COOLING TOWER FILL MODELLING USING ONE-DIMENSIONAL MODEL OF HEAT AND MASS TRANSFER

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The article deals with the numerical modelling of heat and mass transfer in the counterflow wet-cooling tower fill. Due to the complexity of this phenomenon the simplified model based on the set of four ODEs [1] was chosen. The used approach is generally applicable to the simulation of the distribution of moist air temperature, water temperature, specific humidity of air and water mass flow rate. Evaluation of the distribution of heat and mass sources is also done. Boundary condition for outlet water temperature are based on experimentally obtained Merkel number correlation. Numerical solution of chosen model was performed using Dormand-Prince method combined with shooting method. Results are compared with data available in the literature.

Keywords : *evaporative cooling, cooling tower, Merkel number, heat and mass sources, boundary value problem*

1. Introduction

In the counterflow wet-cooling tower fill of film type water flows vertically down through the fill as a liquid film. Air is driven by a tower draft or fan and flows vertically in the opposite direction. Heat and mass transfer occurs at the water and air interface. Evaporation and convective heat transfer cool the water, what leads to increase of air humidity and temperature.

Due to the complexity of two phase flow occurring in the wet-cooling tower fill the one dimensional models of heat and mass transfer are used (e.g. [2, 3, 1]). These models are based on few assumptions which allow to create simplified one dimensional models. The first assumption talks about neglecting of the effects of horizontal temperature gradient in the liquid film, horizontal temperature gradient in air temperature and humidity, see e.g. [3].

Temperatures and humidity are then represented only by their averaged values for each vertical position. We are also assuming that at the interface of two phases, there is a thin vapour film of saturated air at the water temperature, see e.g. [3].

Derivation of every one dimensional model of heat and mass transfer in the fill is based on balance laws [2, 3, 1]. We have four variables in this problem : t_a temperature of air, t_w temperature of water, x specific humidity and $\dot{m}_\mathrm{w}$ water mass flow rate. Mass balance of the incremental step of the fill is given by

$$\mathrm{d}\dot{m}_\mathrm{w} + \dot{m}_\mathrm{a}\,\mathrm{d}x = 0\ , \tag{1}$$

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where $\dot{m}_\mathrm{a}$ is mass flow rate of dry air. The change in water mass flow rate can be expressed using mass transfer coefficient α_m as

$$\mathrm{d}\dot{m}_\mathrm{w} = \alpha_\mathrm{m}\,(x''(t_\mathrm{w}) - x)\,\mathrm{d}A\ , \tag{2}$$

where $x''(t_\mathrm{w})$ is saturated specific humidity at t_w and $\mathrm{d}A$ is infinitesimal contact area. The energy balance can be written in the form

$$\dot{m}_\mathrm{a}\,\mathrm{d}h_{1+x} = \dot{m}_\mathrm{w}\,\mathrm{d}h_\mathrm{w} + h_\mathrm{w}\,\mathrm{d}\dot{m}_\mathrm{w}\ , \tag{3}$$

where h_{1+x} is enthalpy of air water vapour mixture and h_w is enthalpy of water. The change in total enthalpy can be evaluated using interface parameters similarly like in the case of mass balance

$$\dot{m}_\mathrm{a}\,\mathrm{d}h_{1+x} = \alpha\,(t_\mathrm{w} - t_\mathrm{a})\,\mathrm{d}A + h_\mathrm{v}(t_\mathrm{w})\,\mathrm{d}\dot{m}_\mathrm{w}\ , \tag{4}$$

where α is heat transfer coefficient and $h_\mathrm{v}(t_\mathrm{w})$ is enthalpy of water vapour.

2. Merkel's model

The simplest model of heat and mass transfer in the fill is the Merkel's model which is based on previous equations and additional assumptions, see e.g. [3]. The first assumption is about neglecting the change of water flow rate in energy balance. Second assumption states that air exiting the cooling tower fill is saturated and this state can be characterized only by its enthalpy. The last assumption of the Merkel's model states that Lewis factor $Le_\mathrm{f} = 1$. Lewis factor is equal to the ratio of heat transfer Stanton number St to the mass transfer Stanton number St_m, i.e. $Le_\mathrm{f} = St/St_\mathrm{m} = \alpha/(c_\mathrm{p}\,\alpha_\mathrm{m})$, where c_p is constant pressure specific heat capacity of moist air.

The Merkel's model is based on so called Merkel's integral

$$Me = \int\limits_0^A \frac{\alpha_\mathrm{m}}{\dot{m}_\mathrm{w}}\,\mathrm{d}A = \int\limits_{t_{\mathrm{w_o}}}^{t_{\mathrm{w_i}}} \frac{c_{\mathrm{p_w}}\,\mathrm{d}t_\mathrm{w}}{(h''_{1+x}(t_\mathrm{w}) - h_{1+x})}\ , \tag{5}$$

where Me is non-dimensional Merkel number, $t_{\mathrm{w_o}}$ is water temperature at the outlet and $t_{\mathrm{w_i}}$ is water temperature at the inlet. More precise derivation of the Merkel's model together with details connected with numerical calculation of Merkel number and outlet water temperature from known Merkel number can be found e.g. in references [2, 3, 4].

3. Model of Klimanek & Białecky [1]

To derive the system of ODEs we have to choose independent variable. The model of Klimanek & Białecky [1] is based on the selection of spatial coordinate z as independent variable contrary to Poppe's model (e.g. [3]) which is based on the choice of water temperature t_w. The interface area can be expressed using variable z as

$$\mathrm{d}A = a_\mathrm{q}\,A_\mathrm{fr}\,\mathrm{d}z\ , \tag{6}$$

where a_q is the transfer area per unit volume and A_fr is the cross sectional area of the fill. We can derive equation for the change of water mass flow rate from equation (2)

$$\frac{\mathrm{d}\dot{m}_\mathrm{w}}{\mathrm{d}z} = \alpha_\mathrm{m}\,a_\mathrm{q}\,A_\mathrm{fr}\,(x''(t_\mathrm{w}) - x)\ . \tag{7}$$

To obtain the equation for the change of specific humidity in the fill we can substitute equation (7) into equation (1)

$$\frac{\mathrm{d}x}{\mathrm{d}z} = \frac{\alpha_\mathrm{m}\, a_\mathrm{q}\, A_\mathrm{fr}\, (x''(t_\mathrm{w}) - x)}{\dot{m}_\mathrm{a}} \ . \tag{8}$$

Enthalpy of moist air can be expressed like

$$h_{1+x} = c_{\mathrm{p_a}}\, t_\mathrm{a} + x\, (l_0 + c_{\mathrm{p_v}}\, t_a) \ , \tag{9}$$

where $c_{\mathrm{p_a}}$ and $c_{\mathrm{p_v}}$ are constant pressure specific heat capacities of dry air and water vapour and l_0 is latent heat of vaporisation. Differentiation of equation (9) leads to

$$\frac{\mathrm{d}h_{1+x}}{\mathrm{d}z} = (c_{\mathrm{p_a}} + x\, c_{\mathrm{p_v}})\, \frac{\mathrm{d}t_\mathrm{a}}{\mathrm{d}z} + (l_0 + c_{\mathrm{p_v}}\, t_\mathrm{a})\, \frac{\mathrm{d}x}{\mathrm{d}z} \ . \tag{10}$$

The left hand side of equation (10) can be substituted from equation (4), the last term can be expressed using (8) and after application of the Lewis factor definition we can get

$$\begin{aligned} \frac{\mathrm{d}t_\mathrm{a}}{\mathrm{d}z} = \frac{\alpha_\mathrm{m}\, a_\mathrm{q}\, A_\mathrm{fr}}{\dot{m}_\mathrm{a}\, (c_{\mathrm{p_a}}(t_\mathrm{a}) + x\, c_{\mathrm{p_v}}(t_\mathrm{a}))} \big[Le_\mathrm{f}\, (t_\mathrm{w} - t_\mathrm{a}) \big((c_{\mathrm{p_a}}(t_\mathrm{a}) + x\, c_{\mathrm{p_v}}(t_\mathrm{a}) \big) + \\ + (c_{\mathrm{p_v}}(t_\mathrm{w})\, t_\mathrm{w} - c_{\mathrm{p_v}}(t_\mathrm{a})\, t_\mathrm{a})\, (x''\, (t_\mathrm{w}) - x) \big] \ . \end{aligned} \tag{11}$$

By using equation (3) after substitution of equation (7) and equation (4) we derive equation for the change of water temperature

$$\begin{aligned} \frac{\mathrm{d}t_\mathrm{w}}{\mathrm{d}z} = \frac{\alpha_\mathrm{m}\, a_\mathrm{q}\, A_\mathrm{fr}}{\dot{m}_\mathrm{w}\, c_\mathrm{w}(t_\mathrm{w})} \big[Le_\mathrm{f}\, \big(c_{\mathrm{p_a}}(t_\mathrm{a}) + x\, c_{\mathrm{p_v}}(t_\mathrm{a}) \big)\, (t_\mathrm{w} - t_\mathrm{a}) + \\ + \ (x''(t_\mathrm{w}) - x)\, (c_{\mathrm{p_v}}(t_\mathrm{w})\, t_\mathrm{w} - c_{\mathrm{p_w}}(t_\mathrm{w})\, t_\mathrm{w} + l_0) \big] \ . \end{aligned} \tag{12}$$

Application of the Lewis factor in the previous equations simplified the problem to find experimentally only mass transfer coefficient α_m and calculate heat transfer coefficient α using known value of Lewis factor. The most commonly used formula for calculation of the Lewis factor is Bosnjakovic formula [1].

Similar system of ODEs can be derived in the case of supersaturated air [1].

4. Methodology of numerical simulations

Numerical solution of four ODEs mentioned in the previous section represents the boundary value problem. We know air temperature t_ai and specific humidity x_i at air inlet and water temperature t_wi and water mass flow rate $\dot{m}_\mathrm{wi}$ on the opposite site of the fill because air and water flows in the opposite direction. There is additional unknown set of parameters in the system of equations, i.e. $\alpha_\mathrm{m}\, a_\mathrm{q}\, A_\mathrm{fr}$. These parameters have to be solved by using experimentally obtained characteristics of the fill. There are at least two possibilities how to solve this problem.

The fist possibility is based on the calculation of Merkel number of the model of [1] using

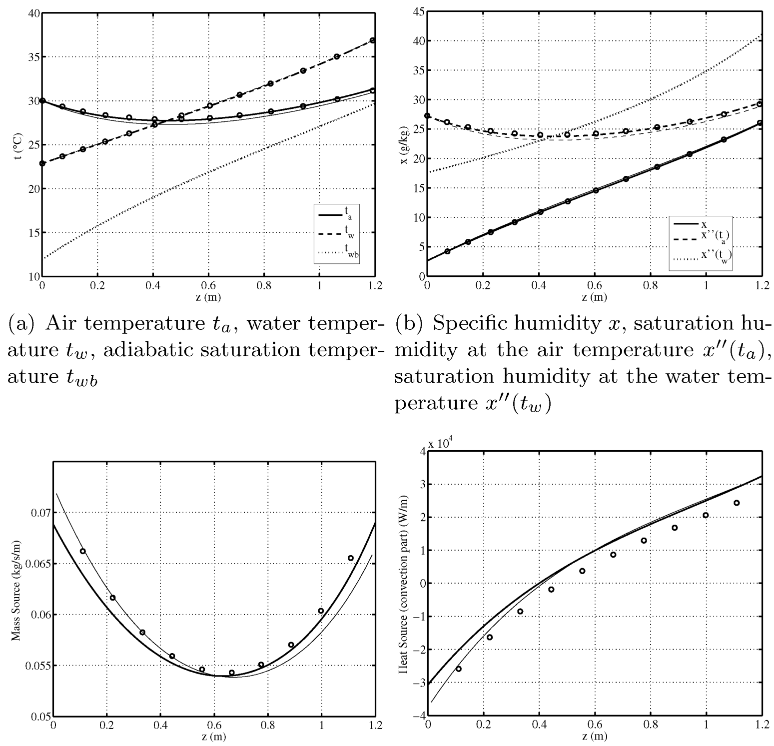
$$\frac{\mathrm{d}Me_2}{\mathrm{d}z} = \frac{\alpha_\mathrm{m}\, a_\mathrm{q}\, A_\mathrm{fr}}{\dot{m}_\mathrm{w}} \ . \tag{13}$$

We can adjust the set of parameters $\alpha_{\mathrm{m}}\, a_{\mathrm{q}}\, A_{\mathrm{fr}}$ until we reach experimentally obtained value of Me_2. It has been shown by [1] that the value of Me_2 calculated by using their model is practically equivalent with Merkel number calculated by using Poppe model. The Merkel number Me_2 has for about few percent higher value than classical Merkel number calculated using Merkel's model (5).

The second possibility is to calculate the outlet water temperature t_{wo} using Merkel's model and adjust $\alpha_{\mathrm{m}}\, a_{\mathrm{q}}\, A_{\mathrm{fr}}$ until we obtain prescribed inlet water temperature t_{wi}. This approach is probably most appropriate because the standard Merkel's model is almost exclusively used in the cooling tower industry and the characteristics of the fill are available as a function of Merkel's model Merkel number (5). Outlet mass flow rate can be adjusted using simple iteration $\dot{m}_{\mathrm{wo}} = \dot{m}_{\mathrm{wi}} - \dot{m}_{\mathrm{a}}\,(x_{\mathrm{o}} - x_{\mathrm{i}})$ and the product $\alpha_{\mathrm{m}}\, a_{\mathrm{q}}\, A_{\mathrm{fr}}$ can be adjusted using regula falsi method.

5. Results

The first test case was taken from the reference [1]. The calculation was performed for a fill of height $H = 1.2\,\mathrm{m}$ and cross-sectional area $A_{\mathrm{z}} = 1\,\mathrm{m}^2$. The air and water mass flow rates are equal to $\dot{m}_{\mathrm{a}} = \dot{m}_{\mathrm{w}} = 3.0\,\mathrm{kg/s}$. Inlet water temperature is $t_{\mathrm{wi}} = 37\,°\mathrm{C}$. Air inlet temperature is $t_{\mathrm{ai}} = 30\,°\mathrm{C}$ and specific humidity at air inlet is $x_{\mathrm{i}} = 2.62\,\mathrm{g/kg}$. These parameters correspond to hot and very dry atmospheric conditions. Reference [1] does not



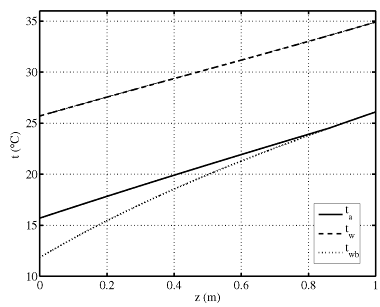
(a) Air temperature t_a, water temperature t_w, adiabatic saturation temperature t_{wb}

(b) Specific humidity x, saturation humidity at the air temperature $x''(t_a)$, saturation humidity at the water temperature $x''(t_w)$

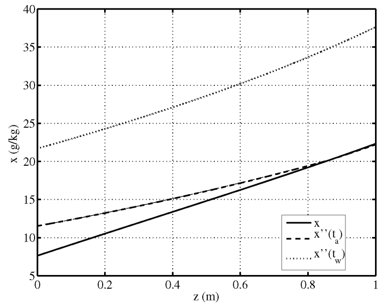
(c) Density of mass source

(d) Density of heat source

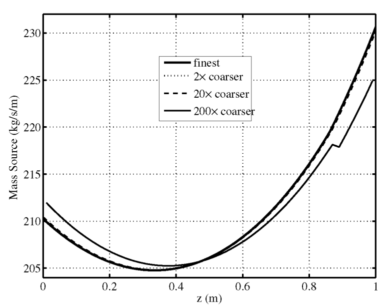
Fig.1: The first test case; solid lines correspond to calculation and circles to data from the reference; thin lines correspond to calculation on coarse grid and thick lines on finest grid



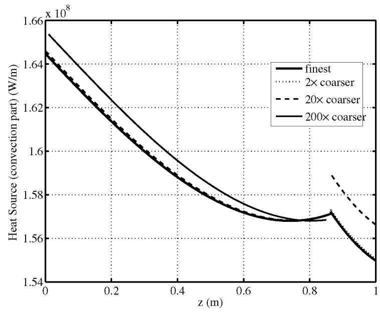
(a) Air temperature t_a, water temperature t_w, adiabatic saturation temperature t_{wb}



(b) Specific humidity x, saturation humidity at the air temperature $x''(t_a)$, saturation humidity at the water temperature $x''(t_w)$



(c) Density of mass source



(d) Density of heat source

Fig.2: Second test case

contain ambient pressure for this particular case but the value of standard atmospheric pressure $p = 101325\,\mathrm{Pa}$ looks relevant and is used.

Figure 1 shows results of numerical simulations and grid sensitivity study. There is an intersection between air temperature curve and water temperature curve. In the bottom half of the fill air temperature decreases and in the upper half slightly increases. The decrease of air temperature in the bottom part of the fill is also connected with the decrease of saturation humidity in the same part of the fill. Water temperature is monotonously decreasing due to the cooling process. Water is also cooled in the bottom part of the fill because the evaporative cooling dominates over convective heat transfer. Distributions of density of heat and mass sources confirms the intensity of evaporative cooling against the convective heat transfer. The negative value of the density of heat source in the bottom part of the fill corresponds to the air cooling in this fill part.

From figure 1 it is visible that the solution on coarse grid shows little differences against the fine grid and the reference solution [1]. There is a good agreement in the distribution of density of mass source with reference solution [1], where in the bottom part of the fill is better correspondence with the solution on coarse grid and in the upper part is better correspondence with solution on fine grid. Calculated distribution of the convective part of density of heat source is slightly overestimated against the reference data as shown in the figure 1.

The second test case was inspired by one case presented in [5]. The calculation was performed for a fill of height $H = 1\,\mathrm{m}$. The air inlet mass flow rate is equal to $\dot{m}_\mathrm{a} = 14333\,\mathrm{kg/s}$

and water inlet mass flow rate is $\dot{m}_\mathrm{w} = 17200\,\mathrm{kg/s}$. Inlet water temperature is $t_\mathrm{wi} = 34.9\,°\mathrm{C}$. Air inlet temperature is $t_\mathrm{ai} = 15.7\,°\mathrm{C}$ and specific humidity at air inlet is $x_\mathrm{i} = 7.622\,\mathrm{g/kg}$. Atmospheric pressure is $p = 98100\,\mathrm{Pa}$. Outlet water temperature is known from the reference $t_\mathrm{wo} = 25.7\,°\mathrm{C}$.

There are monotone changes in variables in the figure 2 and supersaturation in the upper part of the fill. Mesh independence of the solution is questionable, when the figure 2 is taken into consideration. The behavior which is possible to observe in the distribution of mass source density is natural. Solution on the coarse grid is slightly different against other solutions which are almost identical. A problem can be identified on the distribution of convective part of heat source. There is a discontinuity on coarse grids in the place where supersaturation starts. This discontinuity disappears on finer grids with ten thousand steps.

6. Conclusions

Results presented in this article exhibit slight differences from the reference solution. Differences are probably caused by different choice of saturated vapour pressure equation in references and in this article. The difference is also connected with using the more precise equation for calculation of specific humidity in this article. Previous two sentences are based on assumption that the work [1] is based on the same thermodynamics equations as are presented in the book [2] and this is not possible to recognize from their article.

The second test case is shown because of the problematic presence of discontinuity on the distribution of convective part of the density of heat source. It has been shown that the discontinuity is vanishing with the grid refinement. Unfortunately, the article [1] does not mention the distribution of the density of convective part of heat source for the case where supersaturation occurs. Experience gained by author indicates that presence of the discontinuity on the distribution of density of convective part of heat source is not connected with presence of numerical errors, which are connected with discretization of specific ODEs. This problem is connected with the change of the system of ODEs if supersaturation occurs.

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