

## VIBRATION OF THE NEW TURBINE RUNNER OF THE PUMPED-STORAGE HYDRO POWER PLANT AT DLOUHÉ STRÁNĚ

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*The influence of water environment on natural frequencies of a Francis pump-turbine runner with the shaft is analyzed. The computations are executed by ANSYS program package based on finite element method. The fluid 'equivalent mass' matrix concept is used. For solution, the Euler approach is used. Obtained natural frequencies and vibration mode shapes of the turbine runner with shaft in vacuum, in air and in water are presented. Computed natural frequencies and vibration mode shapes of the turbine runner with shaft in vacuum are compared to computed natural frequencies and vibration mode shapes of the turbine runner without shaft.*

Keywords: *fluid-structure interaction, natural frequencies of structures in water, Euler theory, FEM, ANSYS, Francis pump-turbine*

### 1. Introduction

A realistic prediction of the dynamic behaviour of machines or building structures in relation to surrounding influence in different design situations is the basic condition for assessing operational reliability of analyzed structures. Reliable characteristics of the dynamic behaviour of the structure specified in particular by vibration natural frequencies and mode shapes of the structure interacting with its surrounding are significantly important. The interactions are especially significant in the dense fluid surrounding (such as water). In the past, such a problem was solved using computed natural frequencies of the structure with experimentally obtained correction coefficients or using calculation of natural frequencies implementing the finite element method where the so called added mass of interacting water was considered. Both theoretical and experimental determination of the added water mass was realistic at simple shapes of structures.

Current program systems based on FEM allow the computation of the added mass matrix without gross simplification of the geometry of the analyzed structure. Either Lagrange's or Euler's approach is used. The ANSYS program allows a direct solution of natural frequencies of the structure in water using Euler's approach. Here, the added fluid mass matrix is not explicitly determined. This procedure was applied for the further described calculations of natural frequencies and vibration mode shapes of the turboset rotor of the pumped-storage hydro power plant at Dlouhé Stráně.

The computations were carried out alternatively for the runner in vacuum or in air or in water. The computed values of influence coefficients of surrounding air or water on natural

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vibrations (water  $\rightarrow$  vacuum and water  $\rightarrow$  air) are compared for mutually assigned turbine rotor vibration mode shapes.

Further there are shown results of computation of natural frequencies and vibration mode shapes of the water turbine runner alone, i.e. without considering the other parts of the rotor. The computation was performed for the runner in vacuum. The computational results of natural frequencies and vibration mode shapes of the complete rotor with the runner in vacuum and those of the runner alone are analysed and compared.



*Fig.1: Hydro power plant at Dlouhé Stráně – aerial view*

The following facts were taken into account when selecting the tasks of the described analyses. In the course of the design of the turbine rotor system with the runner as a whole, many configurations are investigated. Concerning dynamics, not all design variants can be in due time analysed in full details. The runner design itself proceeds from many points of view independently. Basic and mostly decisive results of the numerical structural analysis of the runner concern free vibration characteristics. Computed natural frequencies can be verified by experiments. However vibration properties of the designed runner variant can be experimentally investigated only using corresponding scaled runner model with limited alternatives of boundary conditions. Conditions for the final verification test of free vibration of the manufactured runner are analogous. The final problem is to relate the limited amount of mentioned computation results to prediction of turbine operation dynamics. In that sense, results of the performed analyses could be useful.

## 2. Equation of motion of the turboset rotor system and assumptions for the vibration problem solution

The ANSYS program system [6] allows the analysis of structure natural vibrations in fluid based on Euler's approach with no need to compute the added mass matrix of the fluid explicitly. The equation of motion describing free undamped vibration of the discretized dynamic system which includes both the complete rotor of the turbine and the fluid domain with its boundary surfaces of the chamber, spiral case, draft tube, etc., modelled in great details can be written in the matrix form :

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{M}_c & \mathbf{M}_p \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{K}_c \\ \mathbf{0} & \mathbf{K}_p \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \quad (1)$$

where  $\mathbf{M}$  – structure mass matrix,  $\mathbf{K}$  – structure stiffness matrix,  $\mathbf{M}_p$  – equivalent fluid 'effective mass' matrix,  $\mathbf{K}_p$  – equivalent fluid 'effective stiffness' matrix  $\mathbf{M}_c$  – matrix of coefficients of 'inertia' based interactions,  $\mathbf{K}_c$  – matrix of coefficients of 'stiffness' based interactions,  $\mathbf{u}$  – nodal displacements vector,  $\mathbf{p}$  – nodal pressures vector.

Variations of the pressure  $p$  in fluid relative to the mean pressure in the modelled fluid domain are described by Navier-Stokes equations and equation of continuity. It is assumed that the mean pressure as well as the fluid density are steady and that the fluid is inviscid and compressible. The fluid is not flowing, there is a permanent contact between the fluid and the structure and the structure displacements are small.

## 3. Differential equations describing the fluid behaviour for the solution of fluid – structure interaction problems

Considerations are based on the law of conservation of momentum and continuity equation as well :

$$\rho \frac{\partial v}{\partial t} + \nabla p + \rho v \nabla v = 0 , \quad (2)$$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0 . \quad (3)$$

In these equations, three main variables of the problem appear, namely the pressure ( $p$ ), the velocity ( $v$ ) and the density ( $\rho$ ). Time variable fields are considered :

$$\rho = \rho_0 + \rho'(t) , \quad v = v'(t) , \quad \text{and} \quad p = p_0 + p'(t) .$$

Subscript 0 denotes the mean value of the variable and superscript ' denotes the fluctuation component of the variable.

Fluctuations are small and consequently, equations (2) and (3) can be written in the form :

$$\rho \frac{\partial v'}{\partial t} + \nabla p' = 0 , \quad \frac{\partial \rho'}{\partial t} + \rho_0 \nabla(v') = 0 . \quad (4)$$

Joining both equations velocity can be eliminated:

$$\frac{\partial^2 \rho'}{\partial t^2} - \nabla^2 p' = 0 . \quad (5)$$

It is useful to express equation (5) only through the pressure. Assuming that the change in density is dependent on the pressure and that the compressibility of the fluid is small, the system of equations can be supplemented by the equation:

$$p' = p - p_0 \approx \left. \frac{\partial p}{\partial \rho} \right|_{\rho=\rho_0} (\rho - \rho_0) = c^2 \rho'(t) \Rightarrow \rho'(t) = \frac{1}{c^2} p', \quad (6)$$

where  $c$  denotes the speed of sound in the fluid.

After substituting equation (6) into equation (5) we obtain the so called Helmholtz acoustic equation:

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = \nabla^2 p. \quad (7)$$

The differential equation (7) must satisfy initial as well as boundary conditions:

- a) At the boundary between the fluid and the solid (structure) is assumed, that the fluid is continuously in contact with the solid:

$$\frac{\partial p}{\partial n} = -\rho \frac{\partial^2 u_n}{\partial t^2}. \quad (8)$$

- b) Free surface condition:

$$p = 0. \quad (9)$$

- c) At the boundary, the energy radiation is assumed:

$$\frac{\partial p}{\partial n} = -\frac{1}{c} \dot{p}. \quad (10)$$

#### 4. Natural frequencies and vibration mode shapes of the complete rotor of the turboset in the pumped-storage hydro power plant Dlouhé Stráně

##### 4.1. Computation model of the analysed dynamic system

The vibration of the complete rotor of the Dlouhé Stráně PSHPP turboset with the new 9-blade runner was analysed considering water environment as well as air or vacuum surroundings ([1] and [2]). The vibration properties were computed using the ANSYS program system. The diagram of the rotor of the Dlouhé Stráně PSHPP turboset is shown in Fig. 2. The analysed dynamic system consists of the complete rotor with the runner surrounded by an adequately limited region of fluid.

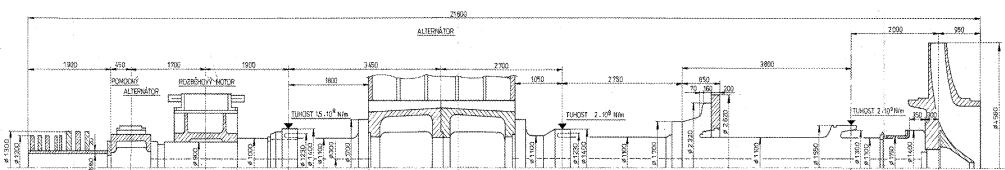


Fig.2: *Diagram of the Dlouhé Stráně PSHPP turboset rotor*

The computation model of the system (see Fig. 3) involves the TG1 turboset rotor with carefully modelled runner. The runner, geometrically modelled in great details, is fixed to the flange. The shaft is flexibly supported by three radial bearings modelled using COMBIN14 type elements with specified, experimentally determined stiffnesses and by one axial bearing modelled using COMBIN14 elements with specified, experimentally determined stiffnesses. The auxiliary alternator, starting engine, alternator, etc., constituting parts of the rotor, are modelled with respect to their inertial properties only, i.e. by concentrated masses. The rotor with the runner was modelled using 166 893 elements of the SOLID187 type, located by 285 379 nodes.

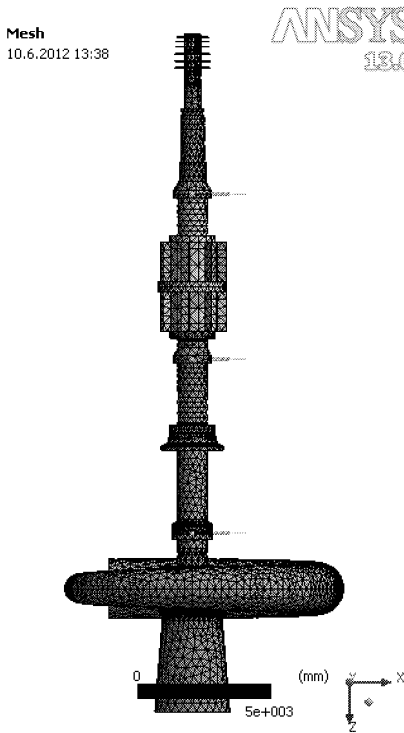


Fig.3: Computation model – finite element mesh

Boundary surfaces of the fluid domain are modelled in details. They are formed by precisely modelled surfaces of turbine covers (upper and bottom covers), vertical surface at the inlet of the spiral (free surface) and by a horizontal surface in draft tube, in a sufficient distance from the end of both the band and the hub cone of the runner. The geometry of boundary surfaces was meticulously respected in zones of the spiral, guide vanes, runner inlet, labyrinths and over the runner. Parts of the turbine (covers, guide vane ring, draft tube, spiral case, etc.) forming the boundary surfaces are considered as stiff, motionless. The fluid domain was modelled using 195 339 elements of FLUID221 type. The elements in contact with the runner have four degrees of freedom per node (three of them correspond to displacements and one corresponds to pressure), the other elements in the domain have one degree of freedom (pressure). The complete computation model of the runner – fluid system

involves 362 292 elements in total, located by 517 332 nodes with 1 300 618 degrees of freedom. The properties of steel parts and properties of fluid are defined by values of Young’s modulus  $E$ , Poisson’s ratio  $\mu$ , density  $\rho$  and speed of sound  $c$ . Alternative values were used in the performed study analyses. The values used in the final analysis are listed below.

**Material characteristics**

ROTOR / RUNNER:	AIR:	WATER:
$E = 210\,000\text{ MPa}$	$c = 340\text{ m s}^{-1}$	$c = 1450\text{ m s}^{-1}$
$\mu = 0.3$	$\rho = 1.25\text{ kg m}^{-3}$	$\rho = 1000\text{ kg m}^{-3}$
$\rho = 7850\text{ kg m}^{-3}$		

**4.2. Computation of natural frequencies of the rotor with the runner in vacuum**

Altogether 150 natural frequencies and natural mode shapes within the frequency range from 13.320 Hz to 563.334 Hz were computed. Natural mode shapes at which runner blades in the area between the runner crown and the runner band are particularly loaded are those

with extreme relative axial vibration displacements of the runner crown toward the runner band (opening of runner channel inlets – the so called fish mouth). Natural mode shapes, at which runner blades in the zones of runner blade – runner crown junctions or runner blade – runner band junctions might be significantly loaded are for example mode shapes corresponding to frequencies  $f_{28} = 172.307$  Hz,  $f_{32} = 202.950$  Hz and  $f_{33} = 203.011$  Hz. The aim of the computation of natural frequencies in vacuum is to verify the correctness of the computation model designed for the computation of the vibration of the runner or that of the complete rotor in air or in water environment. The performed computations proved that the computation model which has been opted for is adequate.

#### 4.3. Computation of natural frequencies of the rotor with the runner in air

Up to 50 natural frequencies and natural mode shapes within the frequency range from 13.319 Hz to 263.002 Hz were computed. Natural mode shapes at which runner blades might be significantly loaded in runner blade – runner crown junction zones or in runner blade – runner band junction zones are for example those, corresponding to frequencies  $f_{31} = 172.739$  Hz,  $f_{35} = 203.651$  Hz and  $f_{36} = 203.729$  Hz.

#### 4.4. Computation of natural frequencies of the rotor with the runner in water environment

Altogether 100 natural frequencies and natural mode shapes within the frequency range from 9.5626 Hz to 270.335 Hz were computed. Natural mode shapes at which runner blades might be significantly loaded in blade – runner crown junction zones or blade – runner band junction zones correspond to natural frequencies above  $f_{17} = 48.820$  Hz. Special attention was paid to natural frequencies of the rotor with the runner near the exciting frequency of 142.857 Hz in the frequency range from 130 Hz to 155 Hz. Here, seven natural frequencies are located ( $f_{36} = 133.197$  Hz,  $f_{37} = 141.416$  Hz,  $f_{38} = 145.694$  Hz,  $f_{39} = 147.293$  Hz,  $f_{40} = 148.195$  Hz,  $f_{41} = 151.169$  Hz and  $f_{42} = 154.951$  Hz). The mode shapes corresponding to frequencies  $f_{37} = 141.416$  Hz,  $f_{38} = 145.694$  Hz,  $f_{41} = 151.169$  Hz and  $f_{42} = 154.951$  Hz are very unfavourable regarding the character of loading of blades in blade – band junction zones as well as in blade – hub junction zones. For these natural mode shapes, possible resonance states should be analysed. Similar situations regarding the given exciting frequency occur in other frequency bands as well. In many cases, such a situation, however, is not interesting due to absent affinity of mode shape to exciting force (pressure) loading feature. Unfavourable shapes correspond for example to natural frequencies  $f_{43} = 158.645$  Hz,  $f_{44} = 161.690$  Hz,  $f_{45} = 166.164$  Hz,  $f_{46} = 170.494$  Hz and  $f_{47} = 171.268$  Hz.

#### 4.5. Comparison of natural frequencies of the rotor with the runner in different environments

The comparison of computed natural frequencies of the rotor – runner system in three environments (vacuum, air, water) was made. The main reason for the computation of natural frequencies in vacuum was to verify the computation model designed primarily for the computation of vibration characteristics of the rotor – runner system in air as well as in water environment. The results of computations of natural frequencies of the rotor – runner system in both air and vacuum prove that the values of natural frequencies and natural mode shapes are not significantly influenced by considering the fluid with a low density.

The inertial interaction (expressed by the so called added mass of interacting air) is weak. Small differences in determined frequency values may occur eventually due to numerical inaccuracies in computations. It is proved that natural frequencies are strongly influenced by water environment. Mutual correlation of the corresponding natural frequencies and modes of vibration of the rotor – runner system in vacuum (in air) and in water is very difficult and not always unambiguous. The procedure of correlation is based on the assessment of affinity of global features of presumed corresponding modes of vibration. Natural frequencies of the structure vibrating in water are due to the effect of the added mass of the interacting water lower than corresponding frequencies of vibration in air. It holds that  $f_{water} = \alpha f_{air}$ , where  $0 < \alpha < 1$ .

For the runner installed in the turbine, the magnitude of the coefficient of reduction of the natural frequency of runner vibration  $\alpha$  depends on the rank of the considered correlated mode shape, on the size of the gap between the runner hub and the cover, on the size of the

Correlation sequence	Natural frequency – vacuum		Natural frequency – air		Natural frequency – water		Natural frequency reduction factor	
	Frequency [Hz]		Frequency [Hz]		Frequency [Hz]		Vacuum – water	Air – water
1	$f_{VA1}$	13.320	$f_{VZ1}$	13.319	$f_{VO1}$	9.5626	0.7179	0.7180
2	$f_{VA2}$	13.321	$f_{VZ2}$	13.322	$f_{VO2}$	9.6272	0.7227	0.7227
3	$f_{VA7}$	19.353	$f_{VZ7}$	19.351	$f_{VO8}$	18.472	0.9545	0.9546
4	$f_{VA8}$	21.248	$f_{VZ8}$	21.791	$f_{VO3}$	13.414	0.6313	0.6156
5	$f_{VA13}$	48.814	$f_{VZ13}$	48.838	$f_{VO18}$	48.839	1.0005	1.0000
6	$f_{VA18}$	80.988	$f_{VZ21}$	81.680	$f_{VO21}$	55.386	0.6839	0.6781
7	$f_{VA26}$	148.527	$f_{VZ29}$	148.484	$f_{VO23}$	79.226	0.5334	0.5336
8	$f_{VA27}$	148.543	$f_{VZ30}$	148.496	$f_{VO24}$	79.334	0.5342	0.5343
9	$f_{VA41}$	248.925	$f_{VZ44}$	249.122	$f_{VO39}$	147.293	0.5917	0.5913
10	$f_{VA42}$	248.957	$f_{VZ45}$	249.126	$f_{VO40}$	148.195	0.5953	0.5949

Tab.1: Correlated selected natural mode shapes and the coefficients of reduction of natural frequency

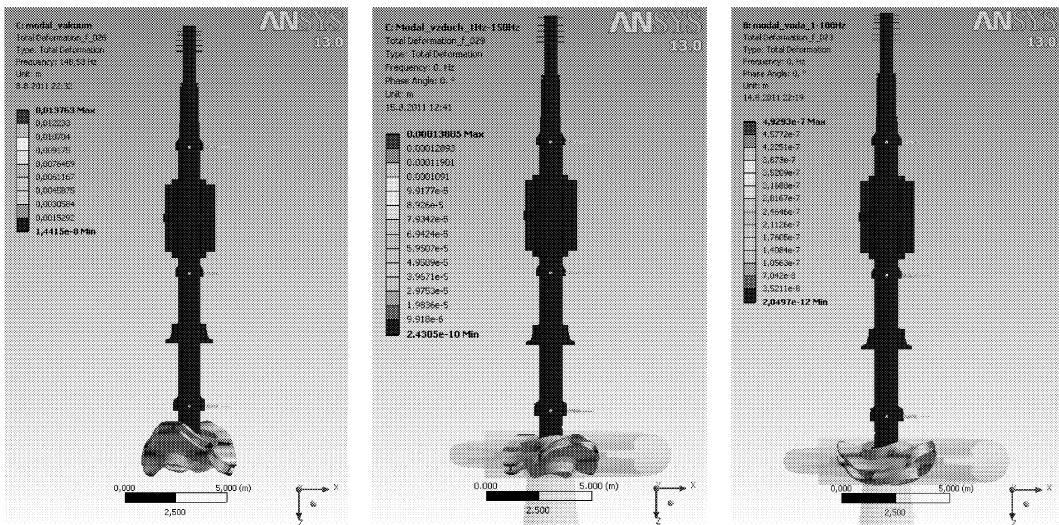


Fig.4: The mutually correlated natural mode shapes no. 7 – 2 nodal diameters

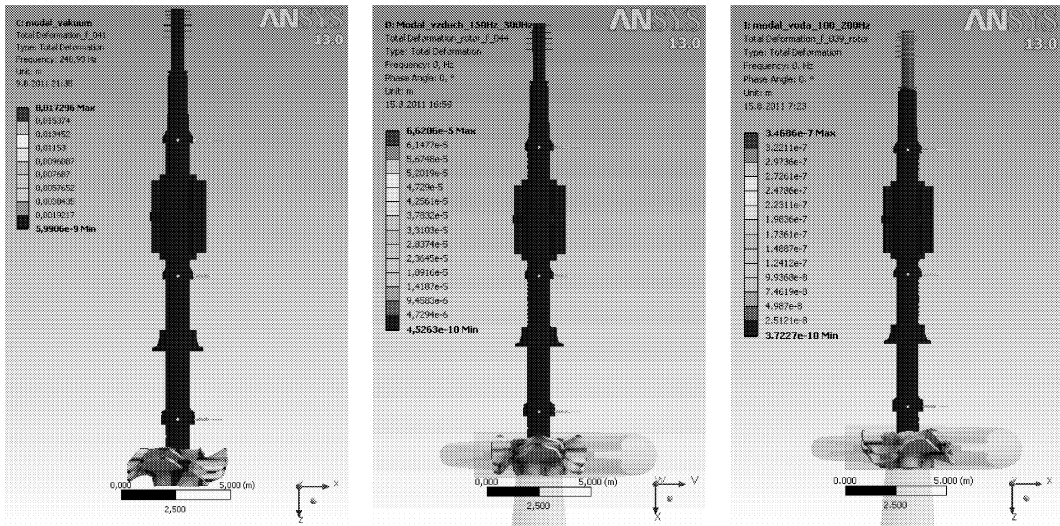


Fig.5: The mutually correlated natural mode shapes no. 9 – 3 nodal diameters

gap between the runner hub and the lower guide vane ring. The mutual correlation of selected natural modes of the rotor – runner system in vacuum, in air and in water environment is shown in Tab.1. The table shows coefficients of reduction of natural frequencies due to water for selected natural modes of the rotor – runner system.

For illustration, Figs. 4 and 5 show mutually correlated natural mode shapes of the runner system.

The coefficients of the influence of water (vacuum – water, air – water) for mutually correlated natural modes of the rotor – runner system given in Tab. 1 are graphically shown in Figs. 6 and 7.

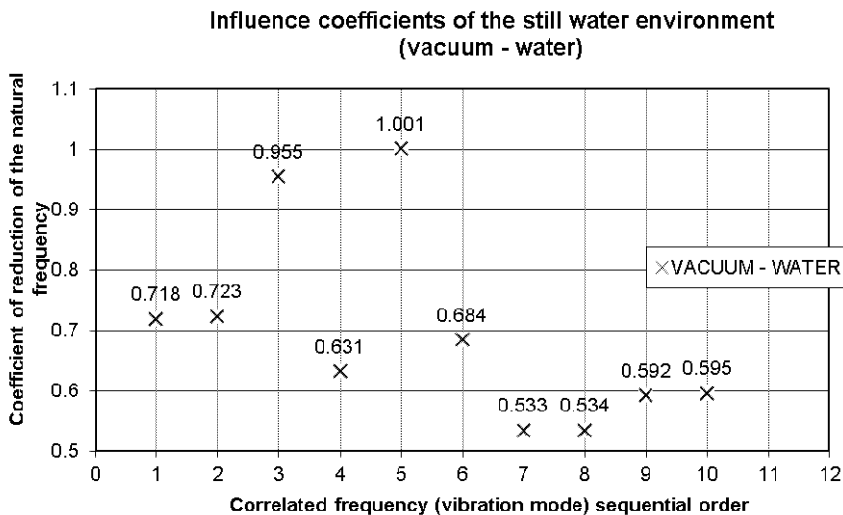


Fig.6



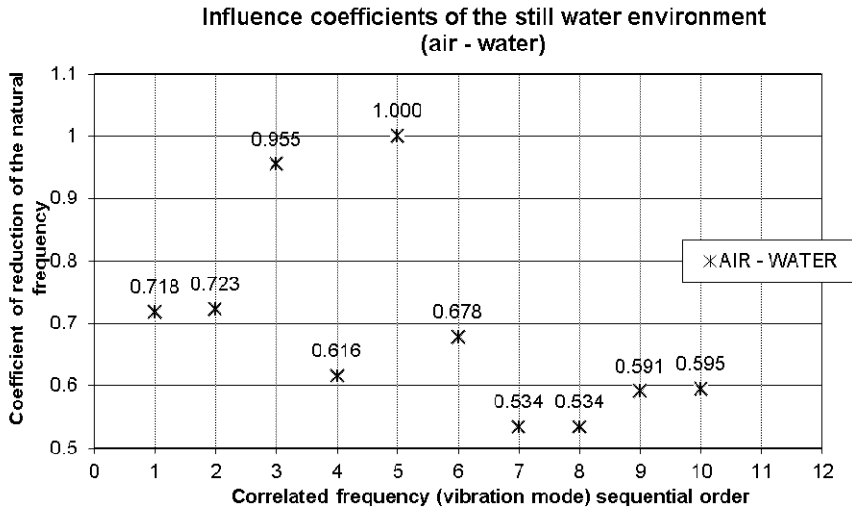


Fig.7

## 5. Natural frequencies and vibration mode shapes of the runner of the turbine in the pumped-storage hydro power plant at Dlouhé Stráně

### 5.1. Computation model

The vibration of the new 9-blade runner of the Dlouhé Stráně PSHPP turbine in vacuum was analysed. The computation was carried out using the ANSYS program system. The analysed runner is shown in Figs. 8 and 9. The runner was modelled using 159 542 SOLID187 elements, located by 264 689 nodes with 793 833 degrees of freedom. The runner was fixed in the bolt holes with respect to the inertial frame of reference.

### 5.2. The computation of natural frequencies of the runner in vacuum

Altogether 50 natural frequencies and natural mode shapes within the frequency range from 67.402Hz to 611.276Hz were analysed. Natural mode shapes at which the blades are more significantly stressed in the blade junction zones are for example those corresponding to natural frequencies  $f_4 = 127.221$  Hz,  $f_{13} = 318.833$  Hz,  $f_{32} = 465.676$  Hz and  $f_{33} = 468.279$  Hz. The analysis of natural frequencies of the runner in vacuum was performed in order to obtain initial assessments of natural mode shapes of the runner.

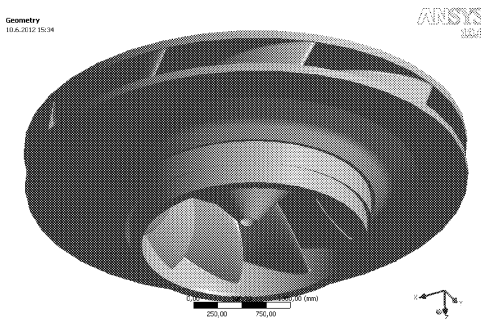


Fig.8: Computation model

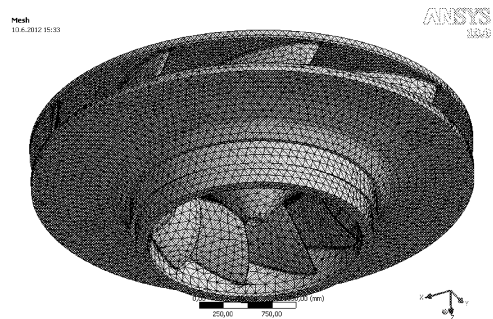


Fig.9: Finite element mesh

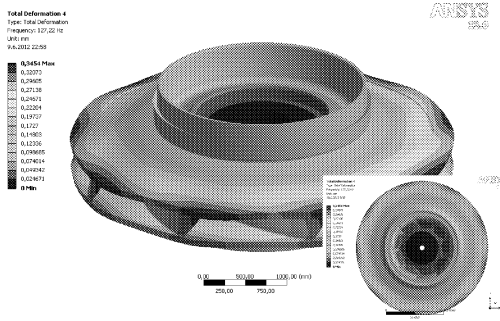


Fig.10: Natural mode shape –  $f_4 = 127.221$  Hz

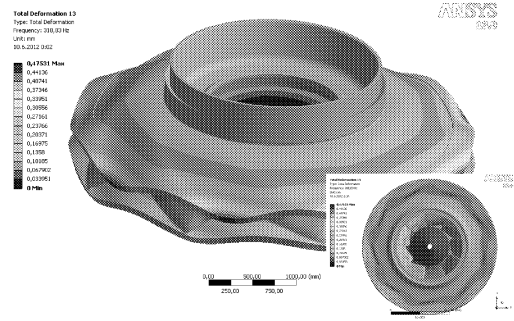


Fig.11: Natural mode shape –  $f_{13} = 318.833$  Hz

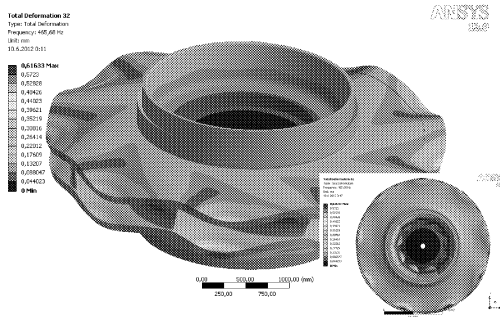


Fig.12: Natural mode shape –  $f_{32} = 465.676$  Hz

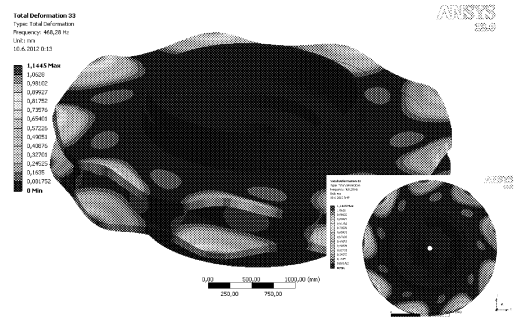


Fig.13: Natural mode shape –  $f_{33} = 468.279$  Hz

Selected natural mode shapes of the runner are shown in Figs. 10 up to 13.

### 6. Comparison of natural frequencies and natural mode shapes of the complete rotor – runner system and those of the runner alone in vacuum

The computed natural frequencies of the complete rotor – runner system in vacuum and those of the runner alone in vacuum were compared. The mutual correlation of corresponding natural mode shapes of the rotor – runner system in vacuum with natural mode shapes of the runner alone is very complicated and not always unambiguous.

Correlation sequence	Natural frequencies – runner		Natural frequencies – rotor – runner system		Description of mode shapes
	Frequency [Hz]		Frequency [Hz]		
1	$f_{SR7}$	19.353	$f_{BR3}$	111.025	Torsional
2	$f_{SR18}$	80.988	$f_{BR4}$	127.221	Umbrella type
3	$f_{SR26}$	148.527	$f_{BR5}$	137.885	2 nodal diameters
4	$f_{SR27}$	148.543	$f_{BR6}$	137.889	2 nodal diameters
5	$f_{SR41}$	248.925	$f_{BR9}$	246.334	3 nodal diameters
6	$f_{SR42}$	248.957	$f_{BR10}$	246.366	3 nodal diameters
7	$f_{SR71}$	315.737	$f_{BR11}$	315.291	4 nodal diameters
8	$f_{SR72}$	315.744	$f_{BR12}$	315.335	4 nodal diameters
9	$f_{SR92}$	395.399	$f_{BR33}$	468.279	fish mouths
10	$f_{SR97}$	414.893	$f_{BR20}$	414.598	5 nodal diameters
11	$f_{SR98}$	415.094	$f_{BR21}$	414.866	5 nodal diameters

Tab.2: Correlation between selected mode shapes of the rotor – runner system and selected mode shapes of the runner in vacuum

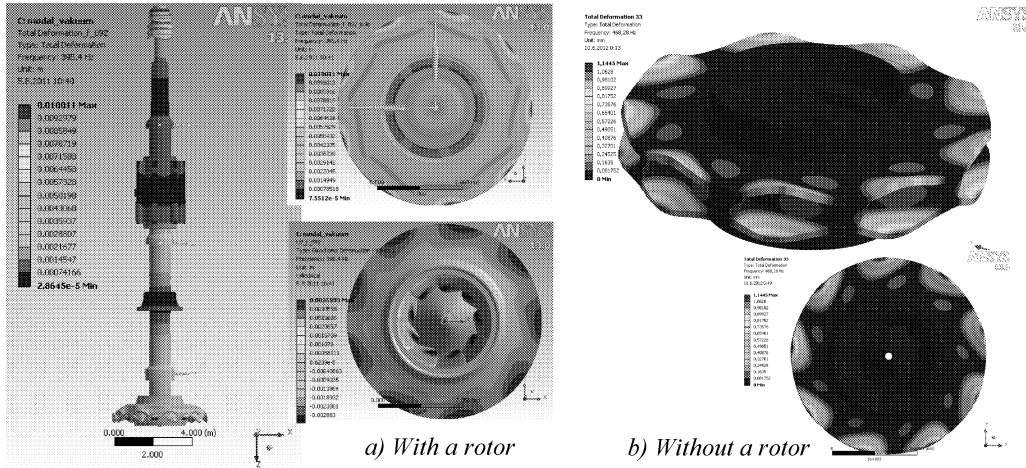


Fig.14: The mutually correlated natural mode shapes no.9 – fish mouths

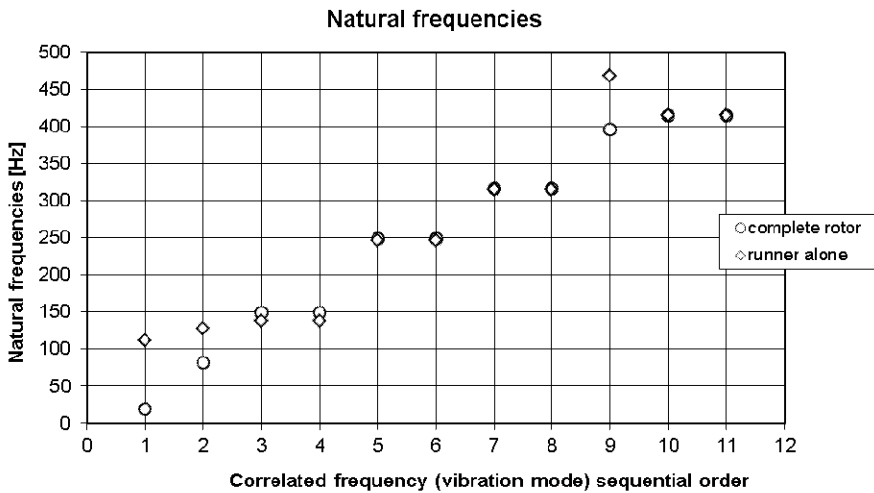


Fig.15: Natural frequencies

The mutual correlation of selected natural mode shapes of the complete rotor – runner system in vacuum with natural mode shapes of the runner alone in vacuum is given in Tab. 2. The mutual correlation was made for the natural mode shapes primarily belonging to the runner. If there is a natural mode shape at which the rotor is deformed, it is not possible to correlate natural mode shapes reliably. The variation in the boundary conditions induces considerable changes in both natural mode shapes and frequencies.

For illustration, Fig. 14 shows the mutually correlated mode shapes of the rotor – runner system and mode shapes of the runner alone in vacuum.

Fig. 15 shows the computed natural frequencies of the complete rotor – runner system and natural frequencies of the runner alone in vacuum.

## 7. Discussion

From the results of studies in which the natural frequencies and natural mode shapes of the complete rotor – runner system and natural frequencies and natural mode shapes of the runner alone in vacuum were analysed, it is obvious that natural frequencies corresponding to correlated normal mode shapes differ in dependence on the natural mode shape as well as on the fact whether they correspond to the runner or to the rotor – runner system. Natural frequencies which are strongly dependent on boundary conditions differ considerably (e.g. torsion, umbrella). On the other hand, natural frequencies corresponding to mode shapes similar to theoretical mode shapes of a circular plate (2–5 nodal diameters) agree very well. The natural frequencies corresponding potentially dangerous natural mode shapes of the runner, i.e. to the so called mouth fish modes show surprisingly gross deviation. For these reasons, it would be necessary to find the so called transition bridge between the computations of natural frequencies of complete rotors of water turbines, on one hand, and the computations of separate runners, on the other hand. That would considerably contribute to both quality and effectiveness of the design procedure.

## 8. Conclusion

Natural frequencies and modes of vibration of the complete rotor system with the new runner for the turboset of the pumped-storage hydro power plant at Dlouhé Stráně were analyzed considering the water environment effects. The developed advanced computation model based on the finite element method of discretization was used. The applied ANSYS program package allowed to perform the modal analysis using the Euler approach without explicit determination of the fluid added mass matrix. There was no need for gross simplifications of the shape of the fluid domain – turbine stator structure boundaries. Comparison of modal properties computed for the rotor system in vacuum and those in the air environment indicated correctness of the computation model. Natural frequencies and modes of vibration of the rotor in water environment have been computed. Consequently, correlated natural frequencies of the rotor for all considered environments (vacuum, air, water) were assessed. For correlated natural frequencies, the coefficients of reduction of natural frequencies due to water environment effects were determined. Those coefficients allow the reduction of the extent of rotor modal analysis (in the first order approach) to the vacuum environment case. Natural frequencies of the complete rotor with runner in the water correlated with those of the runner alone were assessed. Corresponding correlation coefficients could allow the reduction of the complete rotor modal analysis (in the first order approach) to separate runner analysis. However, some vaguenesses due to sudden numerical jumps of sequential coefficient values remained unclear.

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