CRITICAL BUCKLING STRESS OF PATCH LOADING

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Elastic critical buckling stress of the plate girder subjected to transverse force is calculated. The load is applied through the flange and resisted by shear forces in the web. Comparisons of values of buckling coefficient $k_{\sigma,\mathrm{b}} = f(\alpha,\beta)$ *(Tab. 2) obtained by 5 various authors: Petersen, von Berg, Ravinger, Kutzelnigg, Protte (Tab. 1) and calculated by computer program PLII (Tab. 2). Simply supported rectangular plate (a – length, b – width, t – thickness) without flanges and stiffeners is investigated for different aspect ratios* $\alpha = a/b = 1; 2; 3; 4; 5; 8; 10; 20; 30; 40$ *when subjected to transverse uniformly distributed partial load having relative load lengths* $\beta = c/a = 0.005$ *(single concentrated force)*; $0.2; 0.4; 0.6; 0.7; 0.8; 1$ *(uniformly distributed load along a). The values of buckling coefficient* $k_{\sigma,\mathrm{b}} = f(\alpha,\beta,\delta)$ *calculated by program PLII (Tab. 3). Parametrical study of simply supported rectangular plate without stiffeners with* $\alpha = a/b = 4; 5; 8; 10; 20; 30; 40$, $\beta = c/a = 0.005$ *(single concentrated force)*; $0.01; 0.05; 0.1; 0.2; 0.3; 0.4$ *and different relative normal flange rigidity* $\delta = A_f/(b\,t) = 0$ *(without flange)*; $0.3; 0.5; 1; 1.5; 2; 3$. *Torsional rigidity of the flanges is not taken into account.*

Keywords: *patch loading, elastic critical stress, buckling coefficients, parametrical study*

1. Introduction

The rules for the resistance of a web to the patch loading (Fig. 1) in the older editions of the modern codes [2], [4] use the same format as the buckling rules for plates subjected to normal force N, bending moment M or shear force V. In the design procedure it is necessary to calculate:

- the yield resistance in the form of the stress f_y [2], or the force F_y [4],
- the elastic buckling stress σ_cr [2], or the buckling force F_cr [4],
- the relative slenderness $\lambda = \sqrt{f_\mathrm{y}/f_\mathrm{cr}}$ [2], or $\lambda = \sqrt{F_\mathrm{y}/F_\mathrm{cr}}$ [4],
- the reduction factor $\kappa = f(\lambda)$ [2], or $\chi = f(\lambda)$ [4],
- the resistance of web to patch loading $\sigma_\mathrm{R} = \kappa\, f_\mathrm{y}$ [2], or $F_\mathrm{R} = \chi\, F_\mathrm{y}$ [4].

Despite of the formal similarity in the design procedures, there are important differences between the both codes [2] and [4] especially in the details of resistance to patch loading calculation. One of the most important step in a design procedure is calculating of the elastic buckling stress σ_cr.

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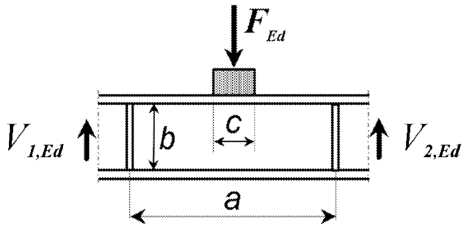

Fig.1: Notation of webplate dimension : a – length, b – width, t – thickness, c – length of loading

2. Elastic buckling stress

The elastic buckling force is written as

$$F_{\mathrm{cr}} = \sigma_{\mathrm{cr}}\, c\, t \tag{1}$$

where c is the length over which the applied transverse force is distributed, t the thickness of the plate, $\sigma_{\mathrm{cr}} = k_\sigma\, \sigma_{\mathrm{E}}$ the critical stress and the Euler critical stress are as follows

$$\sigma_{\mathrm{E}} = \frac{\pi^2 E}{12\,(1-\nu^2)\left(\dfrac{b}{t}\right)^2}\ , \quad \sigma_{\mathrm{E,steel}} = \frac{189\,800}{\left(\dfrac{b}{t}\right)^2}\ , \quad \sigma_{\mathrm{E,aluminium}} = \frac{63\,267}{\left(\dfrac{b}{t}\right)^2}\ , \quad [\mathrm{N/mm^2}] \tag{2}$$

b is the breadth of the plate or depth of the web, b/t the slenderness of the plate or the web, k_σ the buckling coefficient, E Young's modulus of elasticity (210 GPa for steel, 70 GPa for aluminium alloys), ν Poisson's ratio in elastic stage (0.3 for steel and aluminium alloys).

The buckling coefficient k_σ depends generally on the

- type of the action (also on the relative loading length $\beta = c/a$ in the case of transverse action),
- boundary conditions (simply supported plate is investigated in this paper),
- relative normal rigidity of the flange $\delta = A_{\mathrm{f}}/(b\,t)$, where A_{f} is the area of the flange, and torsional rigidity of the flanges (it is neglected in this study),
- longitudinal and/or transverse stiffeners locations and their rigidities,
- shape of the plate (e.g. on the aspect ratio of the plate $\alpha = a/b$, in the case of rectangular plate, where a is the investigated length of the plate – the distance of the transverse stiffeners),

The numerical values of the buckling coefficient k_σ may vary a lot and therefore sometimes for the purpose of diagrams the more convenient forms of the buckling coefficients $k_{\sigma,\mathrm{a}}$ and $k_{\sigma,\mathrm{b}}$ are used. For instance Petersen [9], von Berg [1] and Ravinger [12] use instead of the above defined buckling coefficient k_σ the buckling coefficient $k_{\sigma,\mathrm{a}}$:

$$F_{\mathrm{cr}} = \sigma_{\mathrm{cr}}\, c\, t = k_\sigma\, \sigma_{\mathrm{E}}\, c\, t = \left(k_\sigma\, \frac{c}{a}\right) \sigma_{\mathrm{E}}\, a\, t = k_{\sigma,\mathrm{a}}\, \sigma_{\mathrm{E}}\, a\, t\ . \tag{3}$$

Kutzelnigg [7] and Protte [11] use instead of the coefficient k_σ the buckling coefficient $k_{\sigma,\mathrm{b}}$:

$$F_{\mathrm{cr}} = \sigma_{\mathrm{cr}}\, c\, t = k_\sigma\, \sigma_{\mathrm{E}}\, c\, t = \left(k_\sigma\, \frac{c}{b}\right) \sigma_{\mathrm{E}}\, b\, t = k_{\sigma,\mathrm{b}}\, \sigma_{\mathrm{E}}\, b\, t\ . \tag{4}$$

The following formulae are valid

$$k_{\sigma,\mathrm{a}} = k_\sigma \, \frac{c}{a} = k_{\sigma,\mathrm{b}} \, \frac{b}{a} = \frac{k_{\sigma,\mathrm{b}}}{\alpha} = k_\sigma \, \beta \; , \tag{5}$$

$$k_{\sigma,\mathrm{b}} = k_\sigma \, \frac{c}{b} = k_{\sigma,\mathrm{a}} \, \frac{a}{b} = k_{\sigma,\mathrm{a}} \, \alpha = k_\sigma \, \alpha \, \beta \; . \tag{6}$$

There are many publications, where the values of the buckling coefficients may be found. Some of them are mentioned in Tab. 1 together with the ranges of dimensionless parameters for which the values of buckling coefficient were calculated.

	Petersen [9] (1993)	von Berg [1] (1989)	Ravinger [12] (1979)	Kutzelnigg [7] (1982)	Protte [11] (1994)
Type of k_σ	$k_{\sigma,\mathrm{a}}$			$k_{\sigma,\mathrm{b}}$	
$\alpha = a/b$	0.3–35	0.7–(≥ 10)	1–3	0.5–5	2–10
$\beta = c/a$	0–1	0–1	0, 1	0–1	0–1
$\delta = A_\mathrm{f}/(b\,t)$	0	0	0	0	0, 0.1, 0.3, 0.5, 1
Remarks	simply supported plate (s.s. plate)	simply supported plate $k_\sigma(\alpha \geq 10) = k_\sigma(\alpha = 10)$	4 various boundary conditions incl. s.s. plate	s.s. plate, 0 or 1 or 2 longitudinal stiffeners	simply supported plate

Tab.1: The way of calculation of the buckling coefficient values according to various authors

The numerical values of the buckling coefficient $k_{\sigma,\mathrm{b}}$ computed by the program PLII [10] for the patch loading, simply supported rectangular plate with no flanges ($\delta = 0$) and no stiffeners are given in Tab. 2. They are compared with the results of the authors from Tab. 1. The comparison of the results given in Tab. 2 leads to the following conclusions:

- for the small aspect ratios ($\alpha < 4$) the differences among the values of all authors are negligible ($\leq 10\,\%$),
- the agreement between PLII's [10] and Protte's results [11] is excellent in the whole Tab. 2,
- the greatest difference between Protte's and Kutzelnigg's results in Tab. 2 is 60 % for the case $\alpha = 5$, $\beta = 1$. The reason why the results of Protte [11] and Kutzelnigg [7] differ was explained in Ravinger [12] (p. 34, paragraph 4.1). The reason is, that Kutzelnigg took into account only the influence of the vertical normal stresses σ_y. The influences of the stresses σ_x and τ were neglected in his buckling coefficient calculations. The differences are the greater the greater is aspect ratio α, because with increasing α, the influence of so called beam stresses σ_x on the value of buckling coefficient increases,
- the buckling coefficients for the very long plates ($\alpha > 10$), a case which may be important in the design of the crane runway girders without intermittent transverse stiffeners, can be found only in Petersen [9] and Berg [1]. Ravinger [12] (p. 421), reported problems in finding the minimum value of buckling coefficients for the cases with aspect ratios $\alpha > 3$. Petersen's and von Berg's results are based (see [9] and [1]) on the older Protte's publications. They are each other in good agreement except the cases $\beta = 0$, $\alpha \geq 8$. They differ a lot from PLII's [10] and Protte's [11] results for $\alpha \geq 5$. The difference is the greater the greater is the aspect ratio α. For the

$a = a/b$	Author	$\beta = c/a$						
		0.005	0.2	0.4	0.6	0.7	0.8	1
1	PLII [10] C	3.32	3.42	3.74	4.24	4.55	4.91	5.67
	Kutzelnigg [7] D	3.00	3.30	3.60	4.30	4.70	5.20	6.25
	Petersen [9] D	3.24	3.45	3.76	4.26	4.54	5.04	6.09
	von Berg [1] T	3.20	3.40	3.70	4.20	–	5.00	6.20
	Ravinger [12] D	3.50	3.55	3.79	4.25	4.55	4.90	5.70
2	PLII [10] C	2.37	2.54	2.91	3.46	3.79	4.17	5.09
	Protte [11] D	2.35	2.50	2.90	3.45	3.80	4.20	5.08
	Kutzelnigg [7] D	2.26	2.54	2.9	3.65	4.10	4.50	5.45
	Petersen [9] D	2.30	2.52	2.82	3.31	3.66	4.14	5.21
	von Berg [1] T	2.36	2.60	2.90	3.40	–	4.40	5.10
	Ravinger [12] D	2.40	2.60	3.00	3.64	4.00	4.42	5.28
3	PLII [10] C	2.23	2.54	3.21	4.05	4.52	5.02	6.16
	Protte [11] D	2.21	2.51	3.24	4.05	4.50	5.05	6.13
	Kutzelnigg [7] D	2.20	2.64	3.44	4.40	5.00	5.48	6.68
	Petersen [9] D	2.14	2.61	3.18	3.96	4.47	5.05	6.39
	von Berg [1] T	2.25	2.40	2.94	3.90	–	5.07	6.30
	Ravinger [12] D	2.40	2.70	3.27	4.05	4.50	5.07	6.30
4	PLII [10] C	2.10	2.58	3.58	4.83	5.47	6.02	7.30
	Protte [11] D	2.07	2.55	3.55	4.80	5.46	6.00	7.26
	Kutzelnigg [7] D	2.30	2.90	4.00	5.40	6.00	6.76	8.36
	Petersen [9] D	2.04	2.87	3.72	4.85	5.51	6.22	7.51
	von Berg [1] T	2.20	2.48	3.40	4.60	–	6.20	7.56
5	PLII [10] C	1.98	2.61	3.70	4.38	4.76	5.18	6.23
	Protte [11] D	1.94	2.60	3.75	4.45	4.76	5.20	6.22
	Kutzelnigg [7] D	2.39	3.22	4.65	6.40	7.20	8.10	9.95
	Petersen [9] D	2.00	3.31	4.44	5.91	6.71	8.56	9.83
	von Berg [1] T	2.35	2.70	4.00	5.40	–	7.40	9.15
8	PLII [10] C	1.62	2.17	2.50	2.88	3.10	3.37	4.04
	Protte [11] D	1.58	2.20	2.50	2.90	3.13	3.40	4.10
	Petersen [9] D	2.00	4.46	6.54	8.81	10.16	11.48	14.28
	von Berg [1] T	3.20	3.68	5.44	7.92	–	10.56	13.92
10	PLII [10] C	1.42	1.78	2.02	2.32	2.50	2.71	3.25
	Protte [11] D	1.38	1.80	2.05	2.35	2.55	2.75	3.30
	Petersen [9] D	2.00	5.19	7.90	10.65	12.65	13.82	17.41
	von Berg [1] T	3.60	4.20	6.20	9.50	-	12.70	17.00
20	PLII [10] C	0.81	0.90	1.01	1.14	1.23	1.34	1.60
	Petersen [9] D	2.00	8.49	14.41	20.17	23.86	27.18	33.67
	von Berg [1] T	7.20	8.40	12.40	19.00	-	25.40	34.00
30	PLII [10] C	0.54	0.59	0.63	0.76	0.82	0.89	1.06
	Petersen [9] D	2.00	11.04	20.55	30.25	35.49	40.60	51.15
	von Berg [1] T	10.80	12.60	18.60	28.50	–	38.10	51.00
40	PLII [10] C	0.39	0.45	0.50	0.54	0.58	0.62	0.76
	Petersen [9] D	2.00	14.11	26.49	41.19	46.92	53.00	66.79
	von Berg [1] T	14.40	16.80	24.80	38.00	–	50.80	68.00

Numerical values were: C – computed, D – taken from a diagram, T – taken from a table

Tab.2: Comparison of the values of the buckling coefficients $k_{\sigma,\mathrm{b}} = f(\alpha,\beta)$; patch loading, simply supported rectangular plate, no flanges ($\delta = 0$), no stiffeners case $\alpha = 5$ the Petersen's [9] and von Berg's [1] results does not differ a lot from Kutzelnigg's [7] ones, which are, as it is explained above, not correct. It is therefore believed that Petersen's [9] and von Berg's [1] results for the long plates ($\alpha > 5$) are not correct too and they are, comparing with PLII's [10] and Protte's [11] results, on the unsafe side. Note: PLII gives k_σ-values, from which $k_{\sigma,\mathrm{b}}$-values were calculated according to (6).

The influence of the relative normal rigidity of the flanges $\delta = A_\mathrm{f}/(b\,t)$ on the buckling coefficient was investigated by Protte [11]. The results of program PLII [10] are in excellent agreement with Protte's [11] ones also in this case. The part of a large parametrical study is shown in Tab. 3 and Fig. 2. It may be concluded from the results:

- the influence of the flange normal rigidity δ on the buckling coefficient is negligible in the range $\alpha \leq 4$. Maximum difference is $< 25\,\%$ for the $\beta = 0.005$, $\alpha = 4$,
- for the cases with $\alpha \leq 4$ and any δ, we can use the values of buckling coefficients computed for $\delta = 0$, being slightly on the safe side,
- for the longer plates ($\alpha > 4$) is the influence of the δ on the increasing of the buckling coefficient the greater the greater is aspect ratio α.

Program PLII [10] takes into account the influence of the relative normal $\delta = A_\mathrm{f}/(b\,t)$ and bending rigidity $\gamma = I_\mathrm{f}/(b\,t^3/12)$ of the flange, but it could take into account also torsional flange rigidity, which has greater effect on the increasing of the buckling coefficient. The various boundary conditions may be taken into account by program PLII too.

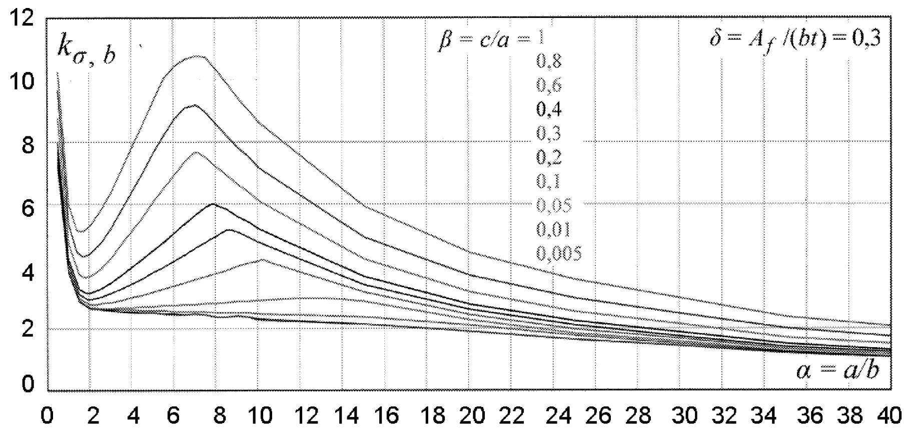

Fig.2: Buckling coefficients $k_{\sigma,\mathrm{b}} = f(\alpha, \beta, \delta = 0.3)$ computed by the program PLII [10] for aspect ratios $0.5 \leq \alpha = a/b \leq 40$, for $0.005 \leq \beta = c/a \leq 1$ and for relative normal rigidity of the flange $\delta = A_\mathrm{f}/(b\,t) = 0.3$; torsional rigidity of the flanges was not taken into account

The more accurate values k_F, which were calculated for more complex boundary conditions and were calibrated with numerous experiments, may be found in Lagerqvist [8]. These expressions were simplified to formula

$$k_\mathrm{F} = 6 + \frac{2}{\alpha^2}\ , \tag{7}$$

which is used in [4], [5] and [15] in completely different design procedure as it was done in [2]. The values computed according to the formula (7) do not differ a lot from the values

		$\beta = c/a$						
$\alpha = a/b$	$\delta = A_{\mathrm{f}}/(b\,t)$	0.005	0.01	0.05	0.1	0.2	0.3	0.4
4	0	2.098	2.104	2.149	2.246	2.577	3.036	3.581
	0.3	2.497	2.504	2.550	2.644	2.966	3.411	3.935
	0.5	2.552	2.559	2.605	2.697	3.013	3.452	3.971
	1.0	2.607	2.612	2.656	2.744	3.056	3.484	3.996
5	0	1.972	1.979	2.037	2.170	2.610	3.194	3.700
	0.3	2.460	2.470	2.531	2.673	3.139	3.762	4.479
	0.5	2.532	2.540	2.604	2.742	3.201	3.817	4.526
	1.0	2.599	2.610	2.670	2.800	3.250	3.860	4.560
	1.5	2.629	2.636	2.693	2.825	3.270	3.873	4.571
	2.0	2.648	2.655	2.709	2.839	3.280	3.880	4.575
	3.0	2.683	2.688	2.738	2.863	3.295	3.888	4.581
8	0	1.606	1.615	1.697	1.884	2.172	2.338	2.501
	0.3	2.329	2.345	2.473	2.781	3.736	4.931	5.950
	0.5	2.451	2.467	2.598	2.908	3.865	5.050	6.298
	1.0	2.566	2.584	2.712	3.016	3.960	5.136	6.384
	1.5	2.612	2.626	2.750	3.053	3.990	5.156	6.406
	2.0	2.641	2.654	2.774	3.072	4.003	5.166	6.416
	3.0	2.683	2.695	2.807	3.098	4.018	5.176	6.426
10	0	1.427	1.427	1.496	1.647	1.781	1.896	2.021
	0.3	2.288	2.288	2.437	2.863	4.143	4.743	5.176
	0.5	2.449	2.449	2.604	3.046	4.368	5.871	6.826
	1.0	2.576	2.600	2.750	3.190	4.510	6.050	7.590
	1.5	2.651	2.651	2.804	3.242	4.546	6.084	7.647
	2.0	2.682	2.682	2.832	3.266	4.563	6.099	7.670
	3.0	2.725	2.725	2.870	3.294	4.579	6.112	7.689
20	0	0.812	0.812	0.835	0.856	0.902	0.954	1.013
	0.3	1.826	1.826	2.078	2.254	2.434	2.600	2.774
	0.5	2.162	2.162	2.574	3.074	3.386	3.646	3.906
	1.0	2.468	2.508	3.020	4.300	5.500	6.060	6.560
	1.5	2.634	2.634	3.166	4.462	7.074	7.890	8.562
	2.0	2.700	2.700	3.232	4.526	7.532	8.780	9.362
	3.0	2.772	2.772	3.296	4.578	7.676	9.488	9.988
30	0	0.545	0.545	0.553	0.565	0.595	0.629	0.667
	0.3	1.402	1.402	1.478	1.534	1.631	1.732	1.842
	0.5	1.832	1.833	2.053	2.150	2.305	2.454	2.614
	1.0	2.339	2.400	3.210	3.570	3.903	4.200	4.500
	1.5	2.619	2.620	3.629	4.827	5.430	5.889	6.330
	2.0	2.726	2.728	3.771	5.799	6.771	7.434	8.070
	3.0	2.834	2.836	3.885	6.132	8.460	9.102	9.612
40	0	0.408	0.408	0.414	0.423	0.445	0.470	0.500
	0.3	1.092	1.096	1.125	1.160	1.229	1.302	1.383
	0.5	1.487	1.500	1.578	1.638	1.744	1.850	1.968
	1.0	2.135	2.200	2.632	2.788	3.000	3.204	3.444
	1.5	2.527	2.436	3.558	3.867	4.224	4.524	4.832
	2.0	2.590	2.692	4.264	4.856	5.392	5.812	6.228
	3.0	2.748	2.854	4.512	6.552	7.384	8.040	8.696

Tab.3: Buckling coefficients $k_{\sigma,\mathrm{b}} = f(\alpha,\beta,\delta)$ computed by the program PLII [10]; patch loading with relative loading length β, simply supported rectangular plate having aspect ratio α, flanges with relative normal rigidity δ; torsional rigidity of the flanges was not taken into account

of buckling coefficient computed for the plate with upper edge fixed in loaded flange when the load is applied on the short relative length β (compare them for instance with the results for $\beta = 0$ in Ravinger [12], p. 410) and are not comparable with $k_{\sigma,\mathrm{b}}$ values given in this paper.

As usually, of course, one cannot mix the parts of design procedures taken from the different codes. This rule is valid also for this topic and the codes [2] and [4, 5, 15]. In the latter codes, which use the formula (7), the design procedure was calibrated with the numerous experimental values given in Lagerqvist [8]. See also Johansson et al [14].

Acknowledgements

Project No. 1/0748/13 was supported by the Slovak Grant Agency VEGA.

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Received in editor's office: July 21, 2013
Approved for publishing: January 3, 2014