ANALYTICAL INVESTIGATIONS AND DESIGN CHARTS FOR RECTANGULAR REINFORCED CONCRETE STIFFENED-PLATES

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An improved theoretical formulation is proposed here to predict the moment field induced in laterally loaded RC rectangular stiffened plates including the effect of strength and stiffness of the internal stiffening beams. Design charts are also presented for their quick proportioning. The study reveals the effect of strength and stiffness of the internal beams on the moment field induced in the plate-system which was not considered, currently, by various methods prescribed by different design codes. The use of stiffened-plates becomes mandatory in buildings to accommodate some architectural constraints as well as for satisfying the serviceability criterion of design codes. This type of a structural system is efficient, economical and readily constructible in most of common materials. Moreover, it can be built as a monolithic unit or as a composite system comprising a plate cast in concrete and beams constructed in prestressed concrete, fabricated sections in steel, and so forth. A working example is presented to demonstrate the validity and efficiency of the simplified approach in comparison to finite element based design and other code prescribed methods.

Keywords : *collapse load, stiffened-plate, beam, yield line, mechanism, concrete, design charts*

1. Introduction

Plate stiffened by the internal beams along its one direction or two orthogonal directions has been used for many years in bridges and buildings. This type of a structural system is efficient, economical and readily constructible in most of common materials. Moreover, it can be built as a monolithic unit or as a composite system comprising a plate cast in concrete and beams constructed in prestressed concrete, fabricated sections in steel, and so forth. The behavior of stiffened reinforced concrete (RC) plates is different from that of normal proportioned plates without internal beams. Strength and stiffness of internal beams has a significant effect on the moment field induced in stiffened plates. Plates stiffened using highly stiff beams are prone to a local mode of failure whereas, plates stiffened using the shallow beams are susceptible to a failure in the global collapse mechanism. Singh et al. [1] experimentally established the influence of strength and stiffness of internal beams on the behavior of RC plates. Reduction in the strength and/or stiffness of internal beams significantly increase the moment-demand of the plate monolithically cast with these beams and leads to the transition of failure mode from a local to the global one. Proper estimation of critical value of beam strength and/or stiffness is necessary to predict the behavior more accurately. Even though design codes [2–5] specify a design procedure for proportioning

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rectangular RC plates but these codes, presently, do not provide any guidelines to account for the effect of strength and stiffness of internal beams on the moment field induced in rectangular plates. However, some empirical expressions are prescribed in these codes to preliminarily decide the thickness of plates to meet the serviceability criterion. Most of these design guidelines have been suggested based upon the results of an extensive series of tests [6–10] and well-established performance record of various plate systems. These guidelines have a number of inherent limitations in the form of assumptions. These are mandatory to be satisfied by all panels of a plate-system for the satisfactory performance, thereby, forcing the designers to proportion the plate-system within the domain of these limitations which sometime fail to satisfy the architectural and/or field constraints.

With regard to the behavior of single panel rectangular plates, significant research [11–22] has been carried out worldwide, and is well documented and appropriate provisions have been incorporated in the codes of practice. These methods have been derived either using the elastic plate theory or are based upon the principles of limit analysis and present a set of analytical expressions giving the moment field etc. for plates of various shapes resting over the non-yielding edges at the outer boundaries. However, these formulations fail to address the effect of internal beams on the moment field induced in plates. The provision of these internal members, sometimes, become necessary to economize the plate section and simultaneously, to satisfy the serviceability criterion of applicable design codes.

Design charts are suggested for a generic rectangular RC plate-system in the present paper. These charts will provide an alternative solution to the finite element based software for the analysis of RC stiffened-plates and alternatively, these can be used for performing a random check in the design process in case a plate-system has been designed using a finite-element method based software. High cost of these proprietary software acts as a big hindrance for their use in the design practice in a small and medium design offices particularly in developing countries. Moreover, these products expect a lot of precision from its users in data preparation, interpretation of the output along with a sufficient expertise to judge the accuracy of the results. The proposed design charts will also supplement the design guidelines recommended by various codes [2–5], which allow that any procedure can be used for designing a plate-system that satisfy the conditions of equilibrium, geometric compatibility and the requirements of strength, and serviceability stipulated by design codes.

2. Research significance

Numbers of design approaches are available in the published literature and design guidelines. Most of these guidelines have been suggested based upon the empirical results and performance studies of number of plates and slabs cast in the past, thereby possessing a number of inherent limitations in the form of assumptions etc. The proposed analytical procedure and design charts give flexibility to the designer to proportion the single-panel or multi-panel plate-systems at any desired strength-level of the stiffening-beams and the plate can be designed to meet the required strength-demand of the plate-system and vice versa. The author believes that this analytical approach is carried out for the first time and will supplement the design guidelines recommended by various design codes and offer a way to economise the plate-system.

3. Analytical modeling

The collapse load of any structural system is characterized by a bending moment distribution that satisfies the equilibrium and the mechanism conditions along with an applicable yield criterion. There exists only one load factor that satisfies all these conditions simultaneously at the collapse. The value of the load factor can be determined uniquely without the necessity of calculating deformations, either at collapse or at any other stage of loading, using the minimum and maximum principles of the limit analysis. According to these principles, a load factor for any structural system is the minimum load factor if it has been obtained by fulfilling the equilibrium condition for all possible collapse mechanisms of the system. It will represent the maximum value of a load factor if it has been calculated by considering all those bending-moment distributions that satisfy the equilibrium and the yield conditions. Therefore, these two conditions will give the lower and the upper bound to the true value of a load factor. However, the true value of a load factor can be obtained by satisfying all these conditions simultaneously.

An orthotropically RC rectangular plate of length $L_{\rm x}$ and width $l_{\rm y}$ has been divided into n-number of panels with length $l_{\rm x}$ each. The plate has been reinforced uniformly along the two orthogonal directions; thereby, providing a moment capacity $m_{\rm ux}$ along the long span $L_{\rm x}$ and moment $m_{\rm uy}$ along the short span $l_{\rm y}$ of the plate with orthotropy $\mu = m_{\rm uy}/m_{\rm ux}$. The plate was cast monolithic along with the $(n-1)$ number of equally spaced internal stiffening beams. The moment-capacity of these beams has been taken as $m_{\rm b} = \alpha_{\rm b}\, m_{\rm ux}\, l_{\rm x}$ each. Here, $\alpha_{\rm b}$ is the *beam-strength parameter* of the plate-system. The plate-system is continuous over its outer non- yielding boundary. The negative moment field per unit width is defined in terms of the positive moment field per unit width in those directions of the plate. The negative moments acting along the long span of the plate has been taken as $i_{\rm x}\, m_{\rm uy}$ and that along the short span of the plate has been defined as $i_{\rm y}\, m_{\rm ux}$. The case of simple supports at the outer boundaries of the stiffened-plate can be obtained by putting these values as zero. The negative resisting moment at the ends of the stiffening beams has been taken as $k\, m_{\rm b}$. The schematic diagram of the stiffened plate-system is shown in Fig. 1.

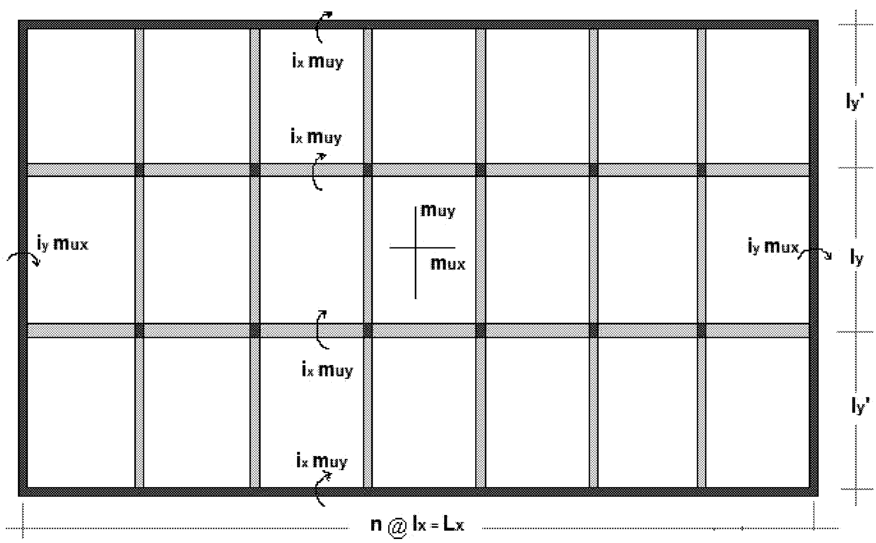

Fig.1: Schematic diagram of the proposed model

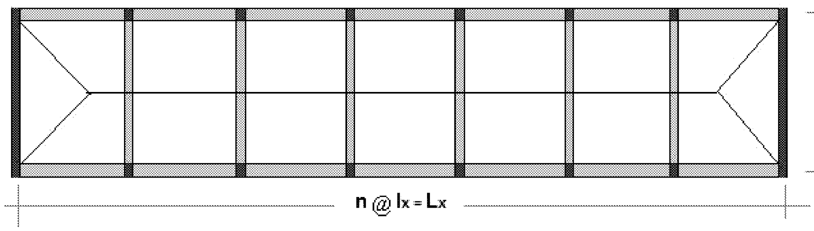

Fig.2: Global-collapse mechanism of the stiffened-plate

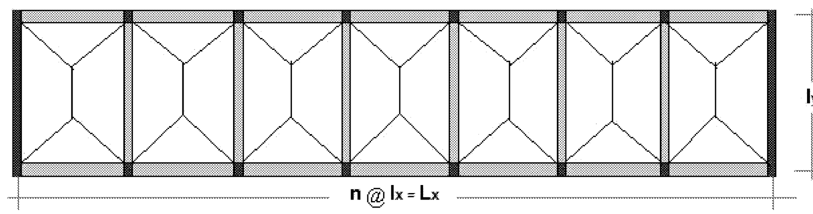

Fig.3: Local-collapse mechanism of the stiffened-plate

The generic plate-system will behave as a one-way or as a two-way plate element depending upon the strength of the internal stiffening beams cast monolithic with the plate. It would behave as a single panel plate of length L_{x} and width l_{y} if the strength of the beams along the short span l_{y} of the plate-system is taken as zero and/or is not adequate to initiate the local collapse in the panels of the plate as shown in Fig. 2. The plate-system would be transformed into a system consisting of a number of smaller rectangular plates of size $l_y \times l_{\mathrm{x}}$ resting over the internal beams with aspect ratio, $r_{\mathrm{p}} = l_{\mathrm{y}}/l_{\mathrm{x}}$ in case these beams are adequately stiff and/or strong e.g. wall-supported plate-system. This has been shown in Fig. 3. This happens mainly due to the formation of negative yield lines along the length of the stiff and/or strong internal beams at collapse load. Most of the available literature contains analytical models for these two extreme failure modes of the plate-systems *viz*: failure of the plate system in a single-panel or as a multi-panel local collapse mechanism [11–16]. However, the proposed analytical model can be used to predict the moment field in the plate-system at any strength level of the internal beams, even if it is zero; thereby representing a plate-system without any internal beams.

3.1. Collapse mechanism

A reinforced concrete plate develops a unique pattern of yield lines at the collapse load. The shape of the yield line pattern of a reinforced concrete plate depends upon the loading, its boundary conditions and various plate-constants. The development of a complete yield line pattern at ultimate state leads to the formation of collapse mechanism and at this stage, RC plate fails to support any further increase in the loading.

Yield lines in any laterally loaded plate, always, develop along the lines of maximum curvature forming in the plate. The stiffness of the support system, whether external or internal, plays a major role in the formation of these lines. A highly stiff support or internal beam of a plate-system causes a sudden change in the curvature across its line and leads to the formation of negative yield lines along its length. A reduction in the stiffness of a beam attracts the lines of maximum curvature in the plate toward the internal beams; whereas, an increase in the beam-stiffness repels these lines away from the beams. At some typical value of the beam stiffness and/or strength, the line of maximum curvature in the plate starts passing through the beam(s) thereby, changing the nature of the collapse mechanism from the local-collapse mechanism (a highly stiff-beam case) to a global-collapse mechanism (relatively a flexible-beam case). The strength of the internal beam(s) and the strength available along the lines of maximum curvature in the plate dictate the collapse load of the plate-system. There exists a coupling between the stiffness and strength of the plate-system. Any change in the one parameter has a significant effect on the other.

The nature and type of the collapse-mechanism in case of a plate cast monolithic with internal beams can be traced depending upon the strength and stiffness of these internal beams. If the internal beams are strong and stiff enough, these do not allow the yield lines developed in the plate to cross through into the adjacent panels and consequently, the plate-system fails with the formation of a yield line pattern, locally and simultaneously, in its all panels under uniform load acting over the entire face. At this stage, the plate-system transforms into a system consisting of a number of interconnected smaller plates resting over the internal stiffening beams at the collapse. This type of plate failure-mechanism is called as a local-collapse mechanism (see Fig. 3).

Otherwise, if the failure of the plate-system occurs with the formation of a plastic hinge in the internal beams simultaneously along with the development of a yield line pattern in the plate, the failure mechanism is called as a global-collapse mechanism. In this case, all the internal stiffening beams allow the yield line pattern developed in the plate to pass through them at the point of the plastic hinge(s) as shown in Fig. 2. This mainly happens because of change in the curvature of the middle plane of the plate caused by the stiffness of the internal stiffening-beams. In the present analytical modeling, the hypothesis collapse mechanism of the plate-system failing in the global-collapse mechanism or in the local-collapse mechanism has been assumed to follow the lines of maximum curvature formed in the plate-system under a uniform area load applied over its entire top face. This has been found to confirm with the guidelines suggested by Johansen [14], Quintas [17], Jennings [18] and Denton [20]. This hypothesis has been experimentally validated by the author [1].

3.2. Equilibrium and yield conditions

The RC plate-system behaves as a rigid-perfectly plastic material after the formation of a complete yield line pattern. Consequently, the work done by the external forces gets completely dissipated in the plastic deformations occurring along the yield lines. Therefore at collapse, the total work done by the external forces and the internal forces acting in the plate-system must disappear for any arbitrary value of a kinematically admissible virtual displacement field δ. This condition is used for ensuring the equilibrium of the hypothesis collapse mechanism of the plate-system. The collapse load so predicted would be on the higher side or at the most equals its true value depending upon the shape of the hypothesis collapse mechanism taken in the analysis. As reinforcement has been uniformly distributed at the tensile face of the plate, it is very unlikely that the yield criterion would be violated in a plate-system. An analytical model developed by satisfying the equilibrium, the true mechanism and yield criterion will always give a true moment field being induced in the plate-system.

4. Solution

A uniform load of intensity w is applied over the entire top face of the plate-system. The work done by the lateral load w can be determined by multiplying the total load acting at the center of gravity of the segmental areas of the collapse mechanism by a corresponding distance moved in the direction of a load, when some arbitrary kinematically admissible displacement δ is imposed over the collapse mechanism. The strength and stiffness of the internal beams is taken in a way to cause the failure of the plate-system in the global-collapse mechanism. The total work done by the external load (EWD) can be obtained by summing the individual contribution of all segments of the collapse mechanism of the plate-system. The final expression of this work EWD is given in Eq. (1).

$$\mathrm{EWD} = \frac{\delta\, w\, r\, l_{\mathrm{x}}^2}{6}\,(3\,n - 2\,p)\ . \tag{1}$$

The internal work performed by the positive moment-field m_{ux} and m_{uy} at the common edges of adjoining collapsed-segments of the mechanism and the beam moment m_{b} at the point of a plastic hinge (see Fig. 2) can be obtained from the work equation: $\sum m_{\mathrm{un}}\,\theta_{\mathrm{n}}\,l_{\mathrm{o}} = = \sum m_{\mathrm{ux}}\,\theta_{\mathrm{x}}\,l_{\mathrm{y}} + \sum m_{\mathrm{uy}}\,\theta_{\mathrm{y}}\,l_{\mathrm{x}}$ [14]. The total internal work done by the positive moment field m_{ux}, m_{uy} and m_{b} acting along the yield lines in the plate and the plastic hinge(s) formed in the beams(s) can be obtained by summing up their individual contribution. The expression for the internal work by the positive moment field is given in Eq. (2).

$$2\,m_{\mathrm{ux}}\,\delta\,r\left[\frac{1}{p} + \frac{2\,\mu\,n}{r_{\mathrm{p}}^2} + \frac{2\,(n-1)\,\alpha_{\mathrm{b}}}{r_{\mathrm{p}}^2}\right]\ . \tag{2}$$

In equation (2), $\alpha_{\mathrm{b}} = m_{\mathrm{b}}/(m_{\mathrm{ux}}\,l_{\mathrm{x}})$ is the beam-strength parameter of the plate-system.

The internal work performed by the negative moment field acting along the outer non-yielding boundaries of the plate-system is given in Eq. (3).

$$2\,m_{\mathrm{ux}}\,\delta\,r\left[(2\,i_{\mathrm{x}})\,\frac{\mu\,n}{r_{\mathrm{p}}^2} + (2\,i_{\mathrm{y}})\,\frac{1}{2\,p} + 2\,(n-1)\,\frac{k\,\alpha_{\mathrm{b}}}{r_{\mathrm{p}}^2}\right]\ . \tag{3}$$

The total internal work (IWD) performed by the moment field m_{ux}, m_{uy} and m_{b} along the positive and the negative yield lines of the collapse mechanism is obtained by adding Eq. (2) and Eq. (3). The simplified expression of the total IWD is given in Eq. (4).

$$\mathrm{IWD} = 2\,m_{\mathrm{ux}}\,\delta\,r\left[\frac{d}{2\,p} + \frac{\mu\,n\,e}{r_{\mathrm{p}}^2} + \frac{2\,f}{r_{\mathrm{p}}^2}\right]\ . \tag{4}$$

In Eq. (4), $d = 2\,(1+i_{\mathrm{x}})$, $e = 2\,(1+i_{\mathrm{y}})$, and $f = \alpha_{\mathrm{b}}\,(n-1)\,(k+1)$ are the continuity-constants, and these define the continuity conditions at outer boundaries of the plate-system.

The moment m_{ux} induced in the plate-system is given in Eq. (5). It has been obtained by equating Eq. (1) and Eq. (4) thereby, ensuring the equilibrium of the plate-system.

$$m_{\mathrm{ux}} = \frac{(3\,n - 2\,p)}{\dfrac{1}{p} + 2\,\beta}\,\frac{w\,l_{\mathrm{x}}^2}{6\,d}\ . \tag{5}$$

In Eq. (5), $\beta = (\mu\, n\, e + 2\, f)/(d\, r_{\mathrm{p}}^2)$ is a stiffened-plate parameter. It is a function of the aspect ratio of the plate panel r_{p}, orthotropy μ, strength of internal beams, continuity conditions i-values at outer boundaries of the plate-system and the number of panels n into which the plate-system has been divided.

The maximum value of the moment m_{ux} induced in the laterally loaded plate-system can be obtained by maximizing the moment function m_{ux}, given in Eq. (5), with respect to the parameter p. The parameter p defines the shape of the yield line pattern of a plate-system failing in the global-collapse mechanism (see Fig. 2). It can be obtained by solving the quadratic expression given in the Eq. (6).

$$\begin{aligned} \frac{\partial m_{\mathrm{ux}}}{\partial p} = 0 \quad \Rightarrow \quad & \frac{\dfrac{\partial}{\partial p}(3\,n - 2\,p)}{\dfrac{\partial}{\partial p}\left(\dfrac{d}{2\,p} + \dfrac{\mu\,n\,e}{r_{\mathrm{p}}^2} + \dfrac{2\,f}{r_{\mathrm{p}}^2}\right)} = \frac{(3\,n - 2\,p)}{\left(\dfrac{d}{2\,p} + \dfrac{\mu\,n\,e}{r_{\mathrm{p}}^2} + \dfrac{2\,f}{r_{\mathrm{p}}^2}\right)} \\ \text{or} \quad & \left(\frac{4\,\mu\,n\,e}{d\,r_{\mathrm{p}}^2} + \frac{8\,f}{d\,r_{\mathrm{p}}^2}\right) p^2 + 4\,p - 3\,n = 0 \\ \text{or} \quad & 4\,\beta\,p^2 + 4\,p - 3\,n = 0\ . \end{aligned} \tag{6}$$

Eq. (6) is solved for a parameter p. The value of the parameter p is given in Eq. (7) by combining two variables of the plate-system *viz*: n and β into a single parameter called as plate-parameter, $A = \sqrt{1 + 3\,n\,\beta}$.

$$p = \frac{3\,n}{2}\,\frac{A-1}{A^2-1} = \frac{3\,n}{2\,(A+1)}\ . \tag{7}$$

Eq. (7) defines the exact shape of the yield line pattern for a RC stiffened-plate failing in the global-collapse mechanism. In this failure mode, the supporting beams allow the yield line in the plate to pass through it at the point of the plastic hinge(s), simultaneously, along with the yield line pattern in the plate. It is interesting to note that Eq. (7) is valid whether branching point of the yield line intersect the internal beam or crosses it. This is due to the fact that the work dissipated along the single plastic hinge in case of an intersection or along two plastic hinges forming at the point of crossing in the beam will be exactly same.

The maximum value of the positive moment m_{ux} induced in the plate-system is obtained by substituting the p-value from Eq. (7) into Eq. (5). The final expression for the maximum value of the moment-field m_{ux}, m_{uy} is given in Eq. (8).

$$m_{\mathrm{ux}} = \left(\frac{3}{A+1}\right)^2 \frac{w\,L_{\mathrm{x}}^2}{12\,d} \qquad \text{and} \qquad m_{\mathrm{uy}} = \mu\, m_{\mathrm{ux}} = \left(\frac{3}{A+1}\right)^2 \frac{\mu\,w\,L_{\mathrm{x}}^2}{12\,d}\ . \tag{8}$$

The value of the plate-parameter A for a single panel plate with aspect ratio of unity becomes equal to two, thereby giving the plate moment m_{ux} that compare favorably well with the results available in the literature [18] at unit value of the orthotropy.

The strength-demand of the internal stiffening-beams of the plate-system depends upon the beam-strength parameter, $\alpha_{\mathrm{b}} = m_{\mathrm{b}}/m_{\mathrm{ux}}\, l_{\mathrm{x}}$ for any given value of the plate moment capacity m_{ux} and the panel length l_{x} of the plate-system. It can be calculated by substituting the expression of the plate moment, m_{ux} from the equation (8). This has been given in Eq. (9) in term of the plate-parameter, A.

$$m_{\mathrm{b}} = \left(\frac{A-1}{A+1}\, r^2 - \frac{3\,\mu}{(A+1)^2}\right) \frac{w\,L_{\mathrm{x}}^3}{(n-1)\,4\,d}\ . \tag{9}$$

Therefore, the design moment field $m_{\rm ux}$, $m_{\rm uy} = \mu\, m_{\rm ux}$, and $m_{\rm b}$ in the laterally loaded stiffened-plate can be obtained from Eq. (8) and Eq. (9) for any valid value of the plate-parameter A. The minimum value of the plate-parameter A, constituting its lower limit $A_{\rm c1}$, can be obtained from the Eq. (10) so that the stiffened-plate system must have a stiffening beam having some positive-non-zero moment capacity $m_{\rm b} > 0$. Otherwise, the same plate-system would start behaving as an ordinary single panel rectangular plate of size $L_{\rm x}\times l_{\rm y}$.

$$A_{\rm c1} = \sqrt{\frac{3\,\mu}{r^2} + 1}\;. \tag{10}$$

The value of plate-parameter, $A > A_{\rm c1}$ should be selected in such a manner that the stiffened-plate system must be able to sustain a load in the global-collapse mechanism mode at collapse. This collapse mechanism is considered desirable in comparison to the local-collapse mechanism of the plate-system because a plate-system designed in the global-failure mechanism is relatively more economical in comparison to a plate designed for the local-collapse mode. In former case, there is no need to provide any reinforcement at the top face of the plate normal to the internal stiffening beams while it is mandatory to use this reinforcement in the latter case as negative yield lines always develop at the top face of the plate due to a sudden change in the curvature across the internal stiff beams.

This condition can be checked by calculating the ultimate resisting moment of a plate-system failing in the local-collapse mechanism at collapse and comparing this value of the moment with a value obtained from the Eq. (8). The plate-system would fail in the global-collapse mechanism only if the value of the plate moment $m_{\rm ux}$ in this failure mode is more than that obtained from the failure of the plate-system in the local-collapse mechanism.

The plate-system converts into a set of number of smaller isolated rectangular plates in case the internal beams are sufficiently strong and/or stiff that lead to the formation of a local-collapse mechanism. These smaller plates rest over the internal stiffening beams after the formation of the local-collapse mechanism. Thus, the plate-system will consist of a number n of smaller isolated rectangular plates with aspect ratio R at collapse. The ultimate moment $m_{\rm ux}$ of these smaller rectangular plates is calculated in following section.

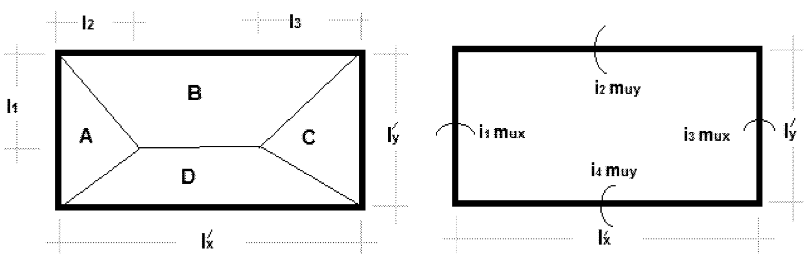

Fig.4: Schematic diagram of a single-panel plate

4.1. Moment Field for a continuous single panel rectangular slab

A single panel rectangular plate along with notations for the geometry and the moment field required to be developed in the plate-system for its equilibrium under a uniform load w is shown in Fig. 4. Three unknown dimensions l_1, l_2 and l_3 define the shape of the local-collapse mechanism being developed in the plate. The plate is assumed to be resting over the non-yielding edges at its outer boundary. Four equations required for the solution of the plate can be obtained by considering the equilibrium of each segment by taking moments about the non-yielding edge of the segment. Numbers of expressions are available in the literature [11to16] to predict the moment capacity of single panel RC slabs/plates resting over the non-yielding edges on its outer boundary. However, an expression [eq. (15)] has been derived in this section in the form of a plate-parameter A and geometrical dimensions of a single panel rectangular plate, shown in Fig. 4, to simplify the mathematical formulations.

1) Segment-A: $(1+i_1)\, m^1_{\mathrm{ux}}\, l^1_{\mathrm{y}} = \dfrac{w\, l^1_y\, l_2^2}{6}$

2) Segment-B: $(1+i_2)\, m^1_{\mathrm{uy}}\, l^1_{\mathrm{x}} = \dfrac{w\, l_2\, l_1^2}{6} + \dfrac{w\, l_3\, l_1^2}{6} + \dfrac{w\,(l^1_x - l_2 - l_3)\, l_1^2}{6}$

3) Segment-C: $(1+i_3)\, m^1_{\mathrm{ux}}\, l^1_{\mathrm{y}} = \dfrac{w\, l^1_y\, l_3^2}{6}$

4) Segment-D: $(1+i_4)\, m^1_{\mathrm{uy}}\, l^1_{\mathrm{x}} = \dfrac{w\, l_2\,(l^1_{\mathrm{y}} - l_1)^2}{6} + \dfrac{w\, l_3\,(l^1_{\mathrm{y}} - l_1)^2}{6} + \dfrac{w\,(l^1_x - l_2 - l_3)\,(l^1_{\mathrm{y}} - l_1)^2}{6}$

By combining the equations at serial no. 1 and 3, relationship between the parameter l_2 and l_3 is obtained. This has been given in Eq. (11).

$$l_2 = l_3 \sqrt{\frac{1+i_1}{1+i_3}} \; . \tag{11}$$

Similarly, a relationship between the parameters l_1 and l^1_{y} is obtained by combining equations at serial no. 2 and 4. This has been given in Eq. (12).

$$l_1 = l^1_{\mathrm{y}} \frac{\sqrt{1+i_2}}{\sqrt{1+i_2}+\sqrt{1+i_4}} \; . \tag{12}$$

By substituting the values of l_1, l_2 into the equations at serial no. 2 and further simplifying this expression along with equation at serial no. 3 will give the expression for the moment, m^1_{uy} in a single-panel plate of aspect ratio R. This has been given below:

$$m^1_{\mathrm{uy}} = \frac{R^2 \left[\sqrt{\left(\dfrac{X}{Y}\right)^2 + \dfrac{3\,\mu}{R^2}} - \left(\dfrac{X}{Y}\right)^2\right]^2}{\mu\, Y^2} \, \frac{w\, {l^1_y}^2}{6} \; .$$

The values of constants X and Y in the above equation can be obtained from following expressions. The i-values define the continuity-moments at outer boundary of the plate.

$$X = \sqrt{1+i_1} + \sqrt{1+i_3} \qquad \text{and} \qquad Y = \sqrt{1+i_2} + \sqrt{1+i_4} \; .$$

The above equation is further simplified to a more workable expression. This has given in Eq. (13).

$$m^1_{\mathrm{uy}} = \frac{R^2 \left(\dfrac{X}{Y}\right) \left[\sqrt{1 + \dfrac{3\,\mu}{R^2}\left(\dfrac{Y}{X}\right)} - 1\right]^2}{\mu\, Y^2} \, \frac{w\, {l^1_y}^2}{6} \; . \tag{13}$$

Let $\gamma = X/Y$ and the plate-constant for a single panel plate, $\beta_1 = \mu^1/(R^2\,\gamma^2)$. By using these notations, Eq. (13) is further simplified.

$$m^1_{\mathrm{uy}} = \frac{\gamma^2\,(\sqrt{1+3\,\beta_1}-1)^2}{X^2\,\beta_1}\,\frac{w\,{l_y^1}^2}{6}\ .$$

The final expression for the plate-moment m^1_{uy} induced under a uniform load w in a single panel rectangular plate continuous over its outer non-yielding boundary can be written in term of the plate-parameter $A_1 = \sqrt{1+3\,\beta_1}$. This has been given in Eq. (14).

$$m^1_{\mathrm{uy}} = \frac{A_1-1}{A_1+1}\,\frac{w\,{l_y^1}^2}{2\,Y^2}\ . \tag{14}$$

Eq. (14) can be made to predict the moment field of the plate-system, shown in Fig. 1, by replacing the long-span l^1_{x} and the short-span l^1_{y} of smaller rectangular plates by the short-span l_{y} and the panel-length l_{x} of the plate-system respectively. It is important to note that the reinforcement provided along the long span L_{x} and that along the short span l_{y} of the plate-system is the reinforcement provided along the short span L^1_{y} and the long span l^1_{x} of the smaller rectangular plates respectively. This condition is used to calculate the orthotropy μ^1 of the smaller rectangular plates from the orthotropy μ of the main plate-system. The aspect ratio R, orthotropy μ^1 and the moment capacity m^1_{uy} of smaller rectangular plates formed in the panel of the plate-system should be determined from Eq. (15).

$$R = \frac{l^1_{\mathrm{y}}}{l^1_{\mathrm{x}}} = \frac{l_{\mathrm{x}}}{l_{\mathrm{y}}} = \frac{l_{\mathrm{x}}}{r\,L_{\mathrm{x}}} = \frac{1}{n\,r}\ , \qquad \mu^1 = \frac{1}{\mu}$$

$$\text{and plate-moment} \qquad m^1_{\mathrm{uy}} = \frac{A_1-1}{A_1+1}\,\frac{w\,l_{\mathrm{x}}^2}{2\,Y^2}\ . \tag{15}$$

Therefore, the value of the plate-constant β_1 and the corresponding value of the plate-parameter, $A_1 = \sqrt{1+3\,\beta_1}$ for these smaller rectangular plates can be calculated form the following expression before determining the moment, m_{uy} from Eq. (15).

$$\beta_1 = \frac{\mu^1}{R^2\,\gamma^2} = \frac{(n\,r)^2}{\mu\,\gamma^2}\ .$$

The relative magnitude of the moment field given by Eq. (8) and Eq. (15) would determine the failure mode of the plate-system. It would fail in the global-collapse mechanism only if the moment capacity of the plate given by Eq. (8) is more than that predicted by Eq. (15). Otherwise, the same plate would fail following the local-collapse mechanism.

Eq. (8) and Eq. (15) indicate that there must exist some value of the plate-parameter A beyond which failure mode of the stiffened-plate transforms from the global-collapse to a local-collapse mechanism. This value of the plate-parameter constitutes an upper limit of plate-parameter $A_{\mathrm{c}2}$. The expressions for the upper limit of plate-parameter $A_{\mathrm{c}2}$ can be determined by equating Eq. (8) with Eq. (15). The simplified expression for $A_{\mathrm{c}2}$ for a general case of a plate-system with edge conditions defined through parameters Y and d is given in Eq. (16).

$$A_{\mathrm{c}2} = n\,Y\,\sqrt{\frac{3\,\mu}{2\,d}\,\frac{A_1+1}{A_1-1}} - 1\ . \tag{16}$$

The value of the beam-strength parameter of the plate system corresponding to A_{c2} is called as critical beam-strength parameter α_{bc}. It has been given in Eq. (17). This parameter defines the minimum strength of the stiffening-beams that leads to the simultaneous formation of the global and local-collapse mechanism in the laterally loaded plate system at collapse.

$$\alpha_{bc} = \frac{r^3}{3} \frac{n}{n-1} [A_{c2}^2 - A_{c1}^2] . \tag{17}$$

The value of the upper-limit of plate-parameter A_{c2} for various boundary conditions at edges of the plate-panels and outer boundary of the plate-system is given in Table 1. These simplified expressions have been obtained by substituting the corresponding i-values in Eq. (16).

S. No.	Boundary condition at edges of a panel	Corresponding i-values*	Expression of A_{c2}	β_1 values for calculating A_1 parameter
1	Discontinuous edges at outer boundary of the plate-system and without any negative reinforcement over the internal beams.	$i_1 = i_2 =$ $= i_3 = i_4 = 0$	$A_{c2} = n\sqrt{3\,\mu\,\frac{A_1+1}{A_1-1}} - 1$	$\beta_1 = \frac{(n\,r)^2}{\mu}$
2	Plate-system with no negative reinforcement over the internal beams and the outer boundaries along its short span (l_y).	$i_2 = 0$, $i_4 = i_y$, $i_1 = i_3 = i_x$	$A_{c2} = n\sqrt{\frac{6\,\mu}{d}\,\frac{A_1+1}{A_1-1}} - 1$	$\beta_1 = \frac{2\,(n\,r)^2}{\mu\,d}$
3	Plate-system with no negative reinforcement over the internal beams and some negative reinforcement over the outer boundaries defined by i_x and i_y.	$i_2 = i_4 = i_i$ $i_1 = i_3 = i_x$	$A_{c2} = n\sqrt{\frac{3\,\mu}{4}\,\frac{e}{d}\,\frac{A_1+1}{A_1-1}} - 1$	$\beta_1 = \frac{(n\,r)^2}{4\,\mu}\,\frac{e}{d}$

*Symbols used in table are illustrated in Fig. 4.

Tab.1: Upper limit of a plate-parameter (A_{c2}) for various boundary conditions

The stiffened-plate system can be designed to sustain a lateral load in the global-collapse mechanism by selecting the value of the plate-parameter A from the range given by the lower limit [Eq. (10)] and the upper limit [Eq. (16)] with the rider that the depth of the stiffening beams must be kept less than $span/12$ for a plate-system discontinuous at its outer boundary and this value must be taken as $span/15$ for a plate-system continuous at the outer boundary. Otherwise, either it would lead to the formation of a local-collapse mechanism in the plate-system [1]. In this case, it is mandatory to provide negative reinforcement at the top face of the plates near the beams to take care of any negative moment field being induced in the plate-system due to sudden change in the curvature across beams and near to beam ends.

4.2. Design charts and analysis of results

The variation of the moment-coefficients for the plate $m_{ux}\, d/(w\, L_x^2)$ and for the internal beams $m_b\,(n-1)\, d/(w\, l_y^2\, L_x)$ with a plate-parameter A is given in Fig. 5 and Fig. 6 respectively. Fig. 5 indicates that the plate moment m_{ux} approaches its maximum value at unit value of the plate-parameter $A = 1$ and the minimum value of the plate moment m_{ux} is possible only at very large value of the plate-parameter $A = \infty$. The plate-system sustains

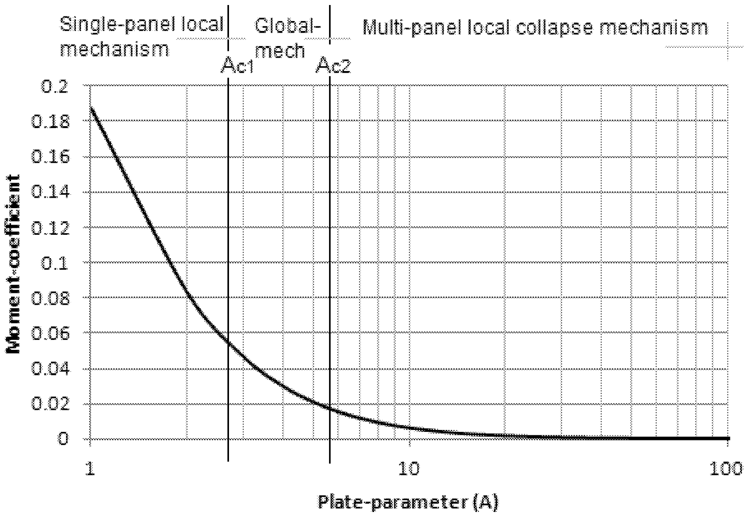

Fig.5: Design chart for a plate moment ($m_{\rm ux}$)

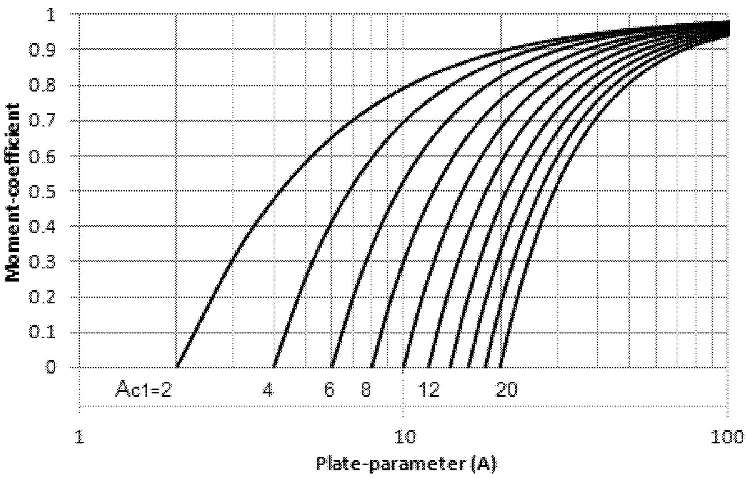

Fig.6: Design chart for a beam moment ($m_{\rm b}$)

lateral load in the global-collapse mechanism mode only in the narrow range, defined by the Eq. (10) and Eq. (16), of the wide spectrum of plate-parameters A-values as shown in Fig. 5. Otherwise, either it would lead to the formation of a local-collapse mechanism, simultaneously, in all panels of the plate-system at value of the plate-parameter, $A > A_{\rm c2}$ [see Eq. (16)] or can results in the non-valid value of the beam strength $m_{\rm b} < 0$ at the value of the plate-parameter, $A < A_{\rm c1}$ [see Eq. (10)]. At this value of the plate parameter, the plate-system transforms into a single panel large plate of size $L_{\rm x} \times l_{\rm y}$. However, the design curve shown in the Fig. 5 can be used very conveniently to predict the design moment field of the single-panel plate $n = 1$ with any arbitrary value of the plate-parameter A irrespective of the lower or the upper limits prescribed by Eq. (10) and Eq. (16). The results predicted from the proposed model for a single-panel plate compares favorably well with the results obtained from the well-established design formulae of the yield line theory [11–16] and the design coefficients recommended by the design code [3].

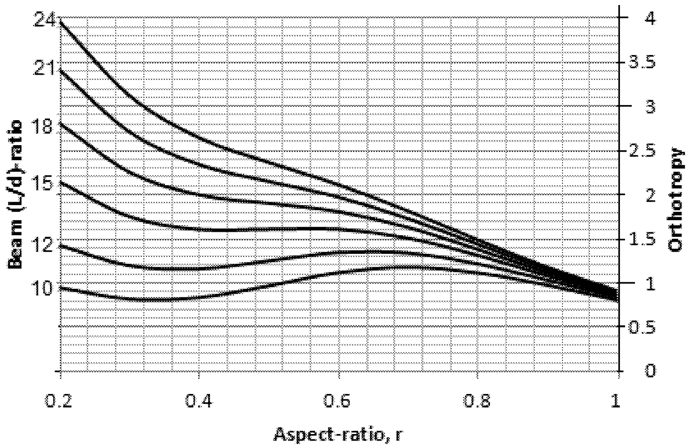

Fig.7: Orthotropy for a stiffened-plate with $n\,r < 2$

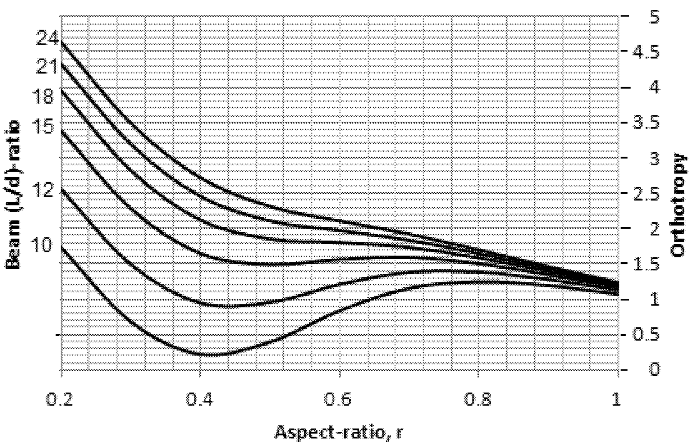

Fig.8: Orthotropy for a stiffened-plate with $n\,r \geq 2$

The lower and the upper limit of a plate parameter A is a fundamental property of any plate-system. These two limits depend upon the aspect ratio r, number of panels n, orthotropy μ and continuity conditions at boundary of the plate-system. The first two parameters *viz*: aspect ratio r and number of panels n of the plate-system are independent variables and are, normally, fixed by the architectural constrains in routine design practice whereas, the third parameter namely the orthotropy of a plate-system is a dependent variable. Its value is influenced by the aspect ratio and number of panels of the plate-system in addition to the stiffness of internal beam(s) and it should be very carefully chosen for the design purpose. A large deviation of value of the orthotropy from that required for an elastic distribution of the moment field will cause a significant cracking at service load on tensile face of the in the plate-system. The value of the orthotropy can be taken from chart in Fig. 7 for a plate-system with $n\,r < 2$ and chart in Fig. 8 can be used for a plate-system with $n\,r \geq 2$. These charts have been developed from a parametric study conducted using a finite element based software on a large number of plate specimens by taking aspect ratio, number of panels, and beam depth as variables. Typical set of cracks formed at top and bottom face of a four panel rectangular plate is given in Fig. 9, when the plate has been

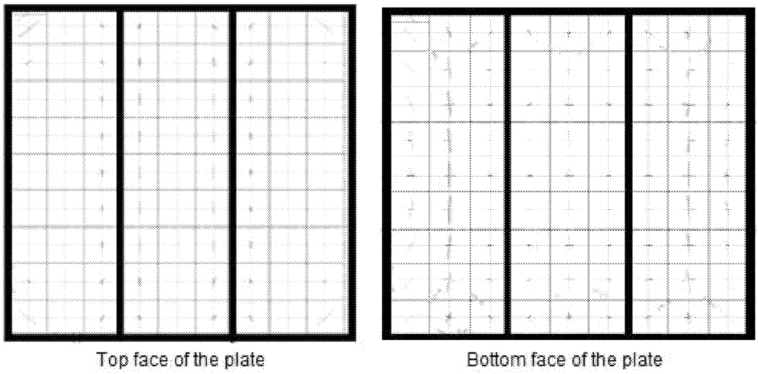

Fig.9: Typical crack pattern for a stiffened-plate with internal non-yielding beams

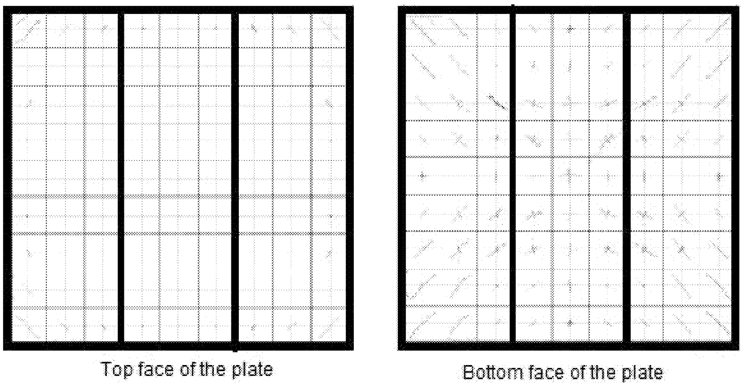

Fig.10: Typical crack pattern for a stiffened-plate with internal shallow beams

cast monolithic with internal non-yielding beams and Fig. 10, when the same plate has been cast monolithic with shallow beams. The crack width in all plate samples has been found to be well within the prescribed limits at the design load, when the reinforcement in the plates was detailed according to the strength-demand predicted from the proposed charts. However, deflection values exceed its prescribed limits but it can be reduced by using a thick plate section or by limiting a beam depth to $span/15$.

A plate-system designed at $A < A_{c1}$ leads to the formation of a single- panel local collapse mechanism of size $L_x \times l_y$ in the plate. If the plate-system has been designed with a very large value of the plate-parameter $A > A_{c2}$, it will lead to the formation of a number of smaller rectangular plates of size $l_x \times l_y$ resting over the internal stiffening-beams at collapse. This is called as a multi-panel local collapse mechanism. In this case, the behavior of the internal stiffening-beams of the plate-system would be quite analogous to the behavior of the internal panels of any two-way slab-beam system with non-yielding edges at outer boundary of the plate-panel. The strength requirement of the stiffening-beams, at this value of the plate-parameter $A = A_{c2}$, matches favorably well with the values obtained from the design procedure recommended by the design code [3]. This fact has been shown in Fig. 6, which indicates that the beam-moment m_b approaches, asymptotically, its maximum value at a very large value of the plate parameter A. At this value of the plate parameter, the plate must be designed as a continuous plate resting over the internal beam(s) of a plate-system. Therefore, the internal stiffening-beams of the plate-system can be made to behave similar to a non-yielding supporting beams and/or walls by the suitable selection of the plate-parameter, $A \geq A_{c2}$. However, detailing of rebars in a reinforced concrete rectangular stiffened plate-system should be done to take care of both positive as well as negative moment field induced in the plate system at this value of the plate-parameter.

Fig. 6 also indicates that the strength requirement of the internal beams increases proportional to the plate-parameter A. Higher the value of the plate-parameter, stronger the beam section must be to support a lateral load along with the plate of the plate-system. When the strength of the stiffening-beams approaches its upper limit A_{c2}, it will repel the branching point of the yield line pattern to an extent that it leads to the failure of the plate-system in the local-collapse mechanism irrespective of the stiffness and/or depth of the stiffening-beams. At this value of the plate-parameter, the beam section would require a large depth or a larger rebar area to provide a requisite flexural capacity. Consequently, the stiffness of the internal beams become large enough to cause a change in the curvature across its member-axis and leads to the formation of negative yield lines in the plate.

Therefore, the purpose of using the internal beams in the plate-system remains only to stiffen the laterally loaded plate to enable it to satisfy the serviceability requirement of the design codes and the strength requirement of these internal beams can be worked out from the valid values of the plate-parameter A. This would enable the designer to use reinforced concrete plates, more judicially with a minimum possible thickness, stiffened by a set of equally spaced internal beams with adequate matching strength required for supporting a lateral load in the global-collapse mechanism mode and vice versa. The depth of these internal beams should be selected in a manner that it must satisfy the serviceability conditions of the applicable design code and the depth criterion required for the initiation of a global collapse mechanism in the plate system.

The strength of the plate materials (reinforcement and concrete) can be utilized to its full allowable capacity by selecting an appropriate value of the plate-parameter A. Because all design codes restricts the maximum and the minimum spacing of the reinforcement bars in the plate section; thereby, providing some inherent minimum flexural-capacity corresponding to the maximum allowable bar spacing. The designer can choose an appropriate value of the plate-parameter A from the valid range of the plate-parameter $A_{c1} < A < A_{c2}$ [highlighted in Fig. 5] for the design purpose corresponding to the minimum available flexural capacity of the plate-system; thereby, utilizing the strength of all reinforcement bars to its full allowable capacity, and simultaneously satisfying the detailing-clauses of the design codes.

These charts therefore, will supplement various design guidelines by providing a generalized approach that could be used both for a single-panel as well as a multi-panel rectangular plate-system. The results from a finite element analysis of a typical stiffened-plate system has been given in appendix-A to demonstrate the validity and efficiency of the proposed charts.

5. Conclusions

- Existing formulations proposed in the literature and design guidelines do not address the effect of stiffness and strength of internal beams on the moment field induced in the RC stiffened plates as established by parametric study conducted using finite element based software.
- An improved theoretical formulation to predict moment field induced in RC stiffened plates is proposed, which attempts to resolve the shortcomings in methods available

in various design codes. This has been validated using finite element based software. Results are found to quite favorable.

– Design charts are suggested for predicting the moment field. These charts can be used to proportion a rectangular RC stiffened-plate system for any strength level of the internal beams, defined by the lower limit A_{c1} and the upper limit A_{c2} of the plate-parameter. As such, this parameter can be used very conveniently to control the participation of the plate in load sharing along with the (stiffening) beams of the plate-system. The stiffened-plate will behave as a single panel plate at its lower limit and it will be divided into a number of smaller rectangular plates resting over the internal stiffening beams at the upper limit. All the standard expressions available in the published literatures are applicable for these two extreme cases of a plate-system.
– A non-dimensional parameter called as plate-parameter A has been suggested to determine the collapse load and/or moment field of a stiffened-plate. The plate-system can sustain the load in the global-collapse mechanism mode only in the narrow range of the plate-parameters, $A_{c1} < A < A_{c2}$ thereby, allowing the designer to proportionate the plate-system with any suitable value of the plate-parameter from this valid range.

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References

[1] Singh H., Kumar M., Kwatra N.: Behavior of Shallow-Beam Supported Reinforced Concrete Rectangular Slabs: Analytical and Experimental Investigations, Advances in Structural Engineering, 13 (6): 1183–1198, (2010)
[2] ACI: Standard 318-08: Building Code Requirements for Reinforced Concrete, American Concrete Institute, Detroit, (2008)
[3] BIS: IS 456 (4th Rev.): Plain And Reinforced Concrete–Code of Practice, Bureau of Indian Standards, India, (2000)
[4] BS: EN 1992-1-1: Design of Concrete Structures (General rules and rules for buildings, Eurocode 2, British Standards, (2008)
[5] CSA: Standard A23.3-94: Design of Concrete Structures, Canadian Standards Association, Ontario, (1994)
[6] Gamble W.L.: Moments in Beam Supported Slabs, Proceedings Journal ACI, 69 (3): 49–157, (1972)
[7] Park R.: Further Test on a Reinforced Concrete Floor Designed by Limit Procedures in Cracking, Deflection and Ultimate Load of Concrete Slab Systems, ACI SP-30, American Concrete Institute, Detroit, pp. 251–269, (1971)
[8] Gamble W.L., Sozen M.A., Siess C.P.: Tests of a Two-Way Reinforced Concrete Slabs, Journal of Structure Division, ASCE, 195 (ST6): 073-1096, (1969)
[9] Sozen M.A., Siess C.P.: Investigation of Multi-panel Reinforced Concrete Floor Slabs: Design Methods-Their Evolutions and Comparison, Proc. ACI, 60: 999–1028, (1963)
[10] Park R.: Limit Design of Beams for Two-Way Reinforced Concrete Slabs, Journal of Institution of Structure Engineers, 146 (9): 269–274, (1968)
[11] Shukla S.N.: Handbook for Design of Slabs by Yield-line and Strip Methods, Structural Engineering Research Centre Roorkee, India, pp. 17–19, (1973)
[12] Jones L.L., Wood R.H.: Yield-Line Analysis of Slabs, Thames and Hudson, London, (1967)

[13] Jones L.L.: Ultimate Load Analysis of Reinforced and Prestressed Concrete Structures, Chatto and Windus, London, (1967)
[14] Johansen K.W.: Yield Line Theory, Cement and Concrete Association, London, (1967)
[15] Wood R.H.: Plastic and Elastic Design of Slabs and Plates, Thames and Hudson, London, (1961)
[16] Park R., Gamble W.L.: Reinforced Concrete Slabs, John Wiley and Sons, New York, (2000)
[17] Quintas V.: Two Main Methods for Yield Line Analysis of Slabs, Journal of Engineering Mechanics, 129 (2): 223–231, (2003)
[18] Jennings A.: On the Identification of Yield-Line Collapse Mechanisms, Engineering Structures, 18 (4): 332–337, (1996)
[19] Timoshenko S.P., Krieger S.W.: Theory of Plates and Shells, McGraw-Hill Book Company, New Delhi, (1959)
[20] Denton S.R.: Compatibility Requirements for Yield Line Mechanisms, International Journal of Solids and Structures, Vol. 38, pp. 3099–3109, (2001)
[21] Nichols J.R.: Statical limitations upon the Steel Requirement in Reinforced Concrete Flat Slab Floors, Transactions, ASCE, 77: 1670–1736, (1914)
[22] Liu Y., Li R.: Accurate Bending Analysis of Rectangular Plates with Two Adjacent Edges Free and Others Clamped or Simply Supported based on New Symplectic Approach, Applied Mathematical Modeling, 34, 856–865, (2010)
[23] Menon D., Pillai U.: Reinforced Concrete Design, Tata McGraw Hill Book Company, New Delhi, (2011)

Appendix A

Illustration: A typical rectangular plate consists of three panels in the two orthogonal directions. The spacing of beams along these two directions is 6 m (236.22 in) and 7.5 m (295.28 in) c/c each. Based upon the preliminary estimates, the columns are of size 400×400 mm (16×16 in) and the beams are of size 400×550 mm (16×21.65 in). The floor to floor height is 3.5 m (137.80 in). Assume a live load of 5 kN/m^2 (0.15 psi) and a finish load of 1 kN/m^2 (0.03 psi). Determine the design moments for the interior panel of the plate system and compare the results from a finite element analysis [23].

Number of panels in each direction of the plate system = 3.

Assuming a thickness of the plate as 180 mm (7.09 in) will satisfy the deflection criterion of design code. Therefore, total factored design load = $1.5\,(5+1+0.18\times25) = 15.75\,\mathrm{kN/m^2}$ (0.4725 psi).

Long span of the plate, $L_\mathrm{x} = 18\,\mathrm{m}$ (708.66 in) and the short span of the plate, $l_\mathrm{y} = 7.5\,\mathrm{m}$ (295.28 in).

Therefore, aspect ratio of the stiffened-plate system, $r = 7.5/18.0 = 0.4167$.

The value of the continuity constants *viz*: i_i, i_x and i_y are taken as 1.25, 1.333 and 0.0 respectively.

Orthotropy, $\mu = 1.40$ [Fig. 7].

The lower limit of the plate-parameter, $A_\mathrm{c1} = 5.022$ [from equation (10)].

The upper limit of the plate-parameter, $A_\mathrm{c2} = 6.915$ [from equation (16)].

The moment field induced in the stiffened-plate is calculated from the proposed charts for various values of plate-parameters A. A comparison of the moment field induced in the plate-system is given in Table A1 against the results obtained from the Direct Design Method prescribed by various design codes and the output of a finite element based software.

It is indicated in Table A1 that the average value of the moment field in the plate-system compare favorably well with the results obtained from the finite element analysis and direct design method prescribed by the design code. However unlike finite element or design code based solutions, the proposed charts gives more flexibility to the designer in choosing the moment field by varying the value of the plate-parameter A. This would enable the designer to use RC plates, more judicially with a minimum possible thickness, stiffened by a set of equally spaced internal beams with adequate matching strength required for supporting a lateral load in the global-collapse mechanism mode and vice versa. The depth of these internal beams should be selected in a manner that it must satisfy the serviceability conditions of the applicable design code and the depth criterion required for the initiation of a global collapse mechanism in the plate-system.

In the present example, span-depth ratio of the internal beams along one direction is 10.91 (thereby acting as rigid beams) and along the orthogonal direction, it is 13.64 (< 15 for a continuous outer boundary), thereby producing negative yield lines in the slab along all beams. The maximum deflection of the plate-system was found to be 15.60 mm (0.614 in) and the value of the maximum crack width in the plate section was found to be 0.254 mm (0.01 in). These two values are well within the code prescribed safe limits.

The moment field induced in the plate-system at the lower limit, and at the upper limit of the plate-parameters matches exactly with the values recommended by design code [3] for a rectangular plate with discontinuous edges at the outer boundary. Therefore, the proposed charts can be used to evaluate the moment field in the stiffened-plate for a single-panel plate of any aspect ratio, orthotropy but resting over the non-yielding edges at outer boundaries. The proposed model will also be helpful for the analysis of a rectangular plate internally stiffened by the some regularly spaced flexural members having any strength level. The strength level of the stiffeners can be controlled through the value of the plate-parameters A.

Λ	$A=\sqrt{\lambda(A_{c2}^2-A_{c1}^2)+A_{c1}^2}$	Moment field from design charts (and kNm)				
		m_{ux} kNm/m (10^3 lb-in/in)	m_{uy} kNm/m (10^3 lb-in/in)	$-m_{ux}$ kNm/m (10^3 lb-in/in)	$-m_{uy}$ kNm/m (10^3 lb-in/in)	m_b kNm/m (10^3 lb-in/in)
0.0	$A_{c1} = 5.022$	22.60 (108.96)	31.67 (152.68)	–	42.12 (203.06)	0.0 (0.00)
0.2	5.453	19.68 (94.88)	27.58 (132.96)	24.59 (118.55)	36.68 (176.84)	46.28 (8.113)
0.4	5.893	17.69 (85.28)	24.45 (117.86)	21.81 (105.15)	32.59 (157.12)	106.37 (18.65)
0.6	6.227	15.69 (75.64)	21.99 (106.02)	19.61 (94.54)	29.24 (140.97)	110.78 (19.42)
0.8	6.580	14.26 (68.75)	19.90 (95.94)	17.83 (85.96)	26.58 (128.14)	134.28 (23.54)
1.0	$A_{c2} = 6.915$	13.08 (63.06)	18.33 (88.37)	16.35 (78.82)	24.44 (117.83)	153.99 (26.99)
Average Moment field		17.15 (82.68)	23.98 (115.61)	21.44 (103.36)	31.94 (153.98)	110.34 (19.34)
Moment field (FEM)		16.47 (79.40)	19.46 (93.82)	24.83 (119.71)	26.57 (128.09)	111.44 (19.54)
Moment field (Direct Design Method)		16.13 (77.76)	19.18 (92.47)	22.02 (106.16)	25.57 (123.27)	128.00 (22.44)

Tab.A1: Comparison of the moment-field at various values of the plate-parameter

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