VIBRATIONS OF TURBINE BLADES BUNDLES MODEL WITH RUBBER DAMPING ELEMENTS

Ladislav Půst*, Luděk Pešek*

A detailed analysis of dynamical properties of five-blade-bundle mathematical model was carried out with the aim to investigate how the damping elements made of rubber inserted into slots between blades heads influence the response curves at different distribution of exciting harmonic forces. Viscous-elastic linear Voigt-Kelvin model was used for modeling the rheological properties of damping elements. Constant values of stiffness and damping parameters were supposed at analysis. The orthogonality of excitation forces distribution to the other eigenmodes of blades bundle is applied at insulated selected resonance.

Keywords: *mathematical model, bundle of five blades, rubber damping elements, eigenmodes*

1. Introduction

A very effective way how to avoid undesirable vibrations of turbine blades is introducing additional damping into blade shroud. For detailed discovering of damping processes and their influence on blades vibrations, the dynamic analysis of five-blade model was performed in Institute of Thermomechanics ASCR. Blades were modeled by 1 DOF slightly damped systems and connected by rubber damping elements. In presented paper, the five-blade model with damping elements described by viscous-elastic linear Voigt-Kelvin model will be investigated.

This theoretical study is focused particularly on the elaboration of a theoretical background for analysis of data gained by measurement on the experimental physical model of blades bundles prepared in laboratories IT ASCR.

2. Vibrations of blades bundle – experimental model

Laboratory measurements of blade bundle will be realized on the experimental set consisting of five models of blades with shroud heads, which were rigidly fastened to a steel plate basement, see Fig. 1. Inserted rubber damping elements connect these heads. The first set of these elements is made of VITON rubber which complex Young modulus $E^* = E_{\mathrm{Re}} + E_{\mathrm{Im}}$ [MPa] depends on temperature T and frequency f. These properties are described by the analytical formula in [1] :

$$E^* = 10.49\,(1 + 0.1\,\mathrm{i}) + 15.88\,(1 + 0.06\,\mathrm{i}) \left(\frac{\mathrm{i}\, f\, \alpha\, T}{1 + 0.0063\,\mathrm{i}\, f\, \alpha\, T} \right)^{0.64} \tag{1}$$

where i is imaginary unit and $\alpha\, T = 10^{-20(T-4.4)/(134.5+T)}$.

* Ing. L. Půst, DrSc., Ing. L. Pešek, CSc., Institute of Thermomechanics ASCR, v.v.i., Dolejškova 5, 182 00 Praha 8

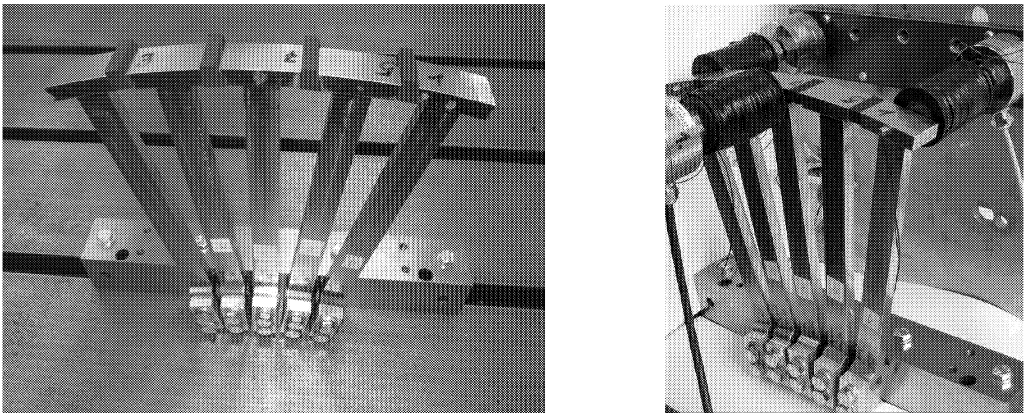

Fig.1: Five-blade model with inserted rubber damping elements

The VITON elements used in the blade bunch have prismatic form and their placement in the blade shroud presumes the shear deformation. Therefore the geometric parameters (base area $A = 0.00025\,\mathrm{m}^2$, height $h = 0.012\,\mathrm{m}$) and volume incompressibility assumption for computing the complex shear stiffness K_s^* of these elements it were used in our numerical simulations:

$$K_\mathrm{s}^* = \frac{E^* A}{3\,h}\ .$$

Rheologic properties of this rubber including stiffness k and damping b coefficient are described also in [6]. Beside VITON rubber also other types of more compliance rubber will be investigated. Therefore before experimental investigation, analytic-numerical analysis of dynamic properties of five blades bundle was realized. A wide range of rubber elements stiffness was selected. As an example of such analysis, some results of dynamic solution focused on mutual influence of resonances and on optimal selection of excitation methods for eigenmode's isolation are included. As the mutual influence of resonances is marked at low stiffness of inserted elements, the rubber element with dynamic stiffness $k_1 = 2000\,\mathrm{N/m}$ and damping coefficient $b_1 = 2\,\mathrm{N\,s/m}$ is applied in the following analysis.

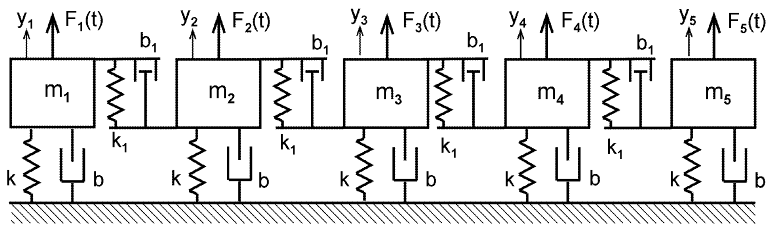

Fig.2: Mathematical model of five blades bundle

3. Simplified mathematical model of five blades bundle

As the experimental research is usually encumbered with a lot of marginal influences and uncertainties, e.g. by impossibility of measurement of deformation of damping elements, uncertainty in contact forces during operation, etc., the additional analytical and numerical solution of simplified mathematical model with exact parameters is very useful and enables to complete knowledge of dynamic behavior of studied system by a lot of new information. Experimental system in Fig. 1 can be modeled by a simple five masses system shown in Fig. 2, where the blades are replaced by 1 DOF systems, the eigenfrequencies of which correspond to the first bending eigenfrequency of real blade. The torsion eigenfrequencies of these blades are much higher than the bending ones (mass m, stiffness k, damping coefficient b) and therefore, in the first approximation, the torsion deformations and torsion vibrations of blades are not taken into account.

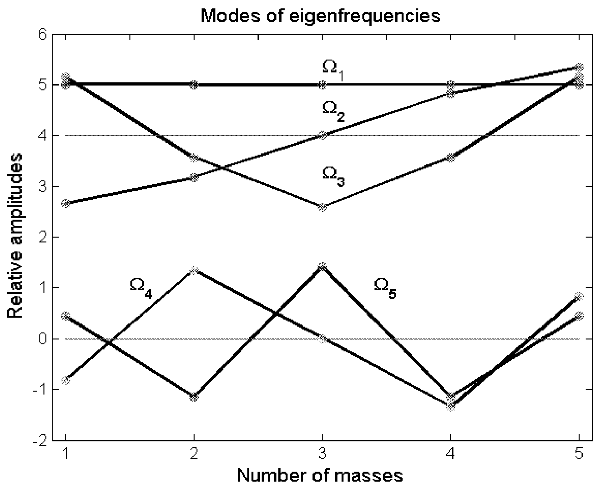

Fig.3: Eigenmodes of bundle

Damping of all individual separated blades is modeled by small viscous damping with very low coefficients $b = 0.4\,\mathrm{N\,s/m}$ in Fig. 2. The reduced mass $m = 0.182\,\mathrm{kg}$ and stiffness $k = 105000\,\mathrm{N/m}$ are ascertained according to the lowest bending eigenfrequency of the real blade structure $f = 120.88\,\mathrm{Hz}$.

The rubber damping elements are loaded on their sides by such sufficiently great friction forces in the contact areas with the neighborhood masses that no slips occur in these connections during operation. The linear Voigt-Kelvin model describes deformation properties of rubber damping elements. Masses m are loaded by the harmonic forces

$$F_i(t) = F_{0i}\,\cos(\omega\,t)\ , \quad i = 1,\dots,5\ , \tag{2}$$

where the amplitudes F_{0i} can be of various values. The excitation frequency ω of force $F_{0i}\,\cos(\omega\,t)$ varies over the whole eigenfrequency spectrum of investigated system.

Differential equations of motion for the excitation by given vector of force amplitudes

$$F = [F_{01}, F_{02}, F_{03}, F_{04}, F_{05}]$$

are:

$$\begin{aligned}
m\,\ddot{y}_1 + b\,\dot{y}_1 + k\,y_1 + g_1(y_1,y_2) &= F_{01}\,\cos(\omega\,t)\ ,\\
m\,\ddot{y}_2 + b\,\dot{y}_2 + k\,y_2 + g_2(y_2,y_3) - g_1(y_1,y_2) &= F_{02}\,\cos(\omega\,t)\ ,\\
m\,\ddot{y}_3 + b\,\dot{y}_3 + k\,y_3 + g_3(y_3,y_4) - g_2(y_2,y_3) &= F_{03}\,\cos(\omega\,t)\ ,\\
m\,\ddot{y}_4 + b\,\dot{y}_4 + k\,y_4 + g_4(y_4,y_5) - g_3(y_3,y_4) &= F_{04}\,\cos(\omega\,t)\ ,\\
m\,\ddot{y}_5 + b\,\dot{y}_5 + k\,y_5 - g_4(y_4,y_5) &= F_{05}\,\cos(\omega\,t)\ ,
\end{aligned} \tag{3}$$

where the linkage functions g_i are described by the linear Voigt-Kelvin parallel viscous-elastic model

$$g_i(y_i, y_{i+1}) = k_1\,(y_i - y_{i+1}) + b_1(\dot{y}_i - \dot{y}_{i+1})\ , \quad i = 1, \ldots, 4\ . \tag{4}$$

4. Free vibrations of blade bundle

First of all, the eigenfrequencies of such 5 DOF system have to be ascertained. Neglecting external and damping forces, the corresponding equations are

$$\mathbf{M}\,\ddot{\mathbf{Y}} + \mathbf{K}\,\mathbf{Y} = \mathbf{0}\ , \tag{5a}$$

where

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k+k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k+2\,k_1 & -k_1 & 0 & 0 \\ 0 & -k_1 & k+2\,k_1 & -k_1 & 0 \\ 0 & 0 & -k_1 & k+2\,k_1 & -k_1 \\ 0 & 0 & 0 & -k_1 & k+k_1 \end{bmatrix}. \tag{5b}$$

For $k_1 = 2000\,\mathrm{N/m}$ are eigenfrequencies $\Omega_i = 120.88, 121.33, 122.47, 123.86, 124.98\,\mathrm{Hz}$.

Corresponding modes belonging to these five eigenfrequencies are plotted in Fig. 3. Damping of the inserted rubber elements is proportional to the differences of relative velocities of neighboring blades, i.e. to the slops of lines connecting the positions of masses in Fig. 3. Therefore the eigenmodes-damping increases with higher frequency in spite of the fact that the material damping coefficient b_1 is constant.

5. Response on external sweep excitation

The simplest way how to gain the response curves of general multi-degrees dynamical systems both with the linear or nonlinear stiffness and damping characteristics is to calculate the response of mathematical model on the sweeping excitations. However due to the non-stationary excitation over a wide frequency range, these response curves are a little distorted against the stationary excited responses at one frequency, but they quickly give a lot of information. It has been shown by the numerical analysis [6] that sweep acceleration $0.2\,\mathrm{rad/s^2}$ causes only $3\,\%$ decrease of resonance peak and $0.03\,\%$ change of resonance frequencies. This value has been therefore applied at the numerical calculation of the next examples.

6. Forced vibrations

The experimental physical model of five blades bundles, prepared in laboratories IT ASCR, will be excited by various combinations of external harmonic forces. Therefore also in this anticipatory analytic-numerical model analysis, the responses on various combinations of external harmonic forces i.e. on the form of excitation force vector $\mathbf{F} = [F_{01}, F_{02}, F_{03}, F_{04}, F_{05}]$ must be investigated.

If the force vector consists of only one force $F_{01} = 1\,\mathrm{N}$, (i.e. $[1, 0, 0, 0, 0]$), then the response curves of the five-blade bundle system with the damping elements stiffness $k_1 = 2000\,\mathrm{N/m}$ and linear damping coefficient $b_1 = 2\,\mathrm{N\,s/m}$ calculated at sweep excitation with angular acceleration $\varepsilon = 0.20\,\mathrm{rad/s^2}$ and passing through the frequency range $f = 120{-}126\,\mathrm{Hz}$ including all eigenfrequencies of the five-blade bundle system are plotted in Fig. 4. Response curve in the place of force application is drawn by the highest curve; the curves of other masses are gradually falling in the beginning near 120 Hz, but due to the influence of other modes they are crossing in the higher frequency range. Small sterns above the curves indicate the positions of eigenfrequencies. The second resonance causes small increase of response curves near 121.4 Hz. In spite of the theoretical possibility of excitation of higher resonances by this one-force action, the other resonances are not indicated.

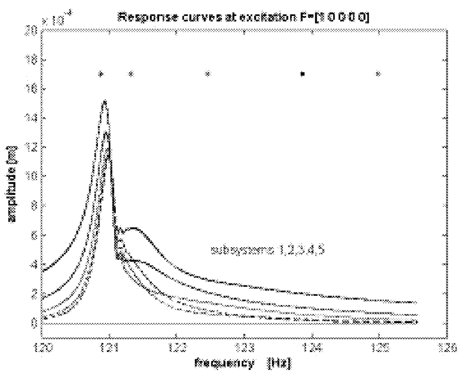

Fig.4: Response curves at $\mathbf{F} = [1,0,0,0,0]$

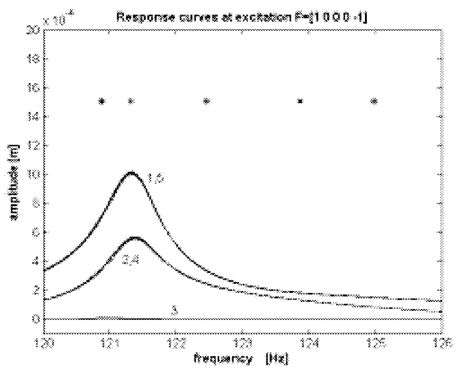

Fig.5: Response curves at $\mathbf{F} = [1,0,0,0,-1]$

Forced oscillations excited by two opposite harmonic forces acting on the corner blades 1 and 5 ($\mathbf{F} = [1,0,0,0,-1]$) are axially anti-symmetric and therefore the blades 1 and 5 as well as 2 and 4 have common response curves. They differ only by different phase shifts. These curves are shown in Fig. 5, where the response curves of the points 1, 5 of forces application are the highest again. Responses on the used anti-symmetric excitation with two opposite forces contain only one resonance peak at $f = 121.4\,\mathrm{Hz}$ in spite of the fact that also the fourth eigenmode is also anti-symmetric and can be theoretically indicated. As it will be seen further, four forces must be used for its accentuation.

By means of three forces distributed according to the force vector $\mathbf{F} = [1,0,-1,0,1]$ and corresponding to the mode of the third eigenfrequency Ω_3, the third resonance should be excited. However, result of numerical solution (Fig. 6) shows that the highest resonance peak has the first resonance and the third one is only indicated by a small increase in $f = 122.46\,\mathrm{Hz}$. Using modified force vector $\mathbf{F} = [1,0,-2,0,1]$, having the balanced force components, the first resonance peak is suppressed and the third one is moderately amplified as shown in Fig. 7. The great damping $b_1 = 2\,\mathrm{N\,s/m}$ is evidently very high for this mode and causes the flat form of third resonance peak.

Force vector $\mathbf{F} = [1,-1,0,1,-1]$ excites the flat fourth resonance peak at 123.86 Hz, but it is also connected with the rise of the comparatively sharp resonance peak at 121.32 Hz as shown in Fig. 8. Suppression of second resonance peak can be reached by transformation of force vector F into orthogonal form to the second eigenmode. Response curves of five-blades bundle excited by the modified force vector $\mathbf{F} = [1,-1.625,0,1.625,-1]$ show only one resonance peak corresponding to the fourth eigenfrequency Ω_4 – Fig. 9.

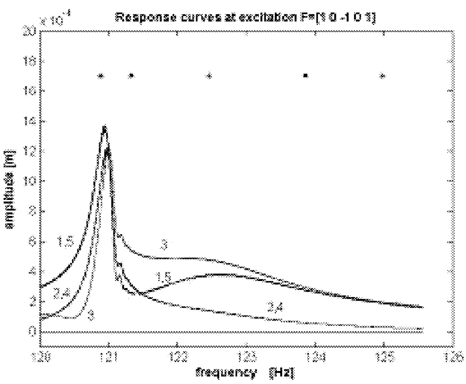

Fig.6: Response curves at $\mathbf{F} = [1, 0, -1, 0, 1]$

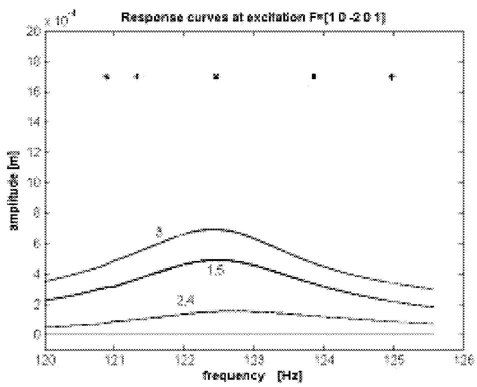

Fig.7: Response curves at $\mathbf{F} = [1, 0, -2, 0, 1]$

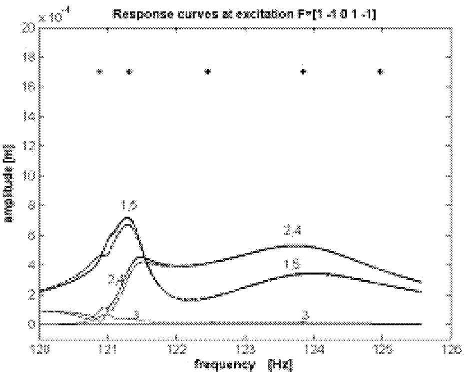

Fig.8: Response curves at $\mathbf{F} = [1, -1, 0, 1, -1]$

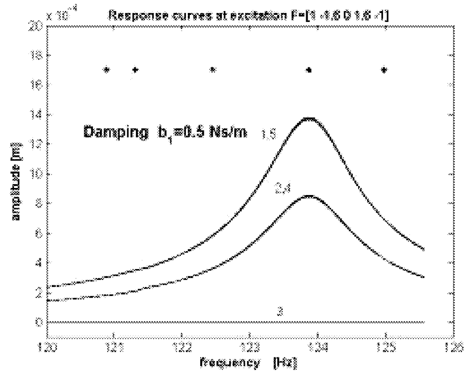

Fig.9: Response curves at $\mathbf{F} = [1, -1.625, 0, 1.625, -1]$

The five-blade bundle has 5 eigenfrequencies, from which three Ω_1, Ω_3, Ω_5 have axially symmetric modes. For separation of the fifth eigenmode, the excitation force vector must be orthogonal to the first and third eigenmodes. Response curves of such system excited by $\mathbf{F} = [1, -2.49, 2.474, -2.49, 1]$ are plotted in Fig. 10. The middle, third mass has the highest resonance amplitude, amplitudes of the second and the fourth masses are lower and both end masses 1, 5 vibrate with minimal amplitudes. The small distortions near the third eigenfrequency $\Omega_3 = 122.46\,\text{Hz}$ show that the applied force vector is not quite correctly orthogonalized to the other modes.

7. Slip in the contact surface

The assumption that no slip occurs between the inserted rubber and shroud heads has been strictly applied in the previous chapters. However, the dynamics of blades bundle change when the dry friction forces in these contacts are limited and slips occur. This can be shown by means of changes of the first resonance peak at decreasing friction force. If the friction force in contact surfaces between the rubber element and blade head is lower than e.g. $F_{\mathrm{t}} = 1\,\mathrm{N}$, then after the vibration reaches some high amplitude, the slip occurs and the additional friction damping causes reduction of resonance peaks. The lower the friction connection in the rubber-steel contacts is, the lower resonance amplitudes are. However, the very low friction can be also dangerous. In Fig. 11 (zoom of Fig. 4), there are shown response

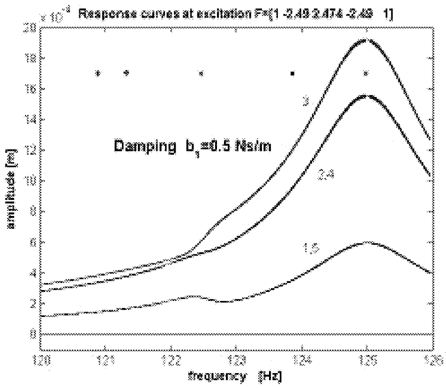

Fig.10: Response curves at $\mathbf{F} = [1, -2.49, 2.474, -2.49, 1]$

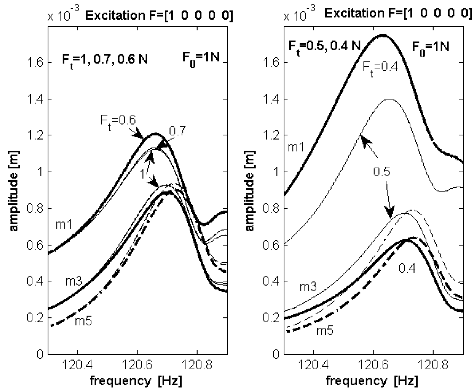

Fig.11: Response curves at low friction contacts left: $F_\mathrm{t} = 1, 0.7, 0.6\,N$*, right:* $F_\mathrm{t} = 0.5, 0.4\,N$

curves of only three masses m_1, m_3, m_5 excited by $\mathbf{F} = [1, 0, 0, 0, 0]\,\mathrm{N}$. The left subfigure shows that the response curves at $F_\mathrm{t} = 1\,\mathrm{N}$ and $0.7\,\mathrm{N}$ are almost the same. However the decrease of friction force on value $F_\mathrm{t} = 0.6\,\mathrm{N}$ (bold curves) increases resonance amplitudes of m_1, and decreases amplitudes of other masses. Further decrease of friction force on $F_\mathrm{t} = 0.5, 0.4\,\mathrm{N}$ (Fig. 11 right) makes these differences more higher; the excited mass m_1 vibrates more due to the weaker sliding connection between m_1 and m_2.

8. Conclusions

A mathematical model of five-blade bundle, connected in the heads slots by damping elements made of special rubber, has been developed and applied for the ascertaining of dynamic behavior of blades bundle at different external excitations.

Properties of the VITON rubber, which was selected as material for the damping elements for the prepared experimental research in laboratory IT AVCR, were investigated also in view of the dependence on frequency of oscillations. The frequency dependences of the Young modulus and loss factor in the range (100–300) Hz are weak and therefore the changes in this frequency range can be neglected for the given temperature. Effect of the temperature and frequency on the forms of eigenmodes in the whole investigated area is also negligible. The analysis of influence of different force distribution on the response curves of blades bundle was realized at stiffness $k_1 = 2000\,\mathrm{N/m}$.

It was proved that due to the more complicated forms at higher eigenmodes the mode's damping increases with frequency even at the constant material viscous damping coefficient. The application of orthogonality of excitation forces distribution to the other eigenmodes of blades bundle is necessary for analysis and isolation of selected resonance.

The influence of friction connections in the contact surfaces between rubber and blades' heads was investigated as well. At the given rheological rubber properties the decrease of friction forces in contact surfaces increases the resonance amplitudes of excited mass and decreases amplitudes of other, non-excited masses.

Note: This article is an extended version of paper [6].

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