EULER-TYPE ALGORITHM APPLIED TO LEVELING PROCESS ANALYSIS

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Presented paper is focused on description of algorithm for fast analysis of leveling process in detail and on verification of obtained results. The main idea is based on using Finite Element Method (FEM) and solving its fundamental equation. The algorithm is programmed using software MATLAB. The program has modular structure, so it is not complicated to modify any setting or utility. Leveling process of long semi products respecting the Eulerian approach is considered. Curved material passes through laterally offset rollers so repeated elasto-plastic bending occurs. Plastic deformation causes redistribution of residual stress. The semi product is straightened when rollers are appropriately positioned. Given problem is complicated due to high inherent nonlinearity and sensitivity, caused by cyclic plasticity. Useful setting of the leveling machine is found by iterative process, based on input measured geometrical data and material characteristics.

Keywords: *long semi product, FEM, plastic bending, leveling, straightening, Eulerian approach*

1. Introduction

The leveling process is based on the elasto-plastic bending which is repeated on every roller [8]. In our case we are talking about seven rollers (Fig. 1) but there is commonly used nine or eleven rollers and for thin materials up to twenty one rollers. The elasto-plastic bending provides redistribution of initial residual stress which brings down curvature of long product. The aim of leveling process analysis is to determine and examine how to set positions of rollers to decrease curvature of long product at possible minimum by convenient redistribution of residual stress using cyclic plasticity of considered material.

The solving process of this problem can be highly unstable and sensitive to material behavior, product inhomogeneity, contact conditions or wear of straightening rollers [6]. Therefore, it is necessary to obtain satisfactory convergence using advanced iterative solvers of nonlinear problem as discussed further.

Historically, one of the first algorithms for solving leveling problem are based on empirical and theoretical knowledge of materials processing technology field, as in [1]. Later, with development of computational technology there is a significant approach along with the use of FEM. There are also many simple and quick analyses in one hand, but due to increasing computer power, more complicated models are used in terms of solving algorithms, used material models or involving other effects and impacts in the other, as in [3]. There are

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many models generally based on the integration of long product's curvature at each roller where the curvature is known, as in [4, 5, 9].

More detailed description and verification of new model for fast leveling process analysis, as proposed by Petruška et al. in [8], and discussion on the number of used elements together with the computing time is published in this paper. Verification of our results is performed on the basis of results published by Nastran and Kuzman in [5]. Developed algorithm starts from the intermeshing of each roller position. Next, deflection, slope, curvature, bending moment and output residual stress with final curvature are computed on the basis of input measured geometrical data and given material characteristics of curved long product. The algorithm and its subroutines are programed using software MATLAB.

2. Suggested algorithm in detail

We suppose leveling of long rods with uniaxial state of stress. Each of top rollers in Fig. 1 is individually adjustable and all bottom rollers are fixed.

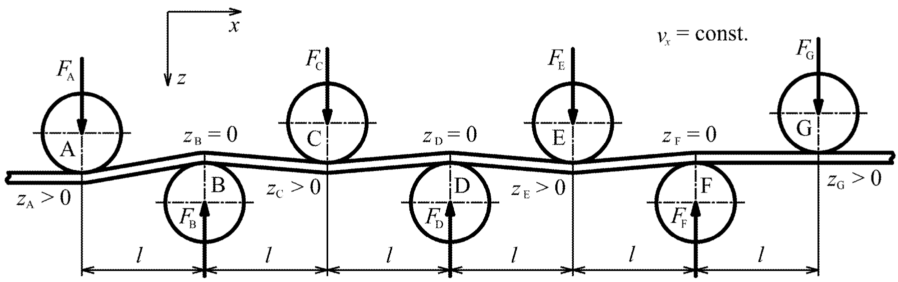

Fig.1: Scheme of leveling machine [8]

Measured geometrical data provide the information about input shape and cross section's dimensions. We consider circular cross section long product so we do not take into account the length dimension but only diameter of leveled material. There is just the condition of minimal length because of design dimensions of the leveling machine. Required mechanical characteristics are E, E_T and σ_k which are Young's modulus, tangential hardening modulus and yield stress, respectively. The isotropic hardening rule is used for cyclic loading.

The main idea of presented algorithm is based on iterative solution. The fundamental nonlinear equation of the FEM in Eq. (1) is solved by the Newton-Raphson method. Index i represents iteration number and $\mathbf{R}_{i-1}$ represents matrix of residual nodal forces, obtained from equivalent nodal forces as $\mathbf{R}_{i-1} = -\mathbf{F}_{i-1}$. Initial external loading at the first iteration $\mathbf{F}_0$ is obtained from prescribed deflections between bar and rollers A–G (Fig. 1). We suppose only point contact between bar and rollers with fixed pitch l (Fig. 1).

$$\mathbf{K}_{\mathrm{T},i-1}\,\Delta\mathbf{U}_i = \mathbf{R}_{i-1}\ . \tag{1}$$

The global increment matrix of nodal displacements and slopes is obtained by Eq. (2). $\mathbf{U}_0$ is equal to zero at first iteration.

$$\mathbf{U}_i = \Delta\mathbf{U}_i + \mathbf{U}_{i-1} \tag{2}$$

The bar is meshed between the first and last roller (A–G in Fig. 1) by beam finite elements with their element stiffness matrices, according to Eq. (3). In this equation j is index of each element, L is length of each element and $(E\,J)_{j,i}$ is beam flexural rigidity, where J is the area moment of inertia.

$$\mathbf{k}_{k,i} = \frac{(E\,J)_{j,i}}{L^3} \begin{bmatrix} 12 & 6\,L & -12 & 6\,L \\ 6\,L & 4\,L^2 & -6\,L & 2\,L^2 \\ -12 & -6\,L & 12 & -6\,L \\ 6\,L & 2\,L^2 & -6\,L & 4\,L^2 \end{bmatrix} . \tag{3}$$

Global tangential stiffness matrix $\mathbf{K}_{\mathrm{T},i-1}$ is assembled from element contributions in Eq. (3) by standard FE procedure.

After solving Eq. (1) we can determine curvatures in all nodes by Eq. (4).

$$w''_{j,i} = \mathbf{B}(x_j)\,\boldsymbol{\delta}_{j,i} \ . \tag{4}$$

In the previous equation $\mathbf{B}(x_j)$ stands out as curvature approximation matrix. This matrix is given by Eq. (5) where x_j is longitudinal coordinate of each element.

$$\mathbf{B}_{j,i}(x+j) = \left[-\frac{6}{L^2} + \frac{12}{L^3}\,x_j \quad -\frac{4}{L} + \frac{6}{L^2}\,x_j \quad \frac{6}{L^2} - \frac{12}{L^3}\,x_j \quad -\frac{2}{L} + \frac{6}{L^2}\,x_j \right]^{\mathrm{T}} \tag{5}$$

The elements have two nodes and four degrees of freedom, two displacements and two slopes, as described by matrix $\boldsymbol{\delta}_{j,i}$ in Eq. (6).

$$\boldsymbol{\delta}_{j,i} = [\, w_{j,i} \quad w'_{j,i} \quad w_{j+1,i} \quad w'_{j+1,i} \,]^{\mathrm{T}} \tag{6}$$

Now, curvature increments between subsequent nodes can be estimated by the following equation.

$$\Delta w''_{j,i} = w''_{j,i} - w''_{j-1,i} \ . \tag{7}$$

The strain and stress distribution is identified considering the Eulerian approach when material flows along the leveling machine through beam elements which are fixed in the space. Total strain increment is defined by Eq. (8) for every layer which the cross section is cut into (Fig. 2). In Fig. 2, z is vertical coordinate and d is diameter of the bar. Then, the testing stress is calculated by Eq. (9). An initial distribution of stress and strain in the first node must be concerned if the bar with initial curvature should be solved.

$$\Delta\varepsilon_{j,i}(z) = \Delta w''_{j,i}\, z \ , \tag{8}$$

$$\sigma_{j,i}(z) = \sigma_{j-1,i} + E\,\Delta\varepsilon_{j,i} \ . \tag{9}$$

Testing stress of each material layer in Eq. (9) is corrected with respect to the elasto-plastic behavior according to algorithm published in [2]. Modified stress distribution is illustrated in Fig. 2. Next, the elastic and plastic components of the total strain can be also evaluated.

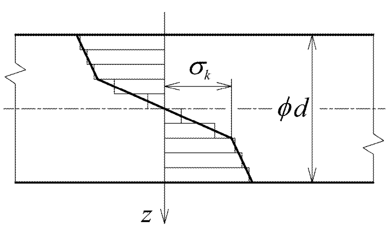

Fig.2: Scheme of material layers and stress distribution [10]

Using stress distribution according to Fig. 2, where plastic deformations occurred, we can modify the bending moment by Eq. (10), where ψ represents the cross section.

$$M_{j,i}(x_j) = \iint\limits_{\psi} \sigma_{j,i}(z)\, z\, \mathrm{d}S \ . \tag{10}$$

Now, new global matrix of equivalent nodal forces $\mathbf{F}_i$ can be assembled from element contributions given by Eq. (11).

$$\mathbf{f}_j = \int\limits_0^L \mathbf{B}_j(x_j)\, M_j(x_j)\, \mathrm{d}x_j \ . \tag{11}$$

Due to plastic deformations we have to modify the stiffness matrix of each element where yielding occurred. It is done by changing the beam flexural rigidity $(E\,J)_{j,i}$ in Eq. (3) using Eqs. (12)–(14).

$$H = \frac{E\,E_\mathrm{T}}{E - E_\mathrm{T}} \ , \tag{12}$$

$$E_\mathrm{m}(z) = E\left(1 - \frac{E}{E+H}\right) \ , \tag{13}$$

$$(E\,J)_{j,i} = \iint\limits_{\psi} E_\mathrm{m}(z)\, z^2\, \mathrm{d}S \ . \tag{14}$$

Variable H in Eqs. (12) and (13) represents the hardening parameter. After that procedure, we can complete new global stiffness matrix for next iteration.

At the end of the iteration, convergence criteria are tested. We use both the global matrix of displacements and slopes in Eq. (15) and the global matrix of residual nodal forces in Eq. (16). In Eqs. (15) and (16) symbols θ_u and θ_f are desired deformation and force tolerances, respectively.

$$\frac{\|\mathbf{U}_i - \mathbf{U}_{i-1}\|}{\|\mathbf{U}_i\|} \le \theta_\mathrm{u} \ , \tag{15}$$

$$\frac{\|\mathbf{R}_i\|}{\|\mathbf{F}_i^\mathrm{react}\|} \le \theta_\mathrm{f} \ . \tag{16}$$

The denominator of a fraction in Eq. (16) represents generally the norm of external forces. As there is only deformation loading in our problem, external loading is represented by reaction forces $\mathbf{F}^\mathrm{react}$ between rollers and leveled material.

Finally, if convergence criteria are satisfied, the procedure is stopped. In the opposite case the iteration number is increased by one and the procedure is returned back to Eq. (1) and repeated.

3. Verification of suggested algorithm

For verification of our algorithm we used results published by Nastran and Kuzman in [5] where the seven roller leveling machine is also used but with different design. Nevertheless, we easily modified our program so it agreed with the one in [5].

Here follows the enumeration of values used in a computation: diameter of the bar 2.1 mm, yield stress 530 MPa, Young's modulus 210000 MPa, no hardening, initial curvature $2.5\times10^{-3}\,\mathrm{mm}^{-1}$, intermesh of roller B 0.4 mm, intermesh of roller D 0.3 mm, intermesh of roller F 0.2 mm, pitch 25 mm and 480 beam elements. For detailed design of the leveling machine and more information see [5].

Analyzed variables along the bar length are depicted in following Figs. 3–6. Mutual comparisons of curvature and bending moment from our program and from [5] are presented in these figures. We can generally state a good correlation between both sets of results. The

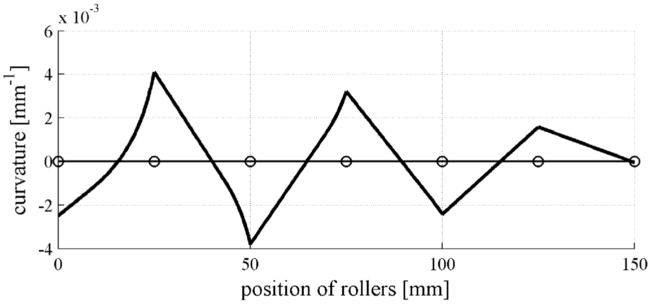

Fig.3: Progress of curvature by our program

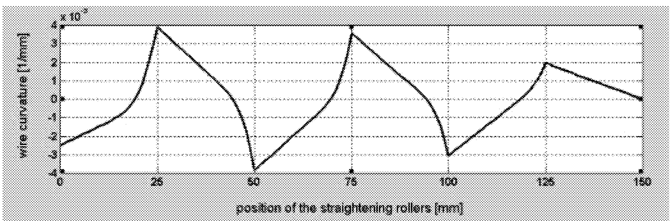

Fig.4: Progress of curvature in [5]

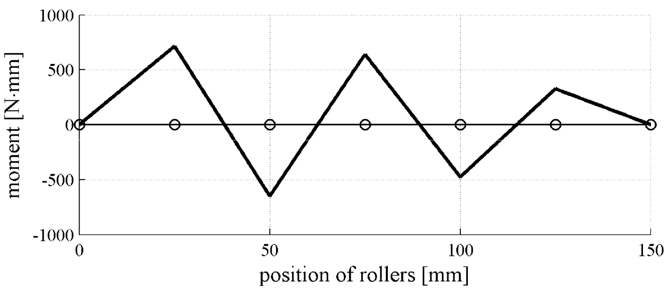

Fig.5: Progress of bending moment by our program

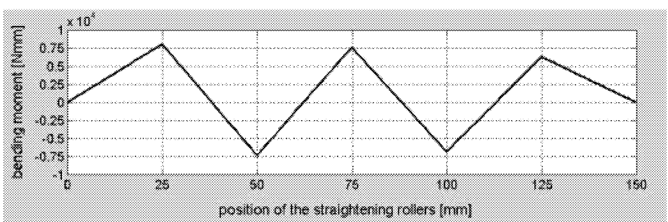

Fig.6: Progress of bending moment in [5]

differences can be credited to more sophisticated material model in [5] with cyclic softening, which results in changes of yield stress during the process, together with smooth transition between elastic and plastic behavior.

4. Discussion

Program with fast algorithm for solving the leveling process of long products has been developed and presented. Suggested algorithm programmed in MATLAB is considerably faster in comparison to multipurpose FEM packages. This fact is illustrated in following Figs. 7 and 8 together with sensitivity analysis of number of used elements.

We can note that suitable number of used elements could correspond to 480. It would be useless to use more elements because the final curvature would not be distinctly refined and the computing time would be unnecessarily extended.

The Fig. 8 shows that the computing time is short indeed. It ranges in hundreds of seconds, thus in tens of minutes. One task is usually solved in less than ten minutes on standard personal computer.

Nowadays literature shows, e.g. in [7], that also the neural network could be used for modeling of manufacturing processes, like straightening, due to rapid development of this field.

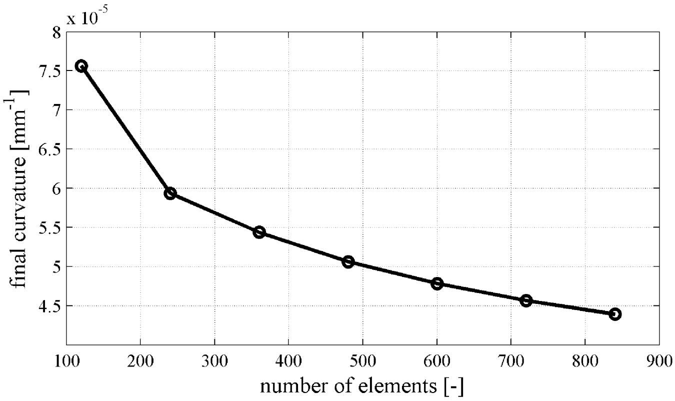

Fig.7: Dependence of number of used elements to final curvature

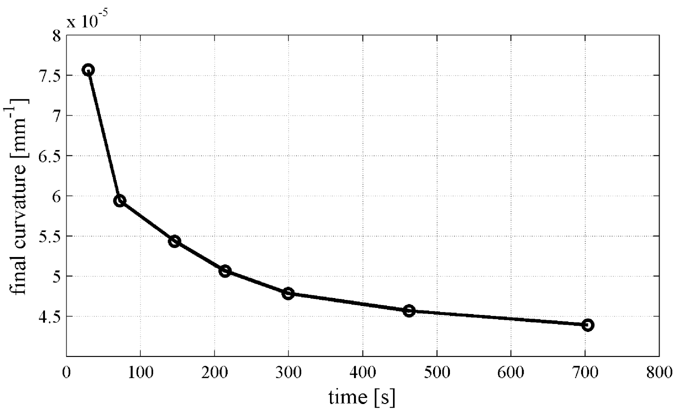

Fig.8: Reached final curvature in correlation to computing time

5. Conclusions

The principle of the algorithm is based on FEM and its fundamental equation using Newton-Raphson method for solving nonlinear problems. Beam elements are used with considering the Eulerian approach when material moves through these fixed elements, thus through leveling machine. Whole suggested algorithm is described in detail. Verification example is shown as well. Modular structure of presented program allows addition of other subroutines such as the one involving rotation of the bar around its axis to enable simulation of cross roll straightening of bars and tubes, plate and tensile leveling, etc. Other improvements can be included as well as more precise material model, especially the cyclic softening, next an influence of shear force to deflection of leveled product or finally the location of contact point between rollers and moving material during leveling process and rate of its effect. Our tendencies are strongly aimed to practical applicability of results in industrial field. The consideration of the bar rotation is being intensively prepared.

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