STABILITY INVESTIGATIONS OF ROTORS MOUNTED ON HYBRID MAGNETORHEOLOGICAL DAMPERS BY THE EVOLUTIVE METHOD

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A frequently used technological solution of reducing the time varying forces transmitted between the rotor and its casing is represented by application of a flexible suspension with damping devices added to the constraint elements. To achieve their optimum performance their damping effect must be controllable. For this purpose a concept of a hybrid damping device working on the principle of squeezing the layers of normal and magnetorheological oils have been developed. Here in this article, there is investigated influence of the proposed damping element on stability of the rotor vibrations induced by its imbalance during the steady state operating regimes. The stability is assessed by the evolutive method which is based on evaluation of eigenvalues of the system linearized in a small neighbourhood of the rotor phase trajectory and calculation of the corresponding damping ratio or logarithmic decrement.

Keywords: *rigid rotors, controllable damping, hybrid magnetorheological dampers, vibration stability, evolutive method*

1. Introduction

Unbalance of rotating parts is a source of time varying forces transmitted between the rotor and its casing. A frequently used technological solution of reducing their magnitudes consists in application of a flexible suspension with added damping devices. Results of the theoretical analysis show that to minimize the force transmission between the rotor and its stationary part the damping effect in the rotor supports must be high for lower rotor angular velocities and as low as possible for velocities higher than approximately the critical speed. This can be achieved by application of controllable magnetorheological damping devices.

Zapoměl et al. [1] developed a mathematical model of a compact short squeeze film damper lubricated with magnetorheological liquid which is applicable for analysis of both the steady state and transient rotor vibrations. Here is investigated behaviour of a rigid rotor supported by hybrid magnetorheological squeeze film dampers with serial arrangement of the damping layers. These devices make it possible to reach lower damping effect at higher rotational speeds than their compact variants as discussed in Zapoměl et al. [2]. The principle objectives of the study are to analyse influence of the proposed damping devices on stability of the rotor vibrations. Results of the computational simulations are evaluated by means of the approach based on application of the evolutive method.

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2. Mathematical model of the proposed rotor support element

The principal parts of the studied damping element (Fig. 1) are three concentric rings mutually separated by clearances filled with normal (inner) and magnetorheological (outer) lubricating oils. The inner ring is coupled with the rotor journal by a rolling element bearing and with the damper's body by a squirrel spring. The outer ring is stationary, fixed to the damper's housing. The lubricating films are mutually separated by the middle ring which is coupled with the damper's housing by a soft spring element. From the physical point of view the lubricating films are in a serial arrangement. The rotor lateral oscillation squeezes the oil films which produces the damping effect. The damper is equipped with an electric coil generating magnetic flux passing through the layer of the magnetorheological liquid. As resistance against its flow depends on magnetic induction, the change of the applied electric current can be used to control the damping force.

The developed mathematical model of the studied damping element is based on utilization of the classical theory of lubrication but with some modifications. The normal oil is considered as Newtonian. The magnetorheological oil is represented by Bingham material whose yielding shear stress depends on magnetic induction. If no magnetic field is applied, the magnetorheological lubricant behaves as normal Newtonian fluid. Further, it is assumed that the geometric and design parameters of the constraint element enable to considered it as short (length to diameter ratio of the rings is small, no or soft sealings are applied at the damper's ends).

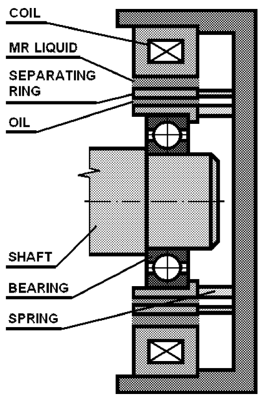

Fig.1: Scheme of the studied damping device

The pressure distribution in the individual oil layers is governed by Reynolds equations, from which those referred to the magnetorheological lubricant was modified for Bingham material. The details on their derivation can be found in Krämer [3] and Zapoměl et al. [1]

$$\frac{\partial^2 p_{\mathrm{CO}}}{\partial Z^2} = \frac{12\,\eta}{h_{\mathrm{CO}}^3}\,\dot{h}_{\mathrm{CO}} \ , \tag{1}$$

$$h_{\mathrm{MR}}^3\, p_{\mathrm{MR}}'^3 + 3\,(h_{\mathrm{MR}}^2\,\tau_{\mathrm{y}} - 4\,\eta_{\mathrm{B}}\,\dot{h}_{\mathrm{MR}}\,Z)\,p_{\mathrm{MR}}'^2 - 4\,\tau_{\mathrm{y}}^3 = 0 \quad \text{for} \quad p_{\mathrm{MR}}' < 0 \ , \quad Z > 0 \ . \tag{2}$$

p_{CO}, p_{MR} denote the pressure in the layers of the normal and magnetorheological oils, η is the normal oil dynamic viscosity, η_{B} is the Bingham viscosity, τ_{y} represents the yield shear stress, h_{CO}, h_{MR} are thicknesses of the classical and magnetorheological oil films, Z is the axial coordinate of points in the lubricating films with the origin in the dampers' middle plane perpendicular to the shaft axis and $(\dot{\ })$, $(')$ denote the first derivatives with respect to time and coordinate Z.

Solving the modified Reynolds equation (2) yields the pressure gradient in the magnetorheological oil film, which after integration gives the pressure profile in the axial direction

$$p_{\mathrm{MR}} = \int p'_{\mathrm{MR}}\,\mathrm{d}Z\ . \tag{3}$$

The boundary conditions needed for calculation of the constants of integration of the Reynolds equation (1) and integral (3) express that the pressure at the damper's ends is equal to the pressure in the ambient space. More details on solving equation (2), determination of the yielding shear stress

$$\tau_{\mathrm{y}} = k_{\mathrm{d}} \left(\frac{I}{h_{\mathrm{MR}}}\right)^{n_{\mathrm{y}}} \tag{4}$$

and calculation of the thickness of the lubricating films at position given by the circumferential angular coordinate φ can be found in Zapoměl et al. [1], [4] and Krämer [3]. n_{y} is the material constant of the magnetorheological liquid, k_{d} is the design parameter depending on the number of the coil turns and material properties of the magnetorheological liquid and I is the applied electric current.

In areas where the thickness of the lubricating films rises with time ($\dot{h}_{\mathrm{CO}} > 0$, $\dot{h}_{\mathrm{MR}} > 0$) a cavitation is assumed. In the developed mathematical model pressure of the medium in cavitated regions remains constant and equal to the pressure in the ambient space. In noncavitated areas the pressure distribution is governed by solutions of the Reynolds equation (1) and integral (3) and the pressure gradient in the axial direction is always negative for $Z > 0$.

Consequently, components of the damping forces are calculated by integration of the pressure distributions around the circumference and along the length of the damping device taking into account the cavitation in the oil films.

$$F_{\mathrm{MRy}} = -2\,R_{\mathrm{MR}} \int_0^{2\pi}\!\!\int_0^{\frac{L}{2}} p_{\mathrm{DMR}}\,\cos\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi\ , \quad F_{\mathrm{MRz}} = -2\,R_{\mathrm{MR}} \int_0^{2\pi}\!\!\int_0^{\frac{L}{2}} p_{\mathrm{DMR}}\,\sin\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi\ , \tag{5}$$

$$F_{\mathrm{COy}} = -2\,R_{\mathrm{CO}} \int_0^{2\pi}\!\!\int_0^{\frac{L}{2}} p_{\mathrm{DCO}}\,\cos\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi\ , \quad F_{\mathrm{COz}} = -2\,R_{\mathrm{CO}} \int_0^{2\pi}\!\!\int_0^{\frac{L}{2}} p_{\mathrm{DCO}}\,\sin\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi\ , \tag{6}$$

F_{COy}, F_{COz}, F_{MRy}, F_{MRz} are the y and z components of the hydraulic forces produced by the layers of normal and magnetorheological oils respectively, R_{CO}, R_{MR} are the mean radii of the layers of normal and magnetorheological oils, L is the axial length of the damping element and p_{DCO}, p_{DMR} denote the pressure distributions (taking into account different pressures in cavitated and noncavitated regions) in the layers of normal and magnetorheological oils.

3. Stability assessment of the investigated rigid rotor

The investigated rotor (Fig. 2) consists of a shaft and of one disc and is coupled with the frame by the studied constraint elements at both its ends. It turns at constant angular speed and is loaded by its weight and excited by the disc unbalance. The squirrel springs of the damping elements can be prestressed to be eliminated their deflection caused by the rotor weight. The whole system is symmetric relative to the disc middle plane. In the computational model the rotor is considered as absolutely rigid and the investigated constraint devices are represented by springs and force couplings. The task was to analyse stability of the rotor vibrations for different magnitudes of the applied current.

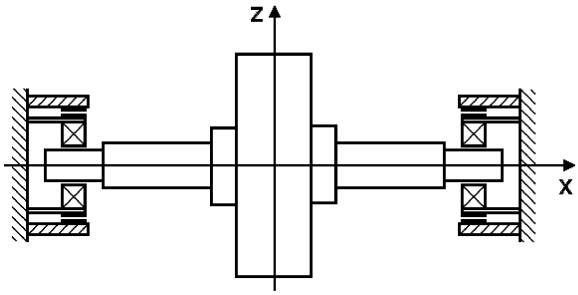

Fig.2: Scheme of the investigated rotor system

The motion equation of the analysed rotor system takes the form

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{B}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} = \mathbf{f}_{\mathrm{A}} + \mathbf{f}_{\mathrm{H}}(\mathbf{x}, \dot{\mathbf{x}}) \tag{7}$$

where $\mathbf{M}$, $\mathbf{B}$, $\mathbf{K}$ are the mass, damping and stiffness matrices, $\mathbf{f}_{\mathrm{A}}$, $\mathbf{f}_{\mathrm{H}}$ are the vectors of applied (unbalance, gravity, prestress) and hydraulic forces, $\mathbf{x}$ is the vector of displacements and $(\ddot{\ })$ denotes the second derivative with respect to time. As the steady state solution of (7) is assumed to be periodic with the period equal to the period of the rotor rotation, a trigonometric collocation method was applied for its determination. More details on its application can be found e.g. in Zapoměl [5], [6] or Zhao et al. [7].

The approach to assessing the vibration stability is based on utilization of the evolutive method. This requires to linearize the nonlinear term represented by the vector of hydraulic forces in the equation of motion (7) in the neighbourhood of the rotor phase trajectory by means of its expansion to the Taylor series neglecting the terms of higher orders

$$\mathbf{f}_{\mathrm{H}}(\mathbf{x} + \Delta\mathbf{x}, \dot{\mathbf{x}} + \Delta\dot{\mathbf{x}}) = \mathbf{f}_{\mathrm{H}}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{D}_{\mathrm{B}}(\mathbf{x}, \dot{\mathbf{x}})\,\Delta\dot{\mathbf{x}} + \mathbf{D}_{\mathrm{K}}(\mathbf{x}, \dot{\mathbf{x}})\,\Delta\mathbf{x}\ . \tag{8}$$

$\mathbf{D}_{\mathrm{B}}$, $\mathbf{D}_{\mathrm{K}}$ are the Jacobi matrices of partial derivatives with respect to velocities and displacements and $\Delta\mathbf{x}$ is the vector of the displacements deviations.

Then substitution of (8) in (7) and expressing the vector of displacements $\mathbf{y}$ of the disturbed vibration as

$$\mathbf{y} = \mathbf{x} + \Delta\mathbf{x} \tag{9}$$

give after some manipulations the equation of motion of the linearized system

$$\mathbf{M}\,\ddot{\mathbf{y}} + [\mathbf{B} - \mathbf{D}_{\mathrm{B}}(\mathbf{x}, \dot{\mathbf{x}})]\,\dot{\mathbf{y}} + [\mathbf{K} - \mathbf{D}_{\mathrm{K}}(\mathbf{x}, \dot{\mathbf{x}})]\,\mathbf{y} = \mathbf{f}_{\mathrm{A}} + \mathbf{f}_{\mathrm{H}}(\mathbf{x}, \dot{\mathbf{x}}) - \mathbf{D}_{\mathrm{B}}(\mathbf{x}, \dot{\mathbf{x}})\,\dot{\mathbf{x}} - \mathbf{D}_{\mathrm{K}}(\mathbf{x}, \dot{\mathbf{x}})\,\mathbf{x}\ . \tag{10}$$

If all its eigenvalues have only negative real parts at any moment of time, then the vibration is stable.

The eigenvalues λ are calculated by solving the characteristic equation

$$|\lambda^2\,\mathbf{M} + \lambda\,(\mathbf{B} - \mathbf{D}_\mathrm{B}) + \mathbf{K} - \mathbf{D}_\mathrm{K}| = 0\ . \tag{11}$$

Because $\mathbf{D}_\mathrm{B}$, $\mathbf{D}_\mathrm{K}$ are functions of displacements and velocities of the undisturbed vibration, the calculated eigenvalues λ are time dependent. The stability rate can be then characterized by a number of damping parameters such as the damping ratio, logarithmic decrement, etc. which can be determined from the real and imaginary parts of the system eigenvalues.

4. Results of the computational simulations

The technological parameters of the investigated rotor system are: mass of the rotor 450 kg, stiffness of the squirrel spring 5 MN/m, length of the constraint elements 60 mm, widths of the clearances filled by the normal and magnetorheological oils 0.2 mm, 1.0 mm, middle radii of the layers of the normal and magnetorheological oils 55 mm, 75 mm, dynamical and Bingham viscosities of the normal and magnetorheological oils 0.004 Pa s, 0.3 Pa s, eccentricity of the rotor centre of gravity 0.1 mm, exponential material constant of the magnetorheological oil 2 and the design parameter 0.001 N/A^2. The force prestressing the squirrel springs is not sufficiently high. Its magnitude reaches only 70 % of those needed for the complete compensation of the springs deflection caused by the rotor weight.

The steady state orbits of the centres of the rotor journal and of the separating ring for three magnitudes of the current (0.0 A, 0.7 A, 1.4 A) are drawn in Fig. 3. The results show that due to the insufficient prestressing of the squirrel springs the orbits are slightly non-circular and shifted in the vertical direction and that the rising current reduces their size.

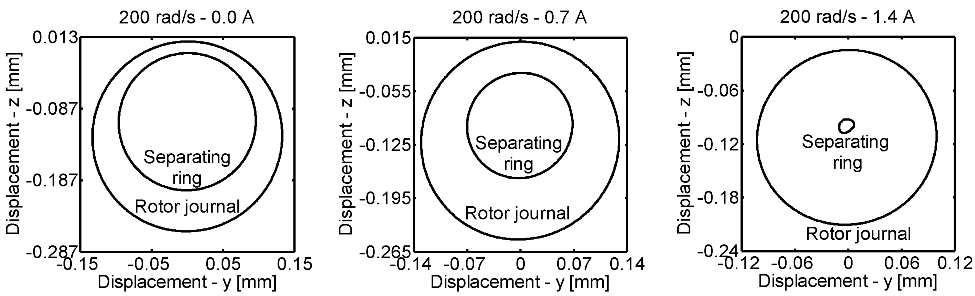

Fig.3: Orbits of the rotor journal and the separating ring

Analysis of the rotor steady state vibrations by means of the evolutive method gives the time histories of eigenvalues of the system linearized in the neighbourhood of the phase trajectory. The results depicted in Fig. 4 show that the real parts of all of them are negative which implies that the rotor oscillations are stable.

In Figs. 5, 6 and 7 there are drawn the time courses of the damped natural frequencies and corresponding damping ratios that belong to two partial motions having oscillatory character during the span of time of two periods. It is evident that their values are not constant. This is caused by insufficient prestress of the squirrel springs leading to the shift of the journal centres in the constraint elements and consequently to anisotropical properties of the lubricating layers.

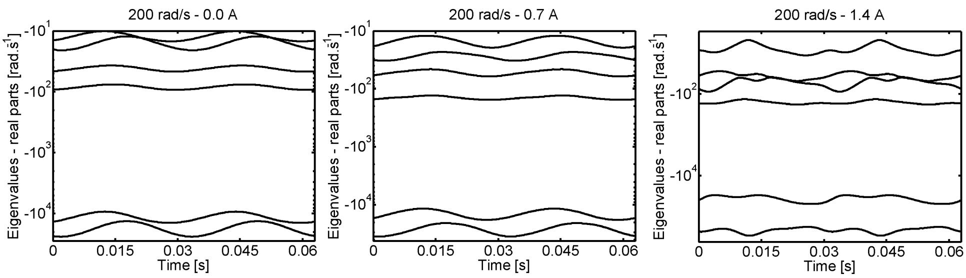

Fig.4: Time evolution of the real parts of the linearized system eigenvalues

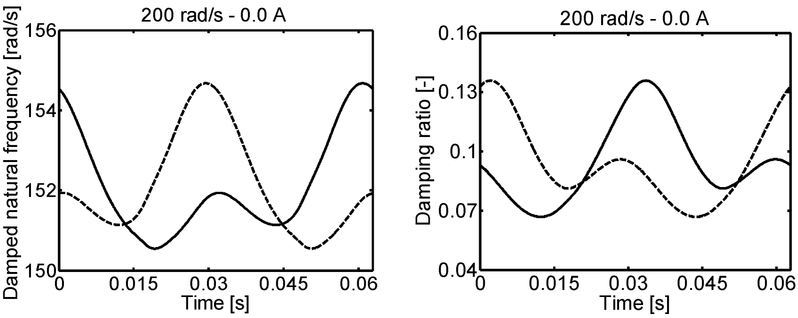

Fig.5: Time evolution of the system frequency and damping parameters (current 0.0 A)

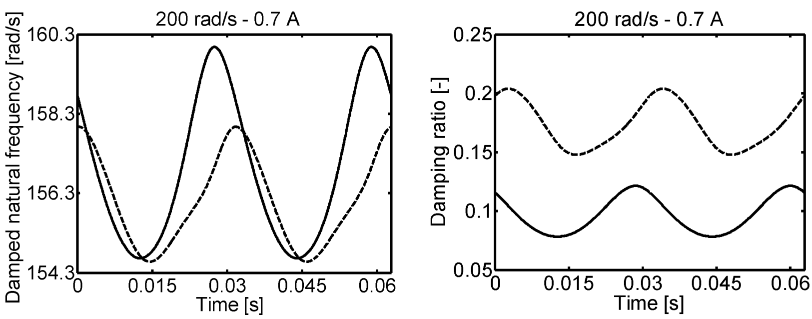

Fig.6: Time evolution of the system frequency and damping parameters (current 0.7 A)

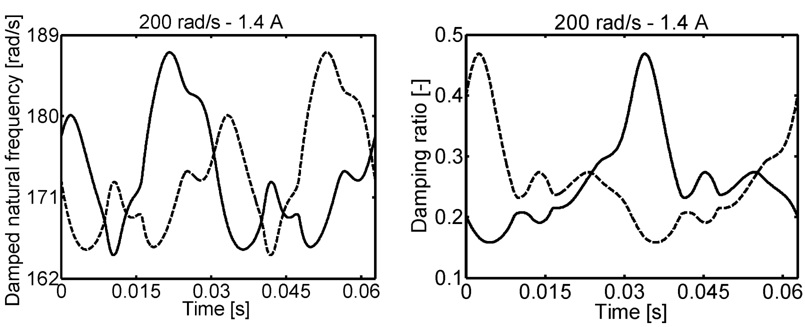

Fig.7: Time evolution of the system frequency and damping parameters (current 1.4 A)

Based on the rotor design parameters the undamped natural frequency of the system is 149 rad/s. As evident from Figs. 5–7, presence of the damping devices increases the natural frequencies which implies that the lubricating films are not only the sources of dissipation of mechanical energy but also they contribute to rising the system stiffness. The results also show that the rising current increases magnitude of the damping ratio which implies it elevates the system stability rate.

5. Conclusions

The carried out computational simulations show that the studied magnetorheological damping element effects stability of the rotor response in dependence on operating conditions. Stability of the induced oscillations are evaluated by utilization of the evolutive method which is based on determination of the time course and changes of the natural frequencies of the rotor system linearized in the small neighbourhood of its undisturbed phase trajectory. The stability rate which expresses how far from the stability limit the system finds itself is possible to be evaluated from the modal and spectral parameters such as the damping ratio, logarithmic decrement, etc.

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