TRANSIENT CALCULATIONS IN ELASTOHYDRODYNAMICALLY LUBRICATED POINT CONTACTS

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The aim of the paper is to present transient calculations of an elastohydrodynamically lubricated (EHL) point contact. The paper focuses on the description of the numerical algorithm used to model non-smooth contacts with surface feature on one of the contacting bodies. Results of film thickness and pressure distributions of a selected surface feature are presented. Simulations were performed and compared for two different rheology approaches (Newtonian, resp. non-Newtonian).

Keywords: *elastohydrodynamic lubrication, surface roughness, fluid rheology, multigrid method*

1. Introduction

EHL is typical for non-conforming contacts of machine elements, such as rolling bearings, cams, gears, etc. In such contacts, where the contact area is very small, high pressures together with elastic deformation of the bodies are present. These facts make the theoretical investigation of the problem very complicated. The mathematical model of the EHL problem includes a partial differential equation of second order (the Reynolds equation) and an integro-differential equation (film thickness or elastic deformation equation). A third equation, the so-called force balance equation is employed in the model as a control equation, ensuring the equilibrium of forces.

Since the early 1960s several attempts were made to model the phenomena inside an EHL contact. The first numerical models were greatly simplified including assumptions concerning e.g. surface roughness, fluid rheology, etc. Throughout the last decades, however, these models and algorithms were largely improved due to the advances in computer technology.

In first studies, authors were interested in the description of the shape of the pressure and film thickness of a smooth line (one-dimensional) [1], resp. point (two-dimensional) [2, 3] contact. Many different numerical algorithms were applied, e.g. direct [3–7] or inverse methods [1, 8]. Although, for the line contact case acceptable results were obtained, the solution was still limited for a wider range of values of the operating conditions (applied load, mean speed of the surfaces, etc.). By adding another spatial dimension (point contact) the problem became even more challenging.

A breakthrough in the EHL calculations came by applying the so-called multilevel techniques by Lubrecht [9] and later by Venner [10]. This advance enabled to calculate the

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pressure and the film thickness for a wide range of operating conditions, as well as to extend the simulations to transient problems assuming a surface feature on one or both of the surfaces. Lubrecht et al. [11] reports a large decrease in computing times and an improved accuracy by applying multilevel methods. Another advantage of the method is the reduced number of the computational operations from $O(n^3)$ (direct and inverse methods) to $O(n \ln(n))$ operations (with n being the number of nodes in one direction on the discretized domain).

Another issue in the calculations is the accurate description of the surface roughness. Since the inclusion of a real roughness into the computational model is very difficult the roughness was replaced by an artificial one, which can be easily described by e.g. harmonic functions defined by its height (amplitude) and width (wavelength). Several types of surface features can be incorporated to the mathematical models, such as dents [12], longitudinal or transverse roughness [13], or waviness [14]. By assuming non-smooth surfaces the equations of the mathematical model become generally time-dependent, so that a transient solution is necessary. This leads to an increased complexity of the model where more dense grids are required.

Due to the developments in computational techniques recent numerical algorithms can deal with a variety of different problems, such as thermal EHL or starvation. The assumption of Newtonian fluid rheology can be replaced by a non-Newtonian approach. By incorporating non-Newtonian fluid rheology into the mathematical model the EHL problem is becoming more realistic as well as more complicated. Papers [15–18] concentrate on the study of the non-Newtonian behaviour in the contacts. Different non-Newtonian approaches exist, e.g. the limiting shear stress model [16] or the computationally less complicated Ree-Eyring model [17]. In case of pure rolling (i.e. the velocities of both surfaces are equal) the Newtonian model still can be used. It is shown in [18] that the film thickness distributions are similar for both rheology approaches for pure rolling. However, when sliding is introduced (the surface velocities are different) the Newtonian model predicts larger deformations inside the contact which do not agree well with experimental observations [18].

The current paper describes the mathematical model of an isothermal EHL point contact with a single transverse ridge on one of the surfaces. Two different fluid rheology approaches are assumed: the Newtonian and non-Newtonian (Ree-Eyring model) model of the lubricant. The passage of the ridge through the contact zone and the corresponding film thickness and pressure distributions can be calculated by a stable and fast numerical multilevel technique which is presented here, as well as the results obtained by this algorithm.

2. Mathematical model

The applied mathematical model of the EHL problem consists of three main equations.

2.1. Reynolds equation

The Reynolds equation describes the pressure flow of a fluid in a narrow gap. This equation was derived by Reynolds [19] from the Navier-Stokes equations. Here the two-dimensional isothermal transient Reynolds equation with a generalized fluid rheology model is presented

$$\frac{\partial}{\partial x}\left(\frac{\rho\, h^3}{12\,\eta}\,\phi_{\mathrm{x}}\,\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho\, h^3}{12\,\eta}\,\phi_{\mathrm{y}}\,\frac{\partial p}{\partial y}\right)-u_{\mathrm{m}}\,\frac{\partial(\rho\, h)}{\partial x}-\frac{\partial(\rho\, h)}{\partial t}=0\ , \tag{1}$$

with boundary conditions $p(x_a, y, t) = 0$, $p(x_b, y, t) = 0$, $p(x, y_a, t) = 0$ and $p(x, y_b, t) = 0$, where x_a, x_b, y_a, and y_b denote the boundaries of the domain. Moreover, the solution is subject to the so-called cavitation condition that all pressures should be larger than, or equal to the vapor pressure of the lubricant, i.e.

$$p(x, y, t) \geq 0 \ . \tag{2}$$

Solving equation (1) the pressure distribution p at position (x, y) and time t is obtained. The density $\rho(p)$ is given by the Dowson-Higginson density-pressure relation [20]

$$\rho(p) = \rho_0 \, \frac{5.9 \cdot 10^8 + 1.34\, p}{5.9 \cdot 10^8 + p} \ , \tag{3}$$

where ρ_0 is the ambient value of density. The viscosity $\eta(p)$ is obtained by the Roelands viscosity-pressure relation [21]

$$\eta(p) = \eta_0 \, \exp\left((\ln(\eta_0) + 9.67) \left(-1 + \left(1 + \frac{p}{p_0} \right)^z \right) \right) \ , \tag{4}$$

where z is the the viscosity index of the lubricant, α the pressure-viscosity coefficient and $p_0 = 1.96 \cdot 10^8$ [Pa] a coefficient defined in the Roelands relation.

In equation (1), Φ_x and Φ_y are the so-called effective viscosities (or flow factors [15]) which in case of a Newtonian lubricant are equal to $\Phi_x = \Phi_y = 1$. However, to make the model more realistic the assumption of the Newtonian fluid behaviour should be replaced by a non-Newtonian model. In this work the lubricant is described by using the Ree-Eyring theoretical sinh law which relates the shear rate $\dot{\gamma}$ to the mean shear stress τ_m in the following way

$$\dot{\gamma} = \frac{\tau_0}{\eta} \, \sinh\left(\frac{\tau_m}{\tau_0} \right) \ , \tag{5}$$

with τ_0 being the Eyring stress characterized as the value of the shear stress above which the response of the fluid to shear becomes nonlinear. For the Eyring fluid the effective viscosities (flow factors) become

$$\Phi_x = \cosh\left(\frac{\tau_m}{\tau_0} \right) \qquad \text{and} \qquad \Phi_y = \frac{\sinh(\tau_m/\tau_0)}{\tau_m/\tau_0} \ . \tag{6}$$

2.2. Film thickness equation

The film thickness equation reads

$$h(x, y) = h_0(t) + \frac{x^2}{2\, R_x} + \frac{y^2}{2\, R_y} - R(x, y, t) + \frac{2}{\pi\, E_r} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{p(x', y')\, \mathrm{d}x'\, \mathrm{d}y'}{\sqrt{(x - x')^2 + (y - y')^2}} \ , \tag{7}$$

where $h_0(t)$ the mutual approach of the bodies, $x^2/(2\, R_x)$ and $y^2/(2\, R_y)$ describe the geometry of the contact, $R(x, y, t)$ is the function describing the surface feature, and the last term of equation (7) is the elastic deformation. In most of the literature the function $R(x, y, t)$ is described by a harmonic function, see references [12–14]. In the current work

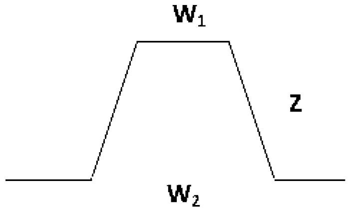

Fig.1: Profile of the ridge

a different description was chosen, a single transverse flat-top ridge is employed similar to the one in [18], see Fig. 1. This feature can be described by the values of its height Z and top and base widths W_1 and W_2.

2.3. Force balance equation

The force balance equation states that the integral of the pressure distribution obtained from the Reynolds equation should balance the externally applied load w

$$w = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y') \, \mathrm{d}x' \, \mathrm{d}y' \; . \tag{8}$$

3. Numerical algortihm

Equations (1)–(8) are solved by means of numerical methods. In order to reduce the number of parameters and to simplify the equations, a set of dimensionless variables is introduced [11]

$$X = \frac{x}{a} \, , \qquad Y = \frac{y}{a} \, , \qquad T = \frac{t\, u_\mathrm{m}}{a} \, ,$$
$$P = \frac{p}{p_\mathrm{H}} \, , \qquad H = \frac{h\, R_x}{a} \, ,$$
$$\bar{\eta} = \frac{\eta}{\eta_0} \, , \qquad \bar{\rho} = \frac{\rho}{\rho_0} \, ,$$

where a is the radius of Hertzian contact, R_x the equivalent radius of curvature ($R_\mathrm{x} = R_\mathrm{y}$), and p_H the maximum Hertzian pressure. The Reynolds equation (1) was discretized on a uniform grid with second-order accuracy both in space and time. The first two terms of (1) (the so called Poiseuille terms) were discretized using a central second-order scheme, while for the advective terms (third and last terms) a combined second-order Narrow Upstream (NU2) scheme was applied. The exact forms of the discrete equations are available e.g. in [22].

As mentioned in the Introduction the iterative methods were not accurate enough, since the complexity of these algorithms limited the number of nodal points that can be used in the solution procedure. For this reason and in order to reduce the computational times, the multilevel techniques were introduced to solve the EHL problem [9, 10]. The Reynolds equation (1) is solved by the Multigrid Techniques. The elastic deformations in equation (7) are solved by the Multilevel Multi-Integration. In these methods, the solution of the problem starts on a fine grid and after a certain number of relaxation cycles the values of the unknowns and residuals of equation (1) are transferred to a coarser grid (usually, the number of the grid nodes is halfed $n_\text{coarse grid} = n_\text{fine grid}/2$). After relaxing the problem to a sufficient accuracy on this coarse grid the values of the residuals are interpolated back to the fine grid and the solution is corrected by these residuals. This process is repeated for a number of cycles until sufficient convergence is obtained. Fig. 2 illustrates the procedure, where four grids are used to solve a given problem. The symbols ν_0, ν_1 and ν_2 represent the numbers of relaxation cycles on a certain grid. The process of moving from the fine grid to a coarse one is called resctriction, while the opposite process is called interpolation. For the transient case, this procedure is repeated for every time step. More details and a precise mathematical description of the multilevel techniques can be found in [9–11] and [24].

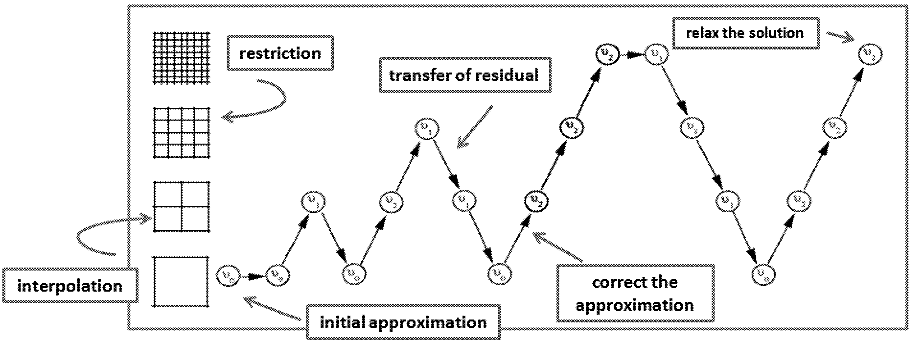

Fig.2: Scheme of a multigrid process [23]

The simulations in the current work were performed on a square domain extending from $-2.5 \leq X \leq 1.5$ and $-2.0 \leq Y \leq 2.0$ using a uniform grid with 257×257 points on the finest grid and 17×17 points on the coarsest grid. The time step was chosen $\Delta T = \Delta X/2$.

3.1. Solution procedure

The solution procedure consists of the following steps and is repeated for every time step :

- Step 1: Initialization
  - Choose the input parameters (applied load w, mean speed u_{m}, lubricant properties, contact geometry, etc.)
  - Calculate the initial dimensionless pressure P and mutual approach H_0
- Step 2: Relaxation on the finest grid and restriction
  - Calculate dimensionless film thickness H and relax ν_1-times P on the finest grid k
  - Transfer P and the residuals to a coarser grid $k-1$
  - Repeat step 2 until the coarsest grid $k=1$ is reached
- Step 3: Relaxation on the coarsest grid
  - Calculate H and relax ν_0-times P on the coarsest grid $k=1$
  - Relax and update the value of H_0
- Step 4: Interpolation and relaxation on the finest grid
  - Transfer P and the residuals from grid $k=1$ to a finer grid $k+1$
  - Correct P with residuals
  - Calculate H and relax ν_2-times P
  - Repeat step 4 until the finest grid k is reached
- Step 5: Output – converged solution of P and H at actual time step

4. Results and discussion

In this section, two sets of results will be shown. Film thickness and pressure distributions will be compared for a Newtonian and a non-Newtonian fluid model. For both cases the passage of a single transient flat-top ridge was simulated where the height of the ridge was $Z = 0.2\,\mu$m, the base width $W_1 = 45\,\mu$m and the top width $W_2 = 20\,\mu$m (see Fig. 1). The velocities of the two surfaces were not equal, the speed of the surface with the ridge $u_1 = 0.04$ m/s was slower than the speed of the smooth surface $u_2 = 0.12$ m/s. This means that slip occured between the two surfaces, so the slide-to-roll ratio was $SRR = (u_1 - u_2)/(u_\mathrm{m}) = 1$. Table 1 summarizes the operating conditions which are except for the flow factors the same for both cases. In case of assuming non-Newtonian behaviour, the Eyring stress of $\tau_0 = 5$ MPa was given.

Parameter	Case 1 & 2
load w [N]	30.0
viscosity index of the lubricant α [GPa^{-1}]	24
viscosity at ambient pressure η_0 [Pa s]	0.22
mean speed u_m [m/s]	0.08
reduced radius of curvature R_x [m]	0.0127
reduced modulus of elasticity E_r [GPa]	123.8
radius of Hertzian contact a [μm]	167
maximum Hertzian pressure p_H [GPa]	0.517

Tab.1: Input parameters of the simulations for both cases

The computations were done on a grid of 257×257 points, this was the finest grid, the coarsest grid had 17×17 nodes. The size of the spatial steps was equal $\Delta X = \Delta Y = 0.015625$, while the time step was half of the spatial steps $\Delta X = \Delta Y = 0.5\,\Delta T = 0.0078125$. For both cases, six cycles of multigrid were performed at each time step to obtain a converged solution. The computational time of one time step was approximately $\approx O(1\,\mathrm{min})$. Based on the value of the slide-to-roll ratio, the number of time steps were corrected and multiplied by a ratio of $1.5\,u_2/u_\mathrm{m}$, i.e. in current case the total number of time steps was 1152.

Results are illustrated in the following two figures. Fig. 3 shows the film thickness distributions at six time steps for Newtonian (dashed line) and non-Newtonian (solid line) models, while Fig. 4 shows the pressure distributions at 2 time steps again for Newtonian (dashed line) and non-Newtonian model (solid line). From a numerical point of view the non-Newtonian case is more complex due to flow factors Φ_x and Φ_y in the model. Therefore, the value of the mean shear stress τ_m is necessary to evaluate at each time step. After that, the flow factors are calculated and included into equation (1). As can be seen from Fig. 3, the transient EHL solution consists of two parts, a particular integral moving with the speed of the ridge and a complementary wave moving with the mean speed of the lubricant. From Fig. 3 it is apparent that the non-Newtonian model predicts lower film thicknesses. Furthermore, the Newtonian model predicts sharper peaks of pressure when the ridge is present in contact, see Fig. 4. The pressure peaks are related to stress concentration in the contact bodies and can lead to contact fatigue. The lower pressure peaks in the non-Newtonian case correspond to lower roughness deformation that agrees with experimental observations [18].

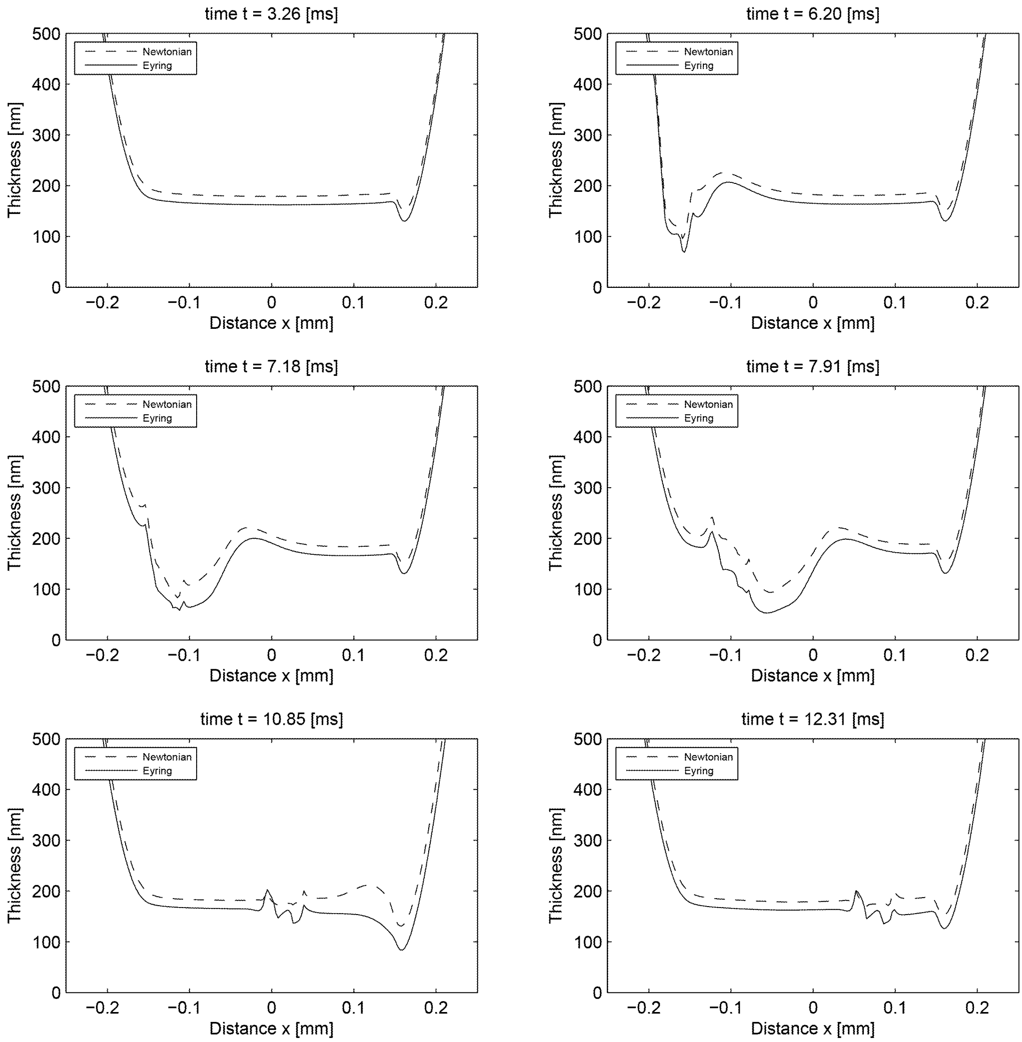

Fig.3: Film thickness in the direction of the rolling speed x at $y = 0$ at six different time steps for Newtonian (dashed line) and Eyring (solid line) model

5. Conclusions

In this article a numerical algorithm of a transient EHL model was presented. The passage of a single transverse ridge across the contact zone was simulated. The multigrid method was used to solve the Reynolds equation (1), i.e. the pressure distribution, while to obtain the film thickness distribution and elastic deformations the Multilevel Multi-Integration technique was applied. A stable and fast multilevel method is able to solve transient EHL problems with high accuracy (in current case) up to $O(10^4)$ spatial grid points. For solving time-dependent problems a second-order discretization scheme is necessary in order to ensure accurate results. Simulations for two fluid rheology models (Newtonian and non-Newtonian) were carried out and compared. In the case when sliding is present a non-Newtonian model is required. In this case, the non-Newtonian model is necessary in order to obtain pressure and film thickness results closer to experimental observations.

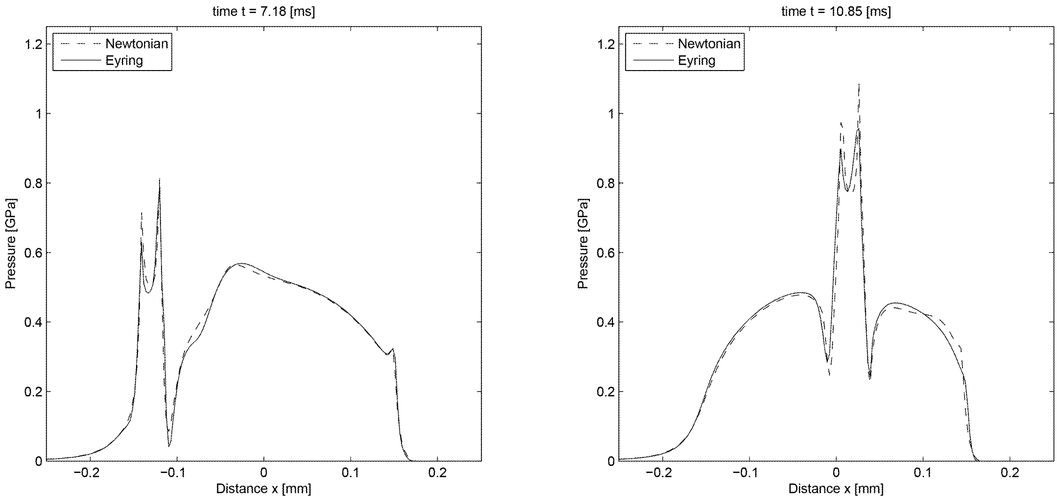

Fig.3: Pressure in the direction of the rolling speed x at $y = 0$ at two different time steps for Newtonian (dashed line) and Eyring (solid line) model

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