INPUT SHAPING CONTROL OF ELECTRONIC CAMS

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The paper deals with the non-vibration control of electronic cams. Described approach is based on shaping of the command input. The goal is precise positioning without residual vibration in final position. The models of different configurations of electronic cams are used – the simple traditional one, the serial one, the parallel one, the multi-input one. The generalized approach to input shaping control is described that allows synthesis of the shaping functions with arbitrary time length. These functions could be further transformed to the shapers with re-entry property. It is demonstrated and explained that some advanced shaping functions are more robust against model misalignment than the simple Heaviside pulse shapers. This generalized input shaping control is applied to different kinds of electronic cams.

Keywords: *input shaping, shaper, electronic cam, residual vibration*

1. Introduction

The term electronic cam means use of a precise controlled servomotor drive instead of the conventional cam [1]. This concept can be further divided into several groups according to the system structure – e.g. serial, parallel or multi-input electronic cams. The demand for fast and precise positioning is common in all mentioned cases. But like other flexible systems electronic cams have to deal with the problem of residual vibration that could corrupt system performance.

To eliminate unwanted dynamics the standard control input could be reshaped in such a way that it doesn't excite flexible modes of the system or, more generally, that all energy

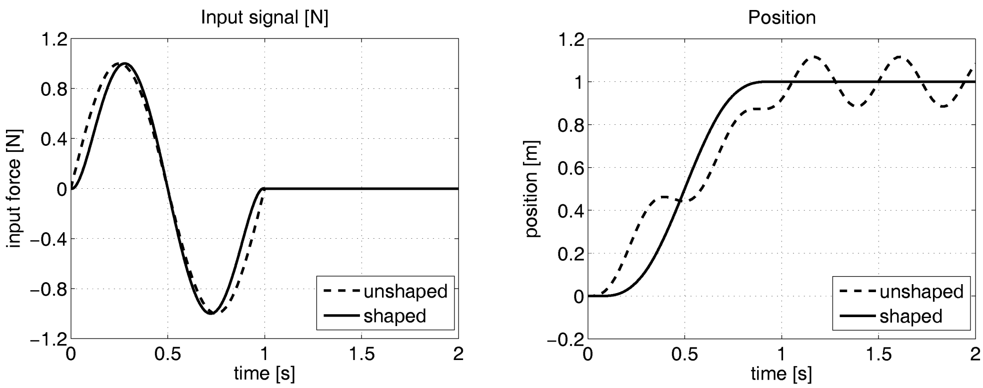

Fig.1: Comparison of shaped and unshaped control input

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put into the flexible modes is totally relieved at the end of the movement [2]. The difference between the original unshaped signal and the shaped one as well as the response of the two-mass model is shown in Fig. 1.

2. Necessary conditions

The control input that ensures non-vibration positioning has to fulfil some necessary conditions. For the system described using the state space formulation as

$$\dot{\mathbf{y}} = \mathbf{A}\,\mathbf{y}(t) + \mathbf{B}\,\mathbf{u}(t) \tag{1}$$

the general solution can be written in the form

$$\mathbf{y}(t_2) = \mathrm{e}^{\mathbf{A}(t_2-t_1)}\,\mathbf{y}(t_1) + \int\limits_{t_1}^{t_2} \mathrm{e}^{\mathbf{A}(t_2-\tau)}\,\mathbf{B}\,\mathbf{u}(\tau)\,\mathrm{d}\tau \ , \tag{2}$$

where $\mathbf{A}$ is the system matrix, $\mathbf{B}$ is the input matrix, $\mathbf{y}$ is the vector of states, $\mathbf{u}$ is the input (control) vector, t_1 and t_2 represent the start and the finish time. Using the finite time Laplace transform [2]

$$U(s) = \int\limits_{t_1}^{t_2} \mathrm{e}^{-s\,\tau}\,u(\tau)\,\mathrm{d}\tau \ , \tag{3}$$

the necessary conditions connecting control input with system states can be derived in the form

$$\sum_{l=1}^{n} U_l(s)\big|_{s=\mathbf{A}}\,\mathbf{b}_l = \mathrm{e}^{-\mathbf{A}\,t_2}\,\mathbf{y}(t_2) - \mathrm{e}^{-\mathbf{A}\,t_1}\,\mathbf{y}(t_1) \ , \tag{4}$$

where $U_l(s)$ is the finite time Laplace transform of the l-th input, $\mathbf{b}_l$ is the corresponding column of $\mathbf{B}$ matrix, n is the number of inputs. The solution $u_l(t)$ in the time domain is the inverse Laplace transform of $U_l(s)$.

Now this approach will be applied to the simple electronic cam that can be modelled as a two mass spring-dumper system in Fig. 2.

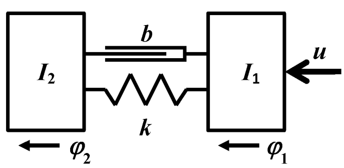

Fig.2: Two-mass model of the electronic cam

This system is described by the equation

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{B}\,\dot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) = \mathbf{F}(t) \ , \tag{5}$$

where

$$\mathbf{x} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} , \quad \mathbf{M} = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} , \quad \mathbf{B} = \begin{bmatrix} b & -b \\ -b & b \end{bmatrix} , \quad \mathbf{K} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} , \quad \mathbf{F} = \begin{bmatrix} u \\ 0 \end{bmatrix} . \tag{6}$$

Non-vibration conditions in the final position Φ_f of point-to-point control problem are

$$\mathbf{x}_f = \begin{bmatrix} \Phi_f \\ \Phi_f \end{bmatrix} , \quad \dot{\mathbf{x}}_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{7}$$

The differential equation of the second order (5) can be rewritten as a set of first order equations and transform to the Jordan canonical form

$$\underbrace{\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix}}_{\dot{\mathbf{y}}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p^* \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}}_{\mathbf{y}} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{B}} u \tag{8}$$

where p and p^* are complex conjugated poles of flexible modes.

The boundary conditions (7) are transformed to the equation

$$\mathbf{y}(t_2) = [\Phi_f; 0; 0; 0]^{\mathrm{T}} . \tag{9}$$

Assuming t_1 and zero initial conditions equation (4) can be rewritten in the component form

$$\begin{aligned} \left.\frac{\mathrm{d}U(s)}{\mathrm{d}s}\right|_{s=0} &= \Phi_f , \\ U(s)\big|_{s=0} &= 0 , \\ U(s)\big|_{s=p} &= 0 , \\ U(s)\big|_{s=p^*} &= 0 . \end{aligned} \tag{10}$$

This simple analytical formulation of necessary conditions for non-vibration positioning used by Bhat & Miu [3] is the result of the system description in the canonical form. Other state space representations usually need a numerical solution of (4).

Described approach leads to the control input in the form of pre-computed curve. However if it is rewritten to the form of a dynamical block it acts like a filter that transform any arbitrary signal to non-vibration one [4]. And in contrast with patented input shaping technique by Singhose & Seering [5] the length of this shaper is not dependent on the system natural frequency and can be set arbitrary.

3. Additional conditions and the control input synthesis

There are an infinite number of input functions $u(t)$ that fulfill equations (10). But these only ensure zero residual vibration. Therefore additional restrictions have to be applied e.g. for the time domain continuity of the input signal

$$u(0) = 0 , \qquad u(t_2) = 0 . \tag{11}$$

Other restrictions are defined by maximal torque and rate of the actuator available etc.

The straight forward method of the control input synthesis is to assume analytical form of the control input with some variable parameters, e.g. the polynomial function with unknown coefficients

$$u(t) = \sum_{i=0}^{n} a_i\, t^i . \tag{12}$$

The exact value of parameters a_i is then calculated with respect to (10) and all other defined restrictions. It is possible to use some optimization methods as well.

Generally the number of parameters should be at least the same as the number of restricted conditions, however a smart choice of an analytical form of the input could automatically filled some of them. For example this form of a control input

$$u(t) = \sum_{i=1}^{n} a_i \sin\left(\frac{2\pi\, i}{t_2}\, t\right) \tag{13}$$

automatically filled conditions (11).

4. Systems with multiple inputs and multiple outputs

The two-mass model in Fig. 2 is probably the most common demonstrator of different input shaping methods. Speaking about electronic cams the two masses represent the actuator (index 1) and the cam (index 2). The solution of the two-mass problem ensures precise positioning of the cam only. But in real systems the cam is connected to the rest of the system that usually has its own flexibility. The following chapters describe an application of the presented approach to different system structures – serial one, parallel one and the system with two inputs.

4.1. Serial electronic cam

The modified serial structure consisting of a three bodies is in Fig. 3.

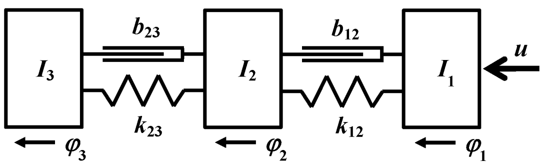

Fig.3: Serial structure of electronic cam

This system has two pairs of complex conjugated flexible modes and the rigid body mode. For simulation experiments it was described according to (5) as

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{bmatrix} + \begin{bmatrix} b_{12} & -b_{12} & 0 \\ -b_{12} & b_{12}+b_{13} & -b_{23} \\ 0 & -b_{23} & b_{23} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{bmatrix} + \\ + \begin{bmatrix} k_{12} & -k_{12} & 0 \\ -k_{12} & k_{12}+k_{13} & -k_{23} \\ 0 & -k_{23} & k_{23} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} . \tag{14}$$

For the simulation purposes the value of all moments of inertia was set to $1\,\mathrm{kg\,m^2}$, stiffness $100\,\mathrm{N\,m/rad}$ and the dumping $1\,\mathrm{N\,m\,s/rad}$. The model was transformed to the set of first order differential equations that is represented in Jordan canonical form

$$\dot{\mathbf{z}} = \mathbf{J}\,\mathbf{z} + \mathbf{c}\,\mathbf{F} \ , \tag{15}$$

where

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5000-9.9875\,\mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5000+9.9875\,\mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.5000-17.2554\,\mathrm{i} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.5000+17.2554\,\mathrm{i} \end{bmatrix} ,$$

$$\mathbf{c} = \begin{bmatrix} 0.0400 \\ 1.0000 \\ 0.9950-0.0999\,\mathrm{i} \\ 0.9950+0.0999\,\mathrm{i} \\ 0.9850-0.1726\,\mathrm{i} \\ 0.9850+0.1726\,\mathrm{i} \end{bmatrix} . \tag{16}$$

The control input was considered in the form of a polynomial function

$$u(t) = \lambda_0 + \lambda_1\,t + \lambda_2\,t^2 + \lambda_3\,t^3 + \lambda_4\,t^4 + \lambda_5\,t^5 \ . \tag{17}$$

Coefficients were calculated using (4) for $t_1 = 0\,\mathrm{s}$, $t_2 = 1\,\mathrm{s}$ and $\Phi_\mathrm{f} = 1\,\mathrm{rad}$ and the solution is

$$\boldsymbol{\lambda} = [52.9868; -529.9711; 2271.6044; -4726.9460; 4399.2114; -1459.8514] \ . \tag{18}$$

The computed input and simulated system response is shown in Fig. 4.

The time domain continuity of the input wasn't required in this case. However it could be reached either by using input function in the form of (13) or adding restrictions (11).

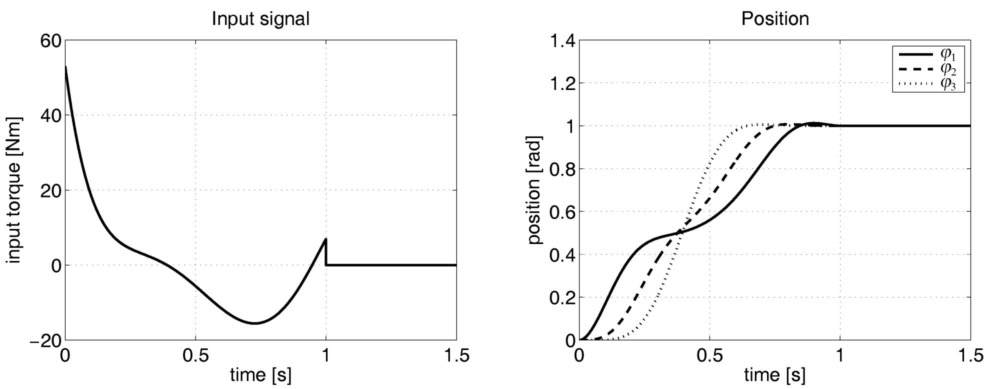

Fig.4: Serial electronic cam – shaped input and system response

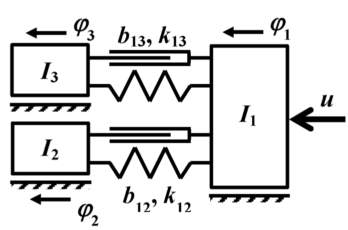

Fig.5: Parallel structure of electronic cam

4.2. Parallel electronic cam

The structure in Fig. 5 describes situation when the load consists of two masses with different resonant frequencies connected parallel to the actuator.

The value of all moments of inertia was set to $1\,\mathrm{kg\,m^2}$, $k_{12} = 100\,\mathrm{N\,m/rad}$, $k_{13} = 50\,\mathrm{N\,m/rad}$ and all the dumping was neglected. The system was described using equation

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{bmatrix} + \begin{bmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12} & 0 \\ -b_{13} & 0 & b_{23} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{bmatrix} + \\ + \begin{bmatrix} k_{12}+k_{13} & -k_{12} & -k_{13} \\ -k_{12} & k_{12} & 0 \\ -k_{13} & 0 & k_{23} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} . \tag{19}$$

As in the previous chapter this model was transform to Jordan canonical form (15) where

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0+15.3819\,\mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0+7.9623\,\mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0-15.3819\,\mathrm{i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0+7.9623\,\mathrm{i} \end{bmatrix} , \\ \mathbf{c} = \begin{bmatrix} 0.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \end{bmatrix} . \tag{20}$$

The shape of the control input was considered in the form of polynomial (17) where the coefficients were calculated using (4) for for $t_1 = 0\,\mathrm{s}$, $t_2 = 1\,\mathrm{s}$ and $\Phi_\mathrm{f} = 1\,\mathrm{rad}$

$$\boldsymbol{\lambda} = [22.4522; -14.8122; -612.5777; 2149.3888; -2611.5054; 1044.6021] . \tag{21}$$

The computed input and simulated system response is shown in Fig. 6.

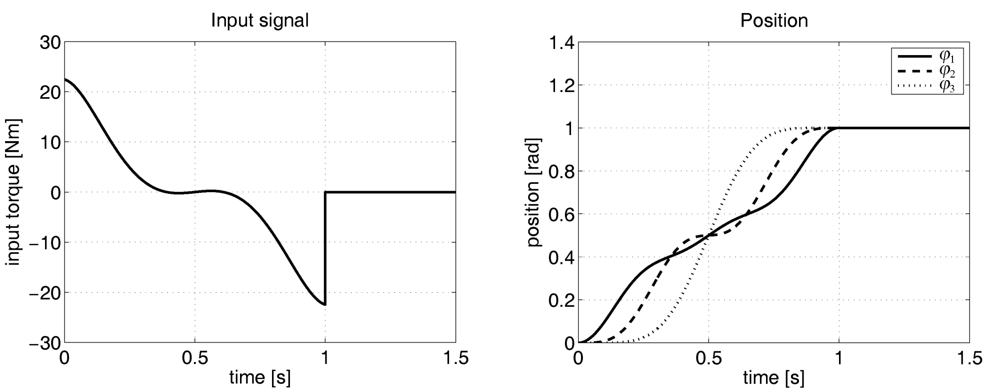

Fig.6: Parallel electronic cam – shaped input and system response

4.3. Two-inputs electronic cam

Input shaping techniques are usually applied only to systems with a single input. However many systems has two or more actuators. The schema of simple electronic cam with two inputs is shown in Fig. 7. The position of the cam I_2 is controlled by actuators I_1 and I_3. Both actuators act on the same axis. The real application of this structure is that I_3 is primary force element, e.g. asynchronous motor, but with low accuracy of positioning. The I_1 is a fast servo motor that ensures precise positioning and/or vibration suppression.

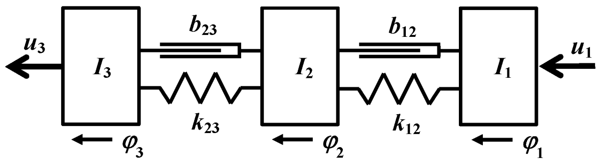

Fig. 7: Two-input structure of electronic cam

The system was described using equation

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{bmatrix} + \begin{bmatrix} b_{12} & -b_{12} & 0 \\ -b_{12} & b_{12}+b_{13} & -b_{23} \\ 0 & -b_{23} & b_{23} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{bmatrix} + \\ + \begin{bmatrix} k_{12} & -k_{12} & 0 \\ -k_{12} & k_{12}+k_{13} & -k_{23} \\ 0 & -k_{23} & k_{23} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ 0 \\ u_3 \end{bmatrix} . \qquad (22)$$

Model parameters were set to the same values as in the chapter 4.1 and for the simulation the input u_3 was defined as a constant load $u_3 = -10\,\mathrm{N\,m}$.

Once again the model was transformed to Jordan canonical form

$$\dot{\mathbf{z}} = \mathbf{J}\,\mathbf{z} + \mathbf{C}\,\mathbf{F} \ , \qquad (23)$$

with

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5000-9.9875\,\mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5000+9.9875\,\mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.5000-17.2554\,\mathrm{i} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.5000+17.2554\,\mathrm{i} \end{bmatrix} ,$$

$$\mathbf{C} = \begin{bmatrix} 0.0400 & 0.0400 \\ 1.0000 & 1.0000 \\ 0.9950-0.0999\,\mathrm{i} & 0.9950-0.0999\,\mathrm{i} \\ 0.9950+0.0999\,\mathrm{i} & 0.9950+0.0999\,\mathrm{i} \\ 0.9850-0.1726\,\mathrm{i} & 0.9850-0.1726\,\mathrm{i} \\ 0.9850+0.1726\,\mathrm{i} & 0.9850+0.1726\,\mathrm{i} \end{bmatrix} , \qquad \mathbf{F} = \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} . \qquad (24)$$

The solution of the control input in the form of polynomial (17) leads to the solution

$$\boldsymbol{\lambda} = [133.607; -1854.280; 8924.832; -18449.097; 16856.130; -5582.174] \ . \qquad (25)$$

The computed input and simulated system response is shown in Fig. 8.

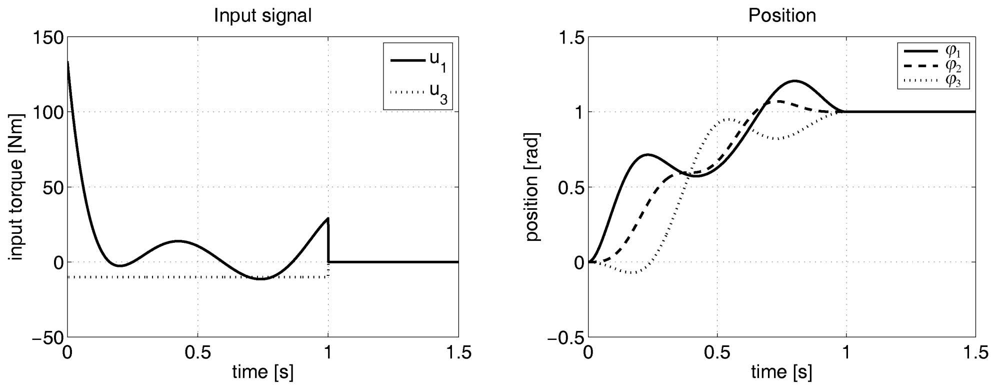

Fig.8: Two-inputs electronic cam – shaped input and system response

Note that there is no residual vibration but the controlled mass no. 2 slightly 'overshoot' the final position during the travel. The reason is that no restrictions were defined to deal with this problem, but it is possible to add them to existing calculating procedure.

5. Robustness

Being a feed-forward method all control shaping techniques need precise system models. The vibration suppression is in fact caused by placing zeros of the control input into the poles of the system. Therefore incorrect system model causes that the control input is not design properly and vibrations are not cancelled. To increase robustness to modelling errors it is possible to formulate additional constrains that either introduced more zeros to the control input or increase the order of existing ones. The price for that is the increase of necessary acting force or longer settling time.

Fig. 9 shows the spectral analysis of the polynomial shaper of 5^{th} order (designed for two-mass model with time domain continuity restrictions) and Fig. 10 shows the robust variant of 7^{th} order that fulfil constraints for zero derivatives in the poles of controlled system.

The density of iso-lines around system poles is considerably higher in the case of robust shaper. That means lower sensitivity to modelling errors.

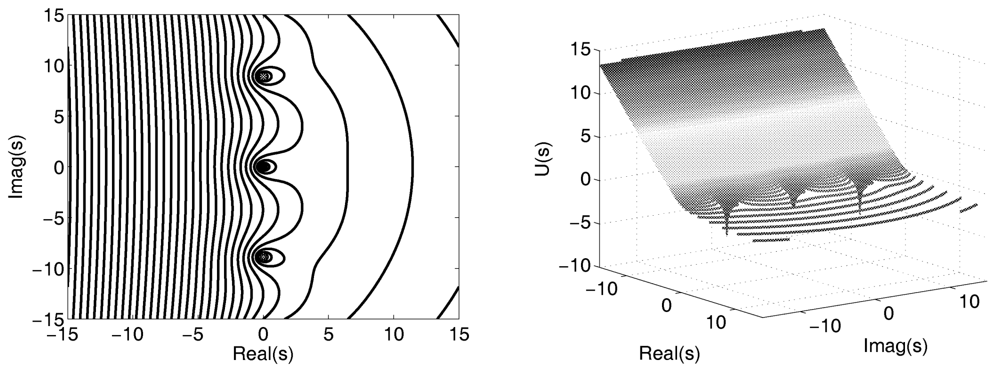

Fig.9: Spectral analysis of 5^{th} order polynomial shaper

Even more robust is the shaper in Fig.11 that was designed by placing two additional zeros nearby the poles of the system. Even if the real system resonant frequency Ω differs from the modelled one Ω_{m} more than 20 % the amount of residual vibration is below 1 % compared to unshaped signal, see Fig. 12. On the other hand to achieve this performance the shaped input command lasts 4 times longer than unshaped one.

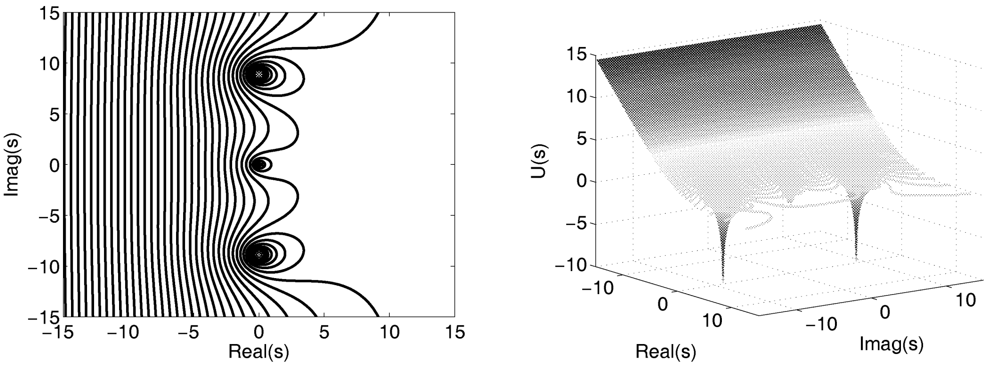

Fig.10: Spectral analysis of 7^{th} order polynomial shaper

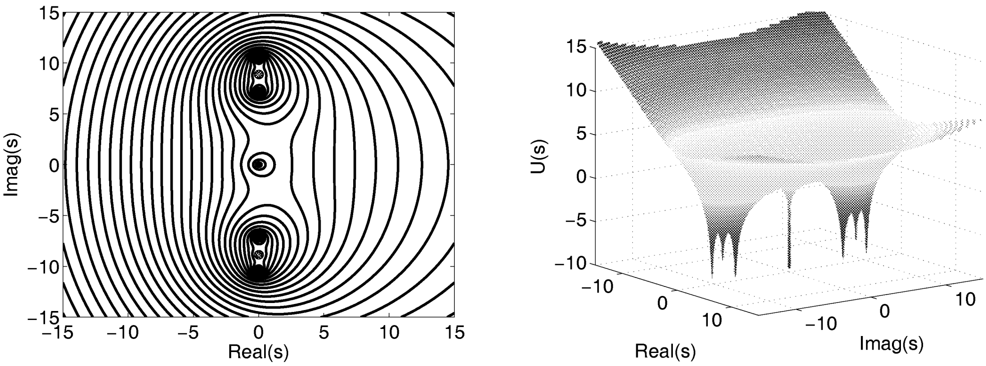

Fig.11: Spectral analysis of 9th order polynomial shaper

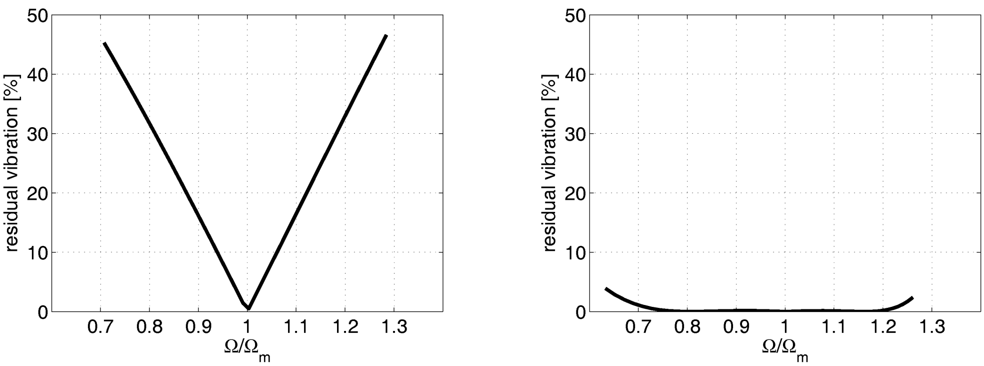

Fig.12: Sensitivity to modelling errors a) 5^{th} order shaper b) 9^{th} order shaper

6. Experiment

The test stand that was used for an experimental evaluation of simulations is in Fig. 13. Its structure is similar to Fig. 2, but the gearbox with ratio 1:5 was added. So for desired position $\varphi_2(t_2) = 2\pi$ rad the motor position has to be $\varphi_1(t_2) = 10\pi$ rad. The settling time was $t_2 = 0.5\,s$. The control input was in the form (13) and two zeros were added to control input nearby modelling system poles.

The computed control input and the system response are in Fig. 14 and Fig. 15. The experiment proved simulation results and no vibration appeared.

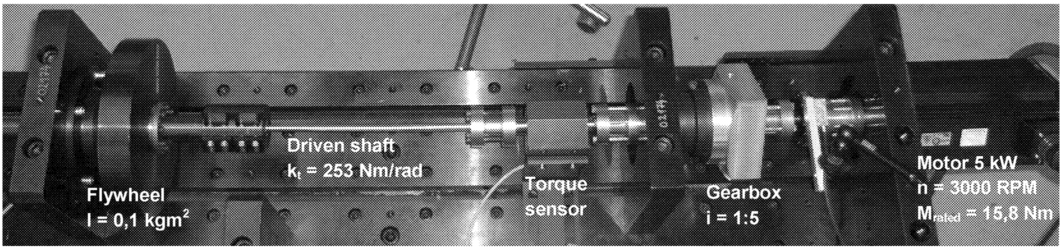

Fig.13: Test stand

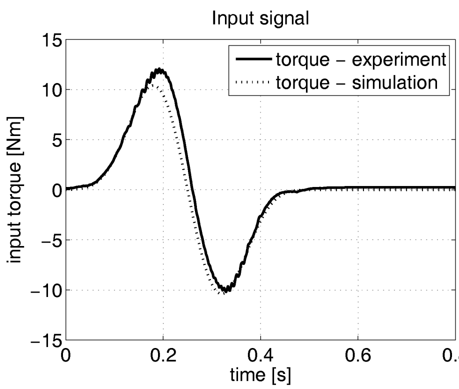

Fig.14: Experiment – control input

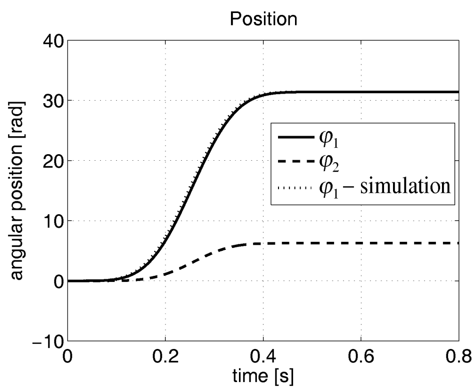

Fig.15: Experiment – system response

7. Conclusions

The presented approach to control of electronic cams describes the design of control curves that suppress residual vibration in final position. Based on the finite time Laplace transform it formulates the set of necessary conditions for non-vibration control input. Further these conditions are modified to increase robustness to model misalignments. The length of the control curve can be set arbitrary. Therefore the transformation of control curve into the dynamical block creates the command shaper that is not strictly determined by the system natural frequency. The results were supported by simulation experiments with models of serial, parallel and multi-input structure of electronic cam and by real experiments using test stand. The formulation is opened for additional constrains and optimization criteria and aside from electronic cams it could be used for other flexible systems as well.

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