MODELING OF THE WEDGE SPLITTING TEST USING AN EXTENDED CRACKED HINGE MODEL

Tomáš Pail, Petr Frantík*

The present paper describes a semi-analytical fracture model based on the cracked hinge approach by Ulfkjær [1]. Some extensions of the original formulation are introduced and also implemented (as JAVA code) to enable the use of any softening function with arbitrary shape for the cracked part of the model, which is considered as a fictitious (cohesive) crack. The application of the model to the wedge-splitting test (WST) is validated, showing the consistency of the adopted formulations with reference data. Furthermore, the capability of the model to integrate various softening curves is verified using FEM simulations.

Keywords: *hinge model, fracture, concrete, tensile softening, fracture energy*

1. Introduction

Nowadays, one of the main approaches commonly applied for the description of the fracture behavior of quasi-brittle materials such as concrete and other cement-based composites is the fictitious crack model (FCM) [2]. The concept of FCM recognizes the experimentally observed cohesive character of crack propagation which these materials exhibit – as so-called material softening – due to microcracking and other related processes that occur ahead of a traction-free crack. The fracture energy dissipated over fracture process zone is approximated there via a particular closing cohesive stress σ_{w} applied to the fracture surface of a fictitious extension of a real crack (called a 'fictitious crack'). The σ_{w} is non-constant along the fictitious crack length and is described as a unique function of the crack opening w by the following formula: $\sigma_{\mathrm{w}} = f(w)$. Function $f(w)$ is termed the softening function and generally expresses the fracture energy G_{F} required to create and fully break a unit surface of the fictitious (cohesive) crack [3].

The popularity and prevalence of the FCM is generally derives from its simple implementation within the framework of the finite element method (FEM). However, the practical use of such an implementation is limited by the amount of trustworthy knowledge regarding the $f(w)$ considered there as the material input. Among the indirect methods of estimating a softening curve from standardized fracture tests – the three-point bending test or the wedge splitting test – the inverse analysis approach plays the key role. In the inverse analysis procedure, a set of parameters of function $f(w)$ is provided as a seed for an iterative process, and a numerical or analytical model is used to determine a corresponding load-displacement (P–d) or load-crack mouth opening displacement (P–CMOD) curve, which is compared with the referenced curve obtained from a laboratory test.

* Ing. T. Pail, Ing. P. Frantík, Ph.D., Institute of Structural Mechanics, Brno University of Technology, Veveří 95, 602 00 Brno, Czech Republic

Since a chosen model is calculated many times during inverse analysis procedures, simplified analytical or semi-analytical models seem to be more adequate for this purpose than available academic or commercial FEM tools, despite the expected loss of accuracy. In the present paper, the author's implementation of the semi-analytical model based on the cracked hinge concept [1] is introduced with some extensions towards the ability of the model to use softening curves without any restriction on their shape.

2. The cracked hinge model

2.1. Theoretical basis

The concept of the cracked hinge model was originally presented as an analytical [1] or semi-analytical [4, 5, 6] solution for the calculation of load-displacement curves for notched and un-notched beams. The basic assumption of the model is the fact that the presence of a crack influences the overall stress and strain field of a structure only in a local manner; this discontinuity is expected to vanish outside a certain bandwidth, s (see Fig. 1, 2). Thus, the propagation of a crack is modeled as a cracked hinge – where the flexural deformation of a structure is concentrated – while outside, the rest of the structure is considered in terms of classical theory of elasticity.

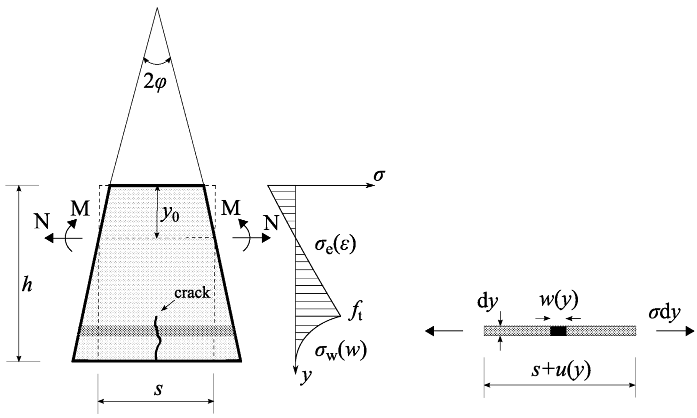

Fig.1: Loading and deformation of the cracked hinge element with a stress distribution (left) and incremental layer inside the hinge considered as the single non-linear spring (right)

The hinge itself can be discretized as a set of independent spring elements which are formed by incremental horizontal strips of the bandwidth. Then, the stress distribution over the hinge consists of the contribution of each spring. For the i-th spring in position y, we can formulate a stress state according to the presence of a crack as

$$\sigma(y) = \begin{cases} \sigma_\mathrm{e}(\varepsilon(y)) = E\,\varepsilon(y) & \text{pre-cracked state}\ , \\ \sigma_\mathrm{w}(w(y)) & \text{cracked state}\ , \end{cases} \tag{1}$$

where E denotes the elastic modulus, $\varepsilon(y)$ elastic strain and $w(y)$ crack opening. For the evaluation of the $\sigma(y)$, it is necessary to determine the deformation state of the hinge. If we assume the hinge has rigid boundaries, this state is uniquely defined by value the of angular deformation φ and by the position of neutral axis, y_0. Based on all the assumptions above, the strain distribution can be simply obtained from the expression

$$\varepsilon(y) = \frac{2\,\varphi\,(y - y_0)}{s} \tag{2}$$

and subsequently the total elongation of each spring, $u(y)$, can be calculated numerically as contributions of $\varepsilon(y)$ and $w(y)$ from the equation

$$u(y) = s\,\varepsilon(y) = s\,\frac{\sigma_\mathrm{w}(w(y))}{E} + w(y)\ . \tag{3}$$

2.2. Application to WS specimens

In general, the hinge model can be incorporated into any structure with a crack subjected to a bending moment (possibly combined with normal force). In the case of the WST, the incorporation of the hinge element in a WS specimen is showed in Fig. 2.

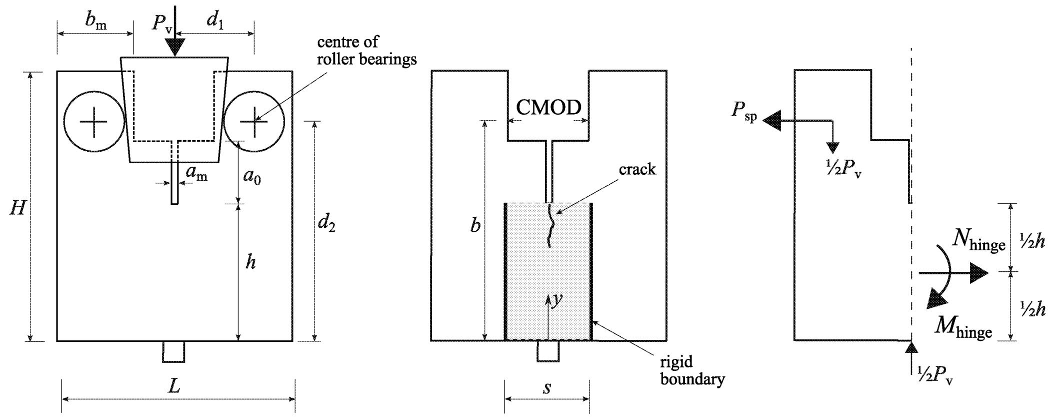

Fig.2: Geometry and the loading of wedge splitting specimen with incorporated hinge element

For the calculation of a P-CMOD diagram as the required model output we proceed from the fact that for given values of φ and y_0, we can simply evaluate the bending moment M_hinge and normal force N_hinge transmitted by the hinge using the relations:

$$\begin{aligned} N_\mathrm{hinge} &= t \int_0^h \sigma(y)\,\mathrm{d}y = \sum_{i=1}^{n} F_i\ , \\ M_\mathrm{hinge} &= t \int_0^h \sigma(y)\,(y - y_0)\,\mathrm{d}y = \sum_{i=1}^{n} F_i\,(y_i - y_0)\ , \end{aligned} \tag{4}$$

where the stress distribution $\sigma(y)$ is defined from (1) – in the discrete form F_i representing the force transmitted by the i-th spring quantified as $F_i = \sigma(y_i)\,t\,\mathrm{d}y$ – and t denotes the thickness of the specimen. Then, by using the equilibrium conditions

$$M_\mathrm{hinge} = M_\mathrm{ext} \qquad \text{and} \qquad N_\mathrm{hinge} = P_\mathrm{sp}\ , \tag{5}$$

we can numerically determine the y_0 for a chosen φ as an iterative process. Here M_ext stands for the bending moment invoked by applied external loading forces P_sp and P_v (see Fig. 2) and can be expressed as

$$M_{\text{ext}} = P_{\text{sp}}\,(d_2 - y_0) + \frac{1}{2}\,P_{\text{v}}\,d_1\ , \tag{6}$$

where the vertical force P_{v} is computed for the specified values of the wedge angle α_{w} and the coefficient of friction in the roller bearings μ_{c}

$$P_{\text{v}} = P_{\text{sp}}\,\frac{2\,\tan\alpha_{\text{w}} + \mu_{\text{c}}}{1 - \mu_{\text{c}}\,\tan\alpha_{\text{w}}}\ . \tag{7}$$

After calculation of force equilibrium conditions, it is necessary to determine the value of CMOD here defined as the opening of the specimen at the line of loading, depending on three different assumed contributions. The first contribution, δ_{e}, is caused via elastic deformation of the specimen. The second, δ_{w}, is due to the crack opening emanating from the starter crack/notch. Finally, the third contribution, δ_{g}, is caused by the fact that there is a certain distance from the crack mouth located at h to the line where the CMOD is measured (located at point b). This geometrical amplification is expressed through the estimation of the 'average' rotation of the crack faces $\theta_{\text{w}} = \delta_{\text{w}}/h$. Thus, CMOD is completely expressed by

$$\text{CMOD} = \delta_{\text{e}} + \delta_{\text{w}} + \delta_{\text{g}}\ . \tag{8}$$

For the evaluation of the first term in (8) the well-known formula [7] can be used

$$\delta_{\text{e}} = \frac{P_{\text{sp}}}{E\,t}\,v_2(x)\ , \tag{9}$$

where $v_2(x)$ represents a geometric function computed for $x = 1 - h/b$ as follows

$$v_2(x) = \frac{x}{(1-x)^2}\,(38.2 - 55.4\,x + 33.0\,x^2)\ . \tag{10}$$

The second term in (8), δ_{w}, can be directly evaluated from (3) at point $y = h$. The last term, δ_{g}, is derived differently than in [7], as a simplified formula: $\delta_g = (b - h)\,\theta_{\text{w}}$.

3. Results and discussion

The above-mentioned procedures were implemented in the JAVA programming language and afterwards validated by referenced data [7]. The considered WS specimen had following dimensions: $L = H = 100\,\text{mm}$; $h = 50$—,mm; $a_0 = 28\,\text{mm}$; $d_1 = 35\,\text{mm}$; $d_2 = 85.2\,\text{mm}$; $a_{\text{m}} = 4.5\,\text{mm}$; $b_{\text{m}} = 35\,\text{mm}$; thickness $t = 100\,\text{mm}$ (the dimensions are indicated in Fig. 2). The angle of the wedge was chosen as $\alpha_{\text{w}} = 15°$ and friction in the roller bearings was ignored, so that $\mu_{\text{c}} = 0$. The elastic un-cracked part of the modeled specimen was prescribed by Young's modulus $E = 30\,\text{GPa}$ and Poisson's ratio $\nu = 0.2$. The fictitious crack part was defined by the referenced softening function as a bilinear diagram with the following parameters: tensile strength $f_{\text{t}} = 2\,\text{MPa}$; critical crack opening $w_{\text{c}} = 0.5\,\text{mm}$; the kink point coordinates $\sigma(w_1) = 0.2\,\text{MPa}$ and $w_1 = 0.045\,\text{mm}$; fracture energy $G_{\text{F}} = 95.5\,\text{J/m}^2$. The hinge model was loaded by an incremental value of the angular deformation $\Delta\varphi = 1.0{\times}10^5$ rad.

The first set of obtained P–CMOD diagrams in Fig. 4 (left) validates the consistency of these results with referenced data [7]. The observable small discrepancies are probably caused by the above mentioned modification of the CMOD calculation. The second set of P–CMOD diagrams in Fig. 4 (right) documents the very fast convergence of the hinge model depending on the increasing number of springs. From these results it is also obvious that it

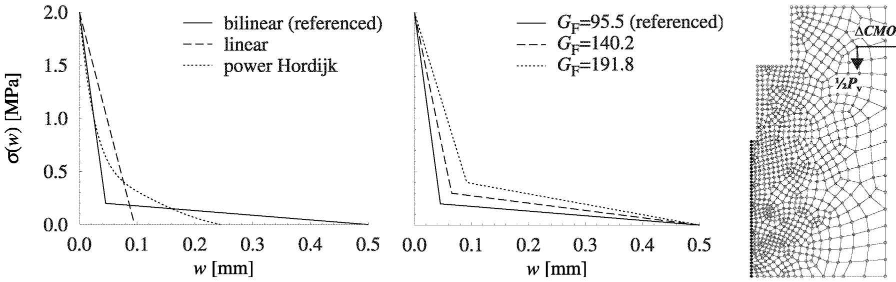

Fig.3: The defined tensile softening functions: with constant (left) and increasing (middle) values of the fracture energy; FE mesh of the WST specimen (right)

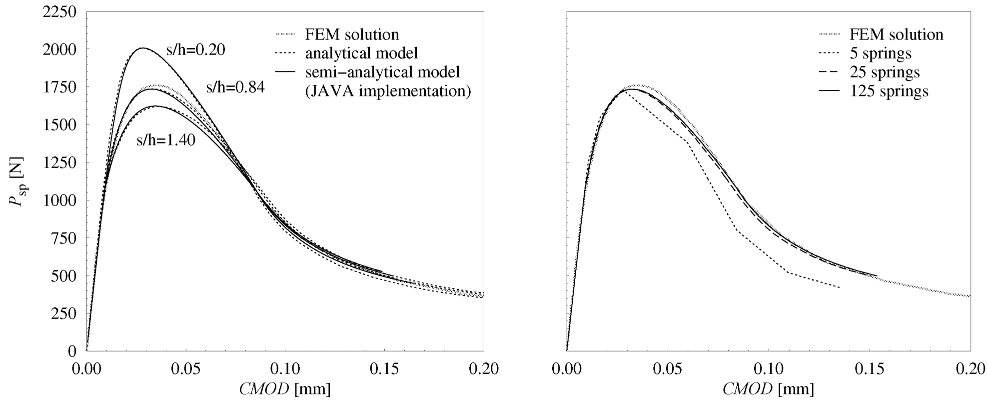

Fig.4: Simulated P–CMOD diagrams of the models using the defined tensile softening functions with the constant (left) and increasing (right) fracture energy

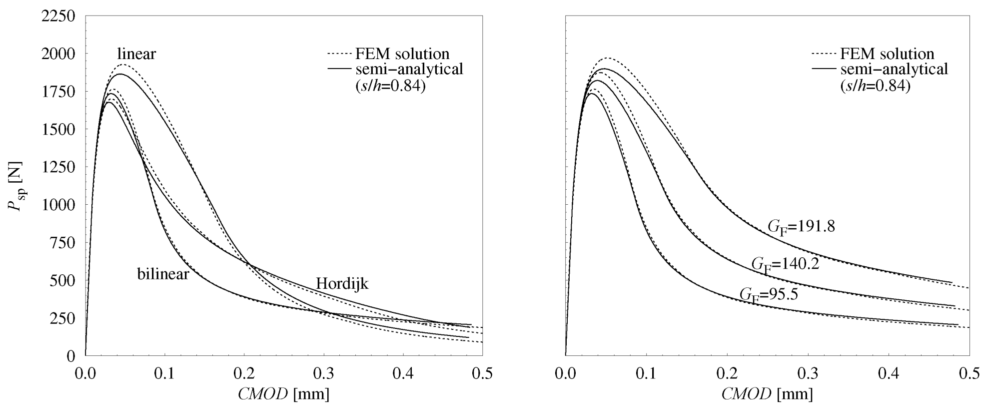

Fig.5: Simulated P–CMOD diagrams of the models using the defined tensile softening functions with the constant (left) and increasing (right) fracture energy

is necessary to calibrate the bandwidth s of the hinge model if we require optimal agreement with the FEM solution (discussed further here [7]).

In order to investigate the capability of implemented model to use any $f(w)$ curve with arbitrary shape, the comparison of two models, the implemented cracked hinge model (with fixed ratio $s/h = 0.64$) and a specialized academic-purpose FE code with FCM developed by the authors, was then performed. In this numerical study, the following selected types of softening function $f(w)$ were used (see Fig. 3): bilinear, linear and Hordijk's power exponential.

The results in Fig. 5 (left) were obtained for the $f(w)$ curves with a considered constant value of $f_\mathrm{t} = 2.0\,\mathrm{MPa}$ and $G_\mathrm{F} = 95.5\,\mathrm{J/m^2}$. It is obvious that a pure change in the shape of the softening curve caused a relatively small deviation from the corresponding FEM solution. Similar behavior viewed in Fig. 5 (right) was also obtained for increasing G_F values in the case of bilinear functions (see Fig. 3). The major discrepancy between the P–CMOD diagrams is concentrated around the peak load and confirms the fact that elastic energy stored in the crack band increases with increasing bandwidth and results in more unstable crack growth.

4. Conclusions

The paper presents a description of the author's implementation of a semi-analytical fracture model based on the cracked hinge approach. A generalized formulation of this model was adopted to enable the use of a softening function with arbitrary shape. The implemented procedures were validated by referenced data [7] for the case of the WST. The obtained results correspond with the referenced data and confirm the ability of the model to use various softening curves.

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