), we get

$$\frac{\partial R_1}{\partial Q_1} = \frac{1+x}{x\,\varepsilon}\ . \tag{31}$$

Thus magnetic field stabilizes the system which is an agreement with the result derived by Bhatia and Stiener [2], Sharma [3], Sharma and Kango [14] and Kango et al. [19]. Also, in fig. 3, $R_1$ increases with the increase in magnetic field parameter $Q_1$. Hence, magnetic field has stabilizing effect, which clearly verifies the analytical result (31).

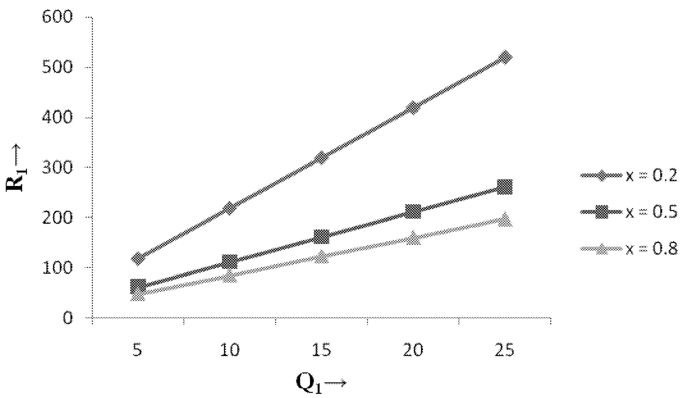


*Fig.3: Variation of Rayleigh number $R_1$ with magnetic field $Q_1$ for $D_{A_1} = 0.05$, $\varepsilon = 0.3$, $P = 0.4$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$*

It is evident from equation (29) that

$$\frac{\partial R_1}{\partial P} = \frac{1+x}{x\,P^2}\left[1 + (1+x)^2\, D_{A_1}\right]\ . \tag{32}$$

From equation (32), we observe that medium permeability has destabilizing effect on the system. Also in fig. 4, $R_1$ decreases with the increase in medium permeability $P$. Hence, medium permeability has destabilizing effect, which clearly verifies the analytical result (32). This destabilizing effect is an agreement of the earlier work of, Sharma [3], Sharma and Rana [12], Kango et al. [19], Rana [20] and Rana and Jamwal [21].

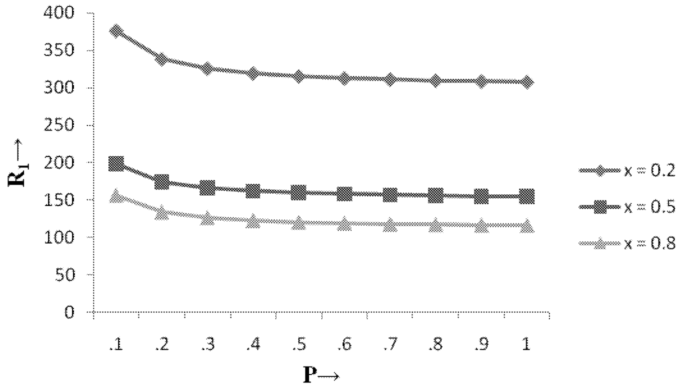


*Fig.4: Variation of Rayleigh number $R_1$ with medium permeability $P$ for $D_{\mathrm{A}_1} = 0.05$, $Q_1 = 15$, $\varepsilon = 0.3$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$*

## 6. The case of overstability

Here, we consider the possibility of whether instability may occur as an overstability. When the marginal state is oscillatory, we must have $\sigma_\mathrm{r} = 0$, $\sigma_\mathrm{i} \neq 0$.

Since for overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pur oscillations, it suffices to find conditions for which (25) will admit of solutions with $\sigma_1$ real.

Equating the real and imaginary parts of equation (25) and eliminating between them, we obtain

$$A_1\,\sigma_1^2 + B_1 = 0\ , \tag{33}$$

where

$$A_1 = \left[\frac{(1+x)\,P\,Q_\mathrm{r}^2}{\varepsilon} + \{E\,P_\mathrm{r} - (1+x)\,\pi^2\,F\}\,Q_\mathrm{r}^2 + (1+x)\,E\,P_\mathrm{r}\,D_{\mathrm{A}_1}\,Q_\mathrm{r}^2\right]\ , \tag{34}$$

$$A_2 = \left[\frac{(1+x)^3\,P}{\varepsilon} + (1+x)^2\,\{E\,P_\mathrm{r} - (1+x)\,\pi^2\,F\} + (1+x)^3\,E\,P_\mathrm{r}\,D_{\mathrm{A}_1}\right] + \frac{Q_1\,P}{\varepsilon}\,(E\,P_\mathrm{r} - Q_\mathrm{r})\ . \tag{35}$$

Since $\sigma_1$ is real for overstability, the two values of $\sigma_1$ are positive. Equation (32) shows that this is clearly impossible if $A_1 > 0$ and $B_1 > 0$. Therefore, $A_1 > 0$ and $B_1 > 0$ gives sufficient condition for the non-existence of overstability which yields

$$E\,P_\mathrm{r} > F\,\pi\,(1+x)^2 \qquad \text{and} \qquad E\,P_\mathrm{r} > Q_\mathrm{r}\ ,$$

which implies

$$\kappa < \eta\left[\varepsilon + (1-\varepsilon)\,\frac{\rho_\mathrm{s}\,C_\mathrm{s}}{\rho_0\,C_\mathrm{f}}\right] \qquad \text{and} \qquad \frac{\nu'}{d}\,(\pi^2 + k^2\,d^2) < \frac{\nu}{\kappa}\left[\varepsilon + (1-\varepsilon)\,\frac{\rho_\mathrm{s}\,C_\mathrm{s}}{\rho_0\,C_\mathrm{f}}\right]\ . \tag{36}$$

Therefore, the conditions (36) are sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply occurrence of overstability. These conditions are good agreement with the earlier results of Sharma and Rana [12].

## 7. Conclusions

The thermal convection in Walters' (model B') elastico-viscous fluid in a Darcy Brinkman porous medium in the presence of uniform vertical magnetic field has been investigated. The dispersion relation, including the effects of Darcy number, magnetic field, medium permeability and viscoelasticity on the thermal convection in Walters' (model B') fluid is derived. From the analysis, the main conclusions are as follows:

(i) For the case of stationary convection, Walters' (model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid.

(ii) The Darcy number and magnetic field stabilize the system while medium permeability destabilizes the system in the stationary convection as shown in figures 2, 3 and 4 respectively.

(iii) The oscillatory modes are introduced due to presence of viscoelasticity, magnetic field and gravity field, which were non- existent in their absence.

(iv) Sufficient conditions for the non-existence of overstability is derived, the violation of which does not necessarily imply occurrence of overstability.

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## Nomenclature

| | |
|---|---|
| $q$ | Velocity of fluid |
| $p$ | Pressure |
| $\vec{g}$ | Gravitational acceleration vector |
| $g$ | Gravitational acceleration |
| $\vec{H}$ | Magnetic field |
| $k_1$ | Medium permeability |
| $T$ | Temperature |
| $t$ | Time coordinate |
| $c_{\mathrm{f}}$ | Heat capacity of fluid |
| $k$ | Wave number of disturbance |
| $k_{\mathrm{x}}, k_{\mathrm{y}}$ | Wave numbers in $x$ and $y$ directions |
| $P_{\mathrm{r}}$ | Thermal Prandtl number |
| $Q_{\mathrm{r}}$ | Magnetic Prandtl number |
| $P_{\mathrm{l}}$ | Dimensionless medium permeability |
| $D_{\mathrm{A}}$ | Darcy-Brinkman number |
| $n$ | Growth rate of the disturbance |

**Greek Symbols**

| | |
|---|---|
| $\varepsilon$ | Medium porosity |
| $\rho$ | Fluid density |
| $\mu$ | Fuid viscosity |
| $\mu'$ | Fluid viscoelasticity |
| $\tilde{\mu}$ | Effective viscosity of porous medium |
| $\nu$ | Kinematic viscosity |
| $\nu'$ | Kinematic viscoelasticity |
| $\kappa$ | Thermal diffusitivity |
| $\alpha$ | Thermal coefficient of expansion |
| $\beta$ | Adverse temperature gradient |
| $\theta$ | Perturbation in temperature |

| | |
|---|---|
| $\delta$ | Perturbation in respective physical quantity |
| $\eta$ | Electrical resistivity |
| $\mu_{\mathrm{e}}$ | Magnetic permeability |

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