OPTIMISATION OF BUILDING STRUCTURES I (DBSO)

Jan Plšek*, Petr Štěpánek*

This contribution deals with a deterministic-based structural optimisation (DBSO). The introductory part of the paper covers a short overview of optimisation algorithms applicable to deterministic-based problems, general DBSO formulation and a target function(s) pattern for structural design. The following part gives attention to particular problem of general RC (reinforced concrete) cross-sectional design subjected to normal force and bending moments (ULS, i.e. ultimate limit state), where basic cross-sectional characteristics (cross-sectional dimensions, steel bars profiles and types of materials) constitute an optimisation space with discrete attributes. The target function (including economical and ecological aspects) and principle problem solution(s) is defined and an illustrative numerical example of a simple rectangular cross-section design is presented. The solution approach is further augmented to RC frame structures problems and a numerical example of a collector tube design is presented.

Keywords: *optimization, building, structures, RBSO*

1. Introduction

The modern concept of structural design should not be result of predetermined material and geometric requirements, but of the assessment of overall design quality. It is necessary to assess structural quality from the point of view of structural reliability, service life, durability, economy and ecology, and this while considering the whole structural lifetime. Therefore, structural design is a matter of complex evaluation based on pre-selected criteria, which then functions as a guideline in the search for a high-quality (optimal) solution. However, even a simple structural design task is commonly, especially in the case of conflicting criteria, so very complicated that the finding of an optimal design is beyond the bounds of human ability. This problem is the subject of optimisation theory, which is inherently connected with computer technology.

The very beginning of optimisation algorithm evolution merely concerned so-called deterministic optimisation (DBSO: Deterministic-Based Structural Optimisation). The DBSO approach is relatively trouble-free and computationally undemanding, which are positive aspects; on the other hand, this is at the cost of the suppression of all the inevitable uncertainties in the mathematical model. The uncertainties are taken into account by the developmentally younger RBSO (Reliability-Based Structural Optimisation) approach. The introduction of uncertainties makes the task of optimisation appreciably more difficult, both from the theoretical and computational point of view (the volume of computational requirements rapidly increases). Nowadays, RBSO is the object of intensive research because its applicability for real structures has only been possible for the last decade, due to the growing

* Ing. J. Plšek, prof. RNDr. Ing. P. Štěpánek, CSc., Department of Concrete and Masonry Structures, Faculty of Civil Engineering, Brno University of Technology, Czech Republic

power and capacity of computers. Even though theoretically RBSO provides qualitatively more valuable results then DBSO, practical arguments guarantee DBSO its significance both now and in the future. The following text is concerned with DBSO, while RBSO issues are dealt with in a follow-up article.

DBSO algorithms can be classified according to computational principle in two classes: standard and stochastic algorithms (stochastic algorithm to solve DBSO tasks). Standard algorithms have their origins in the early period of development, when the small computational capacity available was a substantial factor influencing their fundamental nature. They make demands on the task definition (e.g, the occurrence of constraints, the linearity/non-linearity of target function(s) and constraints, smoothness etc.) to reach maximal effectivity (speed) and minimal resources (memory) utilising required assumptions. By contrast, stochastic algorithms are very robust, slow and require a huge computational effort because they utilise a semi-random search of a design space (space of allowed solutions).

Conventional methods have an invariable calculation progression, as well as results which depend solely on the initial solution. Each iteration improves on the currently achieved solution, usually taking advantage of derivatives, until a locally non-improveable point is reached. Therefore, the search process usually finishes with the detection of local optima, instead of global optima. Tasks without restrictive conditions can be solved with the 'line search' technique, where an originally multi-dimensional problem is transformed via direction vector into a one-dimensional problem which is subsequently solved with appropriate methods such as Fibonacci's Comparision Sequence, Regula Falsi and Newton's methods to achieve an extreme value. The technique of direction vector determination specifies the particular method (e.g. permissible directions method, gradient projection method). Newton's methods approximate functions with first or second order Taylor polynomials; algorithms for linear or quadratic programming can be applied to them (e.g. the well known simplex algorithm for purely linear tasks). Problems with constraints can be mathematically reformulated so that the resulting formulation is constraint-free, and the constraints are taken into account by attaching them into a target function to worsen a target function value if the constraints are violated (penalty method) or close to violation (barrier method). Probably the most successful method for optimisation of smooth non-linear tasks is the GRG-based method (Generalized Reduced Gradient method). For linear integer tasks, there are well known methods like the Bounding Hyperspheres (Hyperplanes) method, Gomory's method and the 'Branch and Bound' method (also applicable to non-linear tasks).

Stochastic algorithms (SA) demonstrate great robustness and the capability to avoid local minima captures; theoretically, some SA algorithms can guarantee the detection of the global optimum by assuming an infinite number of iterations. SA are based on evaluations of randomly generated trial designs. With help of specific techniques (strategies), there is a tendency (not a strict rule) to achieve successively better designs, thus the global optimum should be found in the case of unlimited resources (impossible). These strategies found inspiration in processes observed in nature (biological, physical, chemical), and this is partially reflected in their names. Among the best known algorithms are Genetic Algorithms, Evolution Strategies and Simulated Annealing. Newer ones are e.g. Ant Colony Optimisation, Particle Swarm Optimisation, Greedy Randomized Adaptive Search Procedure algorithm and Harmony Search.

2. General DBSO formulation

2.1. DBSO – monocriterial

DBSO problematics can be formulated as a mathematical programming task. Let the following be valid

$$\vec{x} \in \Omega \subset \mathbb{R}^m(\mathbb{Z}^m) \ , \quad \vec{f} : \Omega \longrightarrow \mathbb{R} \ .$$

Let's find such a vector $\vec{x}_{\text{opt}} \in \Omega$, so that (assuming minimisation) the following requirement holds true

$$\vec{x}_{\text{opt}} = \arg \left\{ \min_{\vec{x}} f(\vec{x}) \right\} \ , \tag{1}$$

where all restrictive conditions in the form of equalities

$$\vec{h}(\vec{x}_{\text{opt}}) = 0 \tag{2}$$

and in the form of inequalities

$$\vec{g}(\vec{x}_{opt}) \leq 0 \tag{3}$$

are satisfied. Symbols in (1)–(3) mean

- $\vec{x}$ .... vector of design variables, generally either continuous or discrete (or both kinds), defined for the design space Ω,
- $f(\vec{x})$ .... target function, which serves to measure the quality of individual designs,
- $\vec{h}(\vec{x})$ .... vector of restrictive conditions in the form of equalities,
- $\vec{g}(\vec{x})$ .... vector of restrictive conditions in the form of inequalities.

2.2. DBSO – multicriterial

In real situations, more then one criterion is taken into account, so a multi-criteria task (MLT)

$$\vec{f} : \Omega \longrightarrow \mathbb{R}^n$$

is obtained. The optimal solution cannot be understood as a simple minimum or maximum of a single function as in the case of mono-criteria tasks (MNT). Symbolically, the requirement can be written as follows

$$\vec{x}_{\text{opt}} = \arg \left\{ \operatorname*{bal}_{\vec{x}} \vec{f}(\vec{x}) \right\} \ , \tag{4}$$

where the operator 'bal' means the requirement for mutually balanced criteria; however, it is not explicitly defined, and thus such formulation lacks definition. To circumvent the lack of task definition, the solution can be required as a pareto-optimal set, consisting of so-called pareto-optimas. A pareto-optimum is a feasible design point, compared with which any

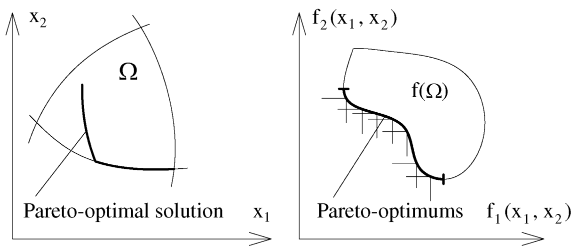

Fig.1: An example of a pareto-optimal set in the case of two design variables and two criteria which are supposed to be minimised

other feasible design point is worse in at least one criterion (e.g. example in Fig. 1). Solution methods consist in the transformation of a MLT into a corresponding MNT. In this manner it is possible to find elements of the pareto-optimal set.

2.3. Target function for building structures

A practical optimisation tool must remember and incorporate all essential aspects. The following multi-criteria objective function fulfils such a principle

$$\vec{f} = \left\{ \begin{array}{c} \max R_{\mathrm{tot}} \\ \min C_{\mathrm{tot}} \\ \min E_{\mathrm{tot}} \\ \max S_{\mathrm{tot}} \end{array} \right\} , \tag{5}$$

where the stated quantities mean

R_{tot} .... the total reliability of the structure under design,
C_{tot} .... investment and operating costs,
E_{tot} .... total impact on the environment,
S_{tot} .... social and cultural quality index.

The term 'cost of building' is often understood as meaning investment cost only, and not the total cost, which consists of items related to the constituent phases of the concrete structure's life cycle

$$C_{\mathrm{tot}} = C_{\mathrm{ini}} + C_{\mathrm{oper}} + C_{\mathrm{m}} + C_{\mathrm{demol}} + C_{\mathrm{recycl}} + \sum p_{\mathrm{f}}\, C_{\mathrm{repair}} + \sum p_{\mathrm{renov}}\, C_{\mathrm{renov}} \ , \tag{6}$$

where the constituent elements represent partial costs related to:

C_{ini} .... plan, projection and construction,
C_{oper} ...... structure operation,
C_{m} .... structure maintenance,
C_{demol} ..... structure demolition,
C_{recycl} ..... recycling of structural material,
C_{repair} ..... repair costs with failure probability p_{f},
C_{renov} ..... reconstruction costs with probability of reconstruction p_{renov}.

A similar formula as in the case of total costs, related to the life-cycle, can be assembled for environmental impacts E_{tot} as well

$$E_{\mathrm{tot}} = E_{\mathrm{ini}} + E_{\mathrm{oper}} + E_{\mathrm{m}} + E_{\mathrm{demol}} + E_{\mathrm{recycl}} + \sum p_{\mathrm{f}}\, E_{\mathrm{repair}} + \sum p_{\mathrm{renov}}\, E_{\mathrm{renov}}, \tag{7}$$

where the constituent elements represent environmental impacts related to:

E_{ini} .... plan, projection and construction,
E_{oper} ...... structure operation,
E_{m} .... structure maintenance,
E_{demol} ..... structure demolition,
E_{recycl} ..... recycling of structural material,
E_{repair} ..... repair costs with failure probability p_{f},
E_{renov} ..... reconstruction costs with probability of reconstruction p_{renov}.

The structural reliability R_{tot} need not necessarily be taken as one design criterion inside the objective function, but it can play the role of restrictive conditions, where a minimal reliability level is given. Consideration of the probability quantities R_{tot}, p_{f} and p_{renov} as optimisation tasks is part of the problematics of RBSO. Some components of socio-cultural criterion S_{tot}, e.g. aesthetics, represent a very soft problem, which is hard to determine well algorithmically, therefore a verbal description is used and their inclusion is not recommended.

3. Formulation of a DBSO RC cross-section task

Let's assume a reinforced concrete (RC) cross-section of any shape and reinforcement, which is allowed to contain inner openings and whose perimeter is approximated by closed polygons, located in a Cartesian coordinate system O_{yz}. It consists of a region filled with concrete Ω_{c} and a region filled with reinforcement bars Ω_{s}. The shape of Ω_{c} depends on geometrical parameters $\vec{r}$, where the reinforcement bars are approximately circular in cross-section, thus are characterised with diameters ϕ_i, where $i \in \{1,2,\dots,N_\phi\}$, and their locations are defined e.g. by the vector of axial cover $\vec{c}$, see Fig. 2. Due to Ω_{c} size, Ω_{s} is considered to be a set of 'mass points' characterised by the cross-sectional area of reinforcement rebar. Material characteristics depend on real numbers R_{c} (concrete) and R_{s} (steel) representing strength classes. Cross-sectional parameters $\vec{r}$, $\vec{\phi}$, R_{c} and R_{s} have not yet been determined, therefore they constitute the vector of design variables $\vec{x}$, i.e. $\vec{x} = (\vec{r}, \vec{\phi}, R_{\mathrm{c}}, R_{\mathrm{s}})^{\mathrm{T}}$. The design feasibility is governed by definition domains of design variables and constraints (see Sect. 3.2); the design quality is measured by the target function (see Sect. 3.1). The cross-section is stressed by a set of given load effects $\vec{LE}$, where $\vec{LE}_j = (N_j, M_{\mathrm{y},j}, M_{\mathrm{z},j})^{\mathrm{T}}$, $j \in \{1,2,\dots,N_{LE}\}$, i.e. by the interaction of normal force N and bending moments M_{y} and M_{z}; positive orientations depicted in Fig. 3. $\vec{LE}_j$ represent possible inner forces acting inside a bar member of a structure subjected to load. The designed cross-section must be strong enough to bear each of the load effects.

Briefly, we are looking for such a design ($\vec{x}_{\mathrm{opt}}$), which:
- minimizes target function (see Sect. 3.1),
- bears given load effects $\vec{LE}_j$ (see Sect. 3.2),
- must meet all additional restrictive conditions.

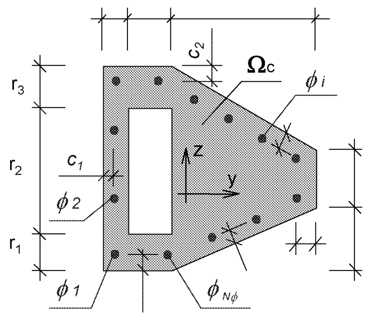

Fig.2: General cross-section; the size is given geometric parameters $\vec{r}$, the amount of reinforcement by bar diameters $\vec{\phi}$ and bar positions by e.g. axial cover $\vec{c}$ (number of bars is pre-selected)

3.1. Target function

We are interested in economic and ecological aspects (acquisition costs and CO_2 and SO_X emissions associated with concrete member formation, respectively). The problem is

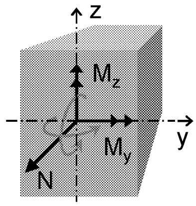

Fig.3: The Cartesian coordinate system and the selected convention of inner forces and moments are for the sake of better aptness both pictured as vectors (double vectors) and arcs in the plane of bending

multi-criteria and all the above-mentioned aspects should be minimised. The method of weighted sums is applied to obtain a single objective function, so the original and resulting tasks are not quite equivalent. The terms of the objective function have different units and thus cannot be directly summed, and for that reason they must first be normalized by chosen reference values. The resulting target function is as follows

$$T_{\mathrm{f}}(\vec{x}) = \alpha_{\mathrm{c}}\,\frac{C(\vec{x})}{C_0} + \alpha_{\mathrm{co}}\,\frac{CO(\vec{x})}{CO_0} + \alpha_{\mathrm{so}}\,\frac{SO(\vec{x})}{SO_0}\ , \tag{8}$$

$$C(\vec{x}) = V_{\mathrm{c}}(\vec{x})\,U_{\mathrm{c}}^{\mathrm{c}}(R_{\mathrm{c}}) + m_{\mathrm{s}}(\vec{x})\,U_{\mathrm{s}}^{\mathrm{c}}(R_{\mathrm{s}}) + S_{\mathrm{f}}(\vec{x})\,U_{\mathrm{f}}^{\mathrm{c}}\ , \tag{9}$$

$$CO(\vec{x}) = V_{\mathrm{c}}(\vec{x})\,U_{\mathrm{c}}^{\mathrm{co}} + m_{\mathrm{s}}(\vec{x})\,U_{\mathrm{s}}^{\mathrm{co}}\ , \tag{10}$$

$$SO(\vec{x}) = V_{\mathrm{c}}(\vec{x})\,U_{\mathrm{c}}^{\mathrm{so}} + m_{\mathrm{s}}(\vec{x})\,U_{\mathrm{s}}^{\mathrm{so}}\ , \tag{11}$$

where the quantities included in the functions are:

$T_{\mathrm{f}}(\vec{x})$ .... the target function dependent on the vector of design variables $\vec{x}$,

$C(\vec{x})$, $CO(\vec{x})$, $SO(\vec{x})$ .... acquisition costs, amount of CO_2 and SO_X emissions,

C_0, CO_0, SO_0 .... reference values set by the problem designer (constants),

α_{c}, α_{co}, α_{so} .... summation weights in the objective function for C, CO_2 and SO_X, respectively,

$V_{\mathrm{c}}(\vec{x})$, $m_{\mathrm{s}}(\vec{x})$, $S_{\mathrm{f}}(\vec{x})$ ..... concrete volume, steel weight and form area of a $1m'$ long beam fragment,

$U_{\mathrm{c}}^{\mathrm{c}}(R_{\mathrm{c}})$, $U_{\mathrm{s}}^{\mathrm{c}}(R_{\mathrm{s}})$, $U_{\mathrm{f}}^{\mathrm{c}}$ .... unit costs of concrete, steel and form, respectively,

$U_{\mathrm{c}}^{\mathrm{co}}$, $U_{\mathrm{s}}^{\mathrm{co}}$ .... unit amounts of CO_2 emissions from concrete and steel, respectively,

$U_{\mathrm{c}}^{\mathrm{so}}$, $U_{\mathrm{s}}^{\mathrm{so}}$ .... unit amounts of SO_X emissions from concrete and steel, respectively.

3.2. Restrictive conditions

In addition to fundamental restrictions following from from the definition domains of design variables, the ultimate limit state (ULS) conditions of a cross-section stressed by the interaction of normal force N and bending moments M_{y}, M_{z} must be met for every load effect $\vec{LE}_j = (N_j^\circ, M_{\mathrm{y}j}^\circ, M_{\mathrm{z}j}^\circ)^{\mathrm{T}}$, kde $j \in \{1,2,\dots,N_{LE}\}$. The assessment methodology proceeds from the concrete standard [3] (possibly from [4], provided methodologies are similar). ULS conditions constitute the main group of constraints and consists of the following system

$$N_j^\circ = N^{\mathrm{R}}(\varepsilon_{\mathrm{c}}^j, K_{\mathrm{y}}^j, K_{\mathrm{z}}^j, \vec{x}) = \iint\limits_{\Omega_{\mathrm{c}}(\vec{r})} \sigma_{\mathrm{c}}(\varepsilon^j(y,z), R_{\mathrm{c}})\,\mathrm{d}y\,\mathrm{d}z + \sum_{i=1}^{N_\phi} F_{\mathrm{s},i}^j\ , \tag{12}$$

$$M^{\circ}_{\mathrm{z},j} = M^{\mathrm{R}}_{\mathrm{z}}(\varepsilon^j_{\mathrm{c}}, K^j_{\mathrm{y}}, K^j_{\mathrm{z}}, \vec{x}) = -\iint\limits_{\Omega_{\mathrm{c}}(\vec{r})} y\,\sigma_{\mathrm{c}}(\varepsilon^j(y,z), R_{\mathrm{c}})\,\mathrm{d}y\,\mathrm{d}z - \sum_{i=1}^{N_\phi} y_i\,F^j_{\mathrm{s},i}\ , \tag{13}$$

$$M^{\circ}_{\mathrm{y},j} = M^{\mathrm{R}}_{\mathrm{y}}(\varepsilon^j_{\mathrm{c}}, K^j_{\mathrm{y}}, K^j_{\mathrm{z}}, \vec{x}) = \iint\limits_{\Omega_{\mathrm{c}}(\vec{r})} z\,\sigma_{\mathrm{c}}(\varepsilon^j(y,z), R_{\mathrm{c}})\,\mathrm{d}y\,\mathrm{d}z + \sum_{i=1}^{N_\phi} z_i\,F^j_{\mathrm{s},i}\ , \tag{14}$$

$$F^j_{\mathrm{s},i} = A_{\mathrm{s},i}\,\sigma_{\mathrm{s}}(\varepsilon^j(y_i,z_i), R_{\mathrm{s}})\ , \tag{15}$$

$$\varepsilon^j(y,z) = \varepsilon^j_{\mathrm{c}} + K^j_{\mathrm{y}}\,y + K^j_{\mathrm{z}}\,z\ , \tag{16}$$

$$\varepsilon^j_{\mathrm{c,\,min}} = \min_{\Omega_{\mathrm{c}}(\vec{r})}(\varepsilon^j(y,z)) \geq \varepsilon^{\circ}_{\mathrm{c,\,min}}(R_{\mathrm{c}})\ , \tag{17}$$

$$\varepsilon^j_{\mathrm{s,\,min}} = \min_{i=1}^{N_\phi}(\varepsilon^j(y_i,z_i)) \geq \varepsilon^{\circ}_{\mathrm{s,\,min}}(R_{\mathrm{s}})\ , \tag{18}$$

$$\varepsilon^j_{\mathrm{s,max}} = \max_{i=1}^{N_\phi}(\varepsilon^j(y_i,z_i)) \leq \varepsilon^{\circ}_{\mathrm{s,max}}(R_{\mathrm{s}})\ , \tag{19}$$

$$y_i = y_i(\vec{r})\ , \tag{20}$$

$$z_i = z_i(\vec{r})\ , \tag{21}$$

$$A_{\mathrm{s},i} = \frac{\pi\,\phi_i^2}{4}, \tag{22}$$

for each $j \in \{1, 2, \ldots, N_{LE}\}$. Variables $\varepsilon^j_{\mathrm{c}}$, K^j_{y}, K^j_{z} are so-called strain parameters corresponding to each load effect $\vec{LE}_j$. The function (16) is linear, which comes from the assumption that the cross-section remains planar after deformation. With steel, both tensile and compressive strain are restricted by inequalities (18) and (19), where $\varepsilon^{\circ}_{\mathrm{s,min}}$ and $\varepsilon^{\circ}_{\mathrm{s,max}}$ are perimissible extremes. With concrete, just compressive strain is restricted by an inequality (17), where $\varepsilon^{\circ}_{\mathrm{c,min}}$ is the permissible extreme. Functions $\sigma_{\mathrm{c}}(\varepsilon)$ and $\sigma_{\mathrm{s}}(\varepsilon)$, used in equations (12) to (15) mathematically describe the shape of stress-strain diagrams of con-

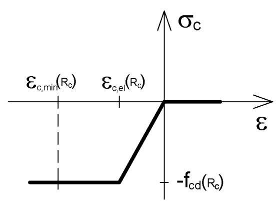

Fig.4: Idealized design stress-strain diagram of concrete used for ULS; the particular characteristic values of the trilinear curve depend on the concrete strength class (represented by the design variable R_{c})

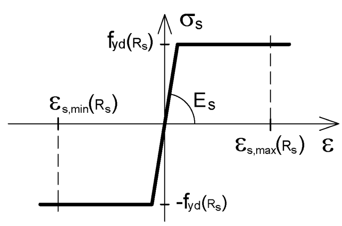

Fig.5: Idealized design stress-strain diagram of steel used for ULS; the particular characteristic values of the trilinear curve depend on the steel strength class (represented by the design variable R_{s})

crete and steel, respectively, see Fig. 4 and 5. The variable $F^j_{\mathrm{s},i}$ is normal force induced in reinforcement rebar i, $i \in \{1, 2, \ldots, N_\phi\}$, corresponding to load effect $\vec{LE}_j$. Reinforcement is characterised by coordinates $[y_i, z_i]$ and the cross-sectional area $A_{\mathrm{s},i}$.

The remaining constraints are additional, and have mainly geometrical significance, related to particular cases, e.g. the maximal permissible ratio of rectangular shape dimensions, the specification of rebar positions in reference to the cross-section border, the diameter uniformity of selected bars etc.

4. Problem solution

The solution was implemented in the GAMS programmatic system (General Algebraic Modelling System, see [1]). GAMS is generally designed for mathematical programming tasks, it provides an IDE (integrated development environment) for task definition according to a special syntax and a set of various optimisation solvers (licences are required). A solver which proved itself effective was Conopt, see [2]. Conopt is one of the so-called GRG (General Reduced Gradient) solvers, which are applicable to continuous (and smooth) nonlinear tasks (continuous definition domains of design variables and smooth continuous functions). With the help of Conopt, tasks with discrete demands were also solved (see 4.2).

4.1. Continuous reformulation

GRG solvers assume the connectivity of all quantities and smoothness of included functions. This requirement is predominantly met, though a problem occurs with the inequalities (17), (19) and (18), because they incorporate the nonsmooth functions 'min' and 'max'. They can be replaced with an equivalent set of inequalities

$$\varepsilon^j(y_k, z_k) \geq \varepsilon^\circ_{\mathrm{c,min}}(R_\mathrm{c}) \ , \quad k \in \{1, \ldots, N_P\} \ , \tag{23}$$

$$\varepsilon^j(y_i, z_i) \geq \varepsilon^\circ_{\mathrm{s,min}}(R_\mathrm{s}) \ , \quad i \in \{1, \ldots, N_\phi\} \ , \tag{24}$$

$$\varepsilon^j(y_i, z_i) \leq \varepsilon^\circ_{\mathrm{s,max}}(R_\mathrm{s}) \ , \quad i \in \{1, \ldots, N_\phi\} \ , \tag{25}$$

where $[y_k, z_k]$, $k \in \{1, \ldots, N_P\}$ are vertexes of the Ω_c polygonal border. Integral functions in (12), (13) and (14) were replaced with numerical integration (Gauss's quadrature). When neglecting the discrete nature of the design variables by substitution of original definition domains with corresponding continuous intervals, a relaxed formulation of the task is obtained. The relaxed form can be solved with standard nonlinear solvers, such as GRG-based Conopt. The relevant 'relaxed' results are then utilised when seeking the correct discrete solution (see 4.2). In the simplest cases, the 'relaxed' results can be rounded to the closest discrete values.

4.2. Satisfaction of discrete demands

For the sake of satisfaction of discrete demands on design variables, the relaxed form of the task and related 'relaxed' results are used, see 4.1. Two strategies were tested, symbolically denoted as 'P-to-S' and 'S-to-P', where 'S-to-P' was designed just for comparative reasons. The calculation consists of cycles which lead to gradual specification of the final solution in the 'variable by variable' manner. In each cycle, the resulting discrete value of one design variable is determined ('fixed') and in the succeeding cycles it acts as a constant.

The number of 'fixed' variables increases until all variables are 'fixed' and the calculation is finished. The order of 'fixations' depends on the influence of the design variables on the target function. The most influential variables (primary) are preferred to the rest of the variables (secondary), thereof comes the denomination 'P-to-S'. Denomination 'S-to-P' expresses the total opposite (secondary variables are preferred to primary); the relevant results should be worse.

5. Example 1 : Cross-section optimisation

Let's assume a rectangular cross-section as shown in Fig. 6, which must sustain three load cases as stated in table 2. For the sake of simplicity fixed materials are considered, i.e. concrete C16/20 and steel B245(K). The rebar numbering is obvious from Fig. 6, the available rebar profile assortment for corner rebars (numbers 1, 3, 5, 7) are 10, 12, 16, 18 and 20 mm, and for inner rebars (numbers 2, 4, 6, 8) they are 0, 10, 12, 16, 18 and 20 mm; for further diameter uniformity $\phi_5 = \phi_7$ and $\phi_6 = \phi_8$ is required. The cross-sectional dimensions b and h range from 400 mm to 600 mm with a difference of 50 mm (400 mm, 450 mm etc.). Multi-criteria optimisation was performed with the target function according to section 3.1, where the summation weights are $\alpha_{\mathrm{c}} = 0.50$ and $\alpha_{\mathrm{co}} = \alpha_{\mathrm{so}} = 0.25$ and the reference values obtained from the mono-criteria optimisation (see table of results 3). For the comparative reasons, four computational methods were utilised. The 'Comb' method denotes searching the design space by testing all possible combinations of the discrete values of the variables. The 'Cont', 'P-to-S' and 'S-to-P' methods denote optimisation as described in sections 4.2.

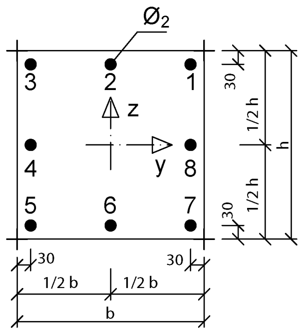

Fig.6: Considered cross-section, design variables are apparent

Item	Unit	C16/20 $1\,\mathrm{m}^3$	B245(K) 1 ton
Price	CZK	1926	12
CO_2	kg	290	768
SO_2	kg	1.01	3.63
$\varepsilon_{\min}$	‰	−3.5	−3.5
$\varepsilon_{\min}$	‰	–	10.0

Tab.1: Parameters of the job (constants)

LE	N kN	M_{y} kNm	M_{z} kNm
1	−350	−50	80
2	−200	−90	−120
3	−150	40	−80

Tab.2: Values of loading effects

6. Example 2 : Collector tube optimisation

A collector tube according to Fig. 8 is optimised. The collector is a linear structure and allows a simplified tube analysis, where a 1m-wide slice of the structure is regarded

Criterion		Price	CO2	SO2	Weights 50 – 25 – 25			
Optim		Cont	Cont	Cont	Cont	Comb	P-to-S	S-to-P
ϕ_1		15.1	20	19.2	17.2	12	20	18
ϕ_3		20	20	20	20	20	12	20
ϕ_5		20	20	20	20	20	20	20
ϕ_7		20	20	20	20	20	20	20
ϕ_2	mm	0	10.9	0	0	0	0	0
ϕ_4		0	16.0	0	0	0	0	0
ϕ_6		20	20	20	20	20	20	20
ϕ_8		20	20	20	20	20	20	20
b	mm	584.9	530.5	546.6	559.6	600	550	550
h	mm	400	419.2	420.6	414.11939	400	450	450
StArea	mm^2	1750	2180	1861	1804	1684	1684	1825
CcArea	mm^2	233974	222387	229926	231730	240000	247500	247500
Price	Kč	**615.5**	633.6	618.1	616.2	620.9	635.3	648.6
CO2	kg	78.4	**77.6**	77.9	78.1	79.8	81.9	82.8
SO2	kg	0.286	0.287	**0.285**	0.285	0.290	0.298	0.302
Target funct.		615.5	77.6	0.285	1.0022	1.0157	1.0411	1.0582

Tab.3: Results of multi-criteria optimisation; reference values for the target function were obtained in the first three columns with the help of mono-criteria optimisation; computational methods and summation weights α are stated in the column headings

a) P-to-S

Variable assigned	/	$\phi_6=\phi_8$	$\phi_5=\phi_7$	b	h	ϕ_2	ϕ_4	ϕ_3	ϕ_1
Value mm	/	20	20	550	450	0	0	12	20
Target function	1.0022	1.0022	1.0022	1.0024	1.0369	1.0369	1.0369	1.0373	1.0411

b) S-to-P

Variable assigned	/	$\phi_5=\phi_7$	ϕ_3	ϕ_2	$\phi_6=\phi_8$	ϕ_4	ϕ_1	b	h
Value mm	/	20	20	0	20	0	18	550	450
Target function	1.0022	1.0022	1.0022	1.0022	1.0022	1.0022	1.0024	1.0024	1.0582

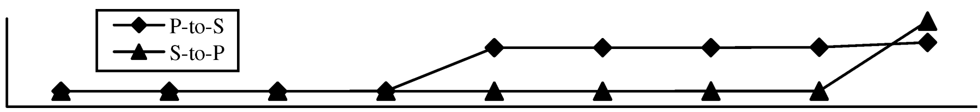

Fig.7: A detailed comparison between the P-to-S (a) and S-to-P (b) methods; the sequential growth in the amount of fixed design variables causes a gradual target function increase (the degree of freedom descends)

as a 2D frame (Fig. 9). The tube's design lies in the determination of the thickness of the walls t_{s}, ceiling slab t_{h} and bottom slab t_{d}, including the reinforcement design of the cross-sections $A_1, \ldots, A_7$ (or more precisely rebar diameters $\phi_1, \ldots, \phi_7$), see Fig. 9. The frame is vertically symmetrical, including the envelope of inner forces, therefore it is possible to only analyse cross-sections of half of the structure (see Fig. 9). The target function criteria involve both economical, ecological and structural aspects (see Sect. 4.1). Three types of reinforcement were considered – metallic (ordinary reinforcing steel and stainless steel) and also non-metallic GFRP (Glass Fibre Reinforced Polymer). The ultimate collector bearing capacity (ultimate limit state) is evaluated in seven critical cross-sections (see Fig. 9), which are stressed by the interaction of normal force and bending moment. With regards to the serviceability limit state, the flexural rigidity of the same cross-sections is monitored. The problem thus involves the extension of the task of optimising a single RC cross-section to deal with a structure where more cross-sections are optimised in parallel. Considerable modification has occurred to loading effects, i.e. the internal forces of certain bars at given cross-sections (see Fig. 9). These forces are not constant, because their distribution in statically indeterminate structures depends on cross-sectional rigidities, thus on their heights (the thickness of the wall's heights). These heights are, however, the object of optimisation, therefore structural analysis has to be a constituent of the optimisation model. Design variables are considered to be continuous and range over given intervals, i.e. t_{d}, t_{h}, $t_{\mathrm{s}} \in \langle 100, 600\rangle$ mm, $\phi_i \in \langle 6, 30\rangle$ mm. The mutual axial distance of load-bearing rebars is fixed at 100 mm.

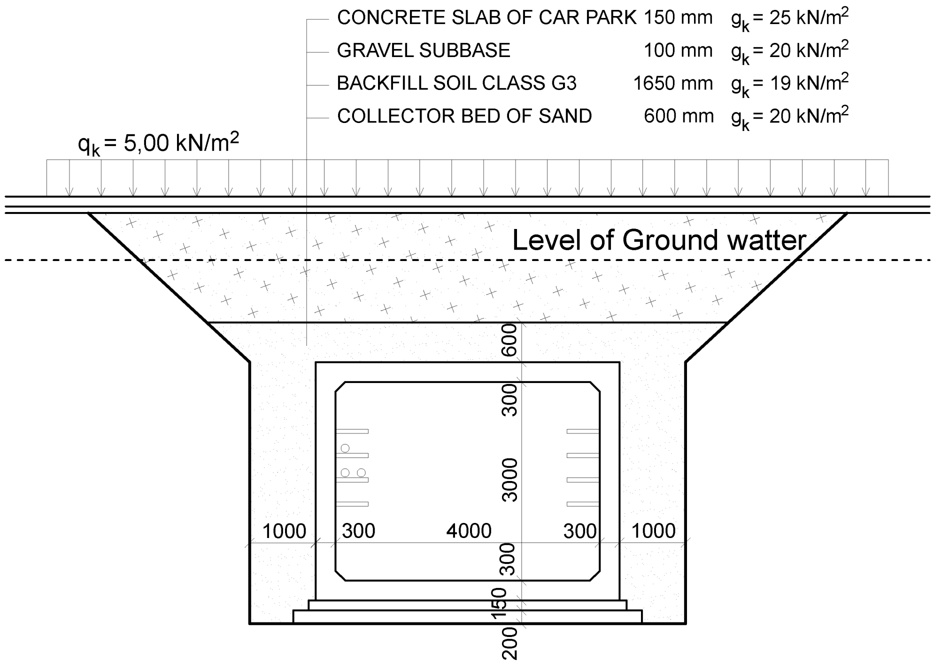

Fig.8: Transversal cross-section of a collector tube; wall, ceiling and bottom thicknesses are chosen for illustrative purposes

6.1. Target function

The target function involves 5 criteria in total (multi-criteria optimisation), i.e. economical (1×), ecological (3×) and structural (1×). Economical criteria include the cost of the concrete necessary for the manufacture of $1m'$ of collector tube, i.e. concrete and reinforcement. In the cost calculation for non-metallic reinforcement, the dependence of unit cost on diameter must be taken into account, unlike with metallic reinforcement. From the ecological point of view, the amounts of emitted CO_2, SO_X and consumed energy E are monitored. The structural aspect is represented by the flexural rigidities B_i of selected critical cross-sections (see Fig. 9). Flexural rigidities are dependent on crack occurrence/development and the strain affecting materials that is caused by loads. In contrast to the other criteria, where we are aiming to minimise costs and ecological impacts, the target is to maximise rigidity, so that structural deformations are reduced. For the sake of integration of the directions of optimisation of the target function, the inverse value $1/B$ is taken into account (should be minimised). The resulting target function is obtained as follows

$$\min T(\vec{x}) = \alpha_{\mathrm{c}} \frac{C}{C_0} + \alpha_{\mathrm{co}} \frac{CO}{CO_0} + \alpha_{\mathrm{so}} \frac{SO}{SO_0} + \alpha_E \frac{E}{E_0} + \alpha_{\mathrm{B}} \sum_i^7 \beta_i \frac{B_{0,i}}{B_i} \,, \tag{26}$$

where C denotes initial costs, CO_2 and SO_X stand for emissions, E denotes consumed energy and B_i is the flexural rigidity of the cross-section number i, $i \in \{1, 2 \ldots, 7\}$, see Fig. 9. Quantities marked with a lower index '0' are corresponding normalising constants, which were derived (calculated) from collector tube at maximal allowable values of the design variables. Factors α and β_i are weighting coefficients, where $\beta_i = 1/7$ and α-s were varied for the sake of parametrical study.

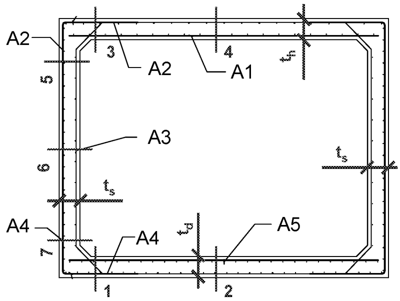

Fig.9: Optimised frame; t_{s}, t_{h}, t_{d}, $A_1, \ldots, A_5$ are design variables; critical sections 1 to 7 are monitored regarding bearing capacity (ultimate limit state) and their flexural rigidities (serviceability limit state)

Material	Energy MJ/kg	CO_2 kg CO_2/kg	SO_x g SO_x/kg	Unit cost	
Concrete C35/45	0.8	0.13	0.5	2385	CZK/m^3
Reinf. steel B490	49	3.2	14.6	23.7	CZK/kg
Stainless steel	113	6.7	303.6	200	CZK/kg
FRP glass	41.6	1.92	14.66	acc. to diam.	CZK/m′

Tab.4: Unit costs and environmental impacts of considered building materials

Material	Diameter mm	Cost CZK/mm^2	Strength MPa	Cost CZK/kN	Relative cost to B490
Steel B490	/	0.186	426	0.437	1.0
Stainless st.	/	1.570	183	8.579	19.6
FRP glass 100 %	5	1.630	423	3.855	8.8
	30	0.308		0.727	1.7
FRP glass 38 %	5	1.630	161	10.143	23.2
	30	0.308		1.914	4.4

Tab.5: Economical-physical comparison of reinforcements; if FRP reinforcement is bent (in corners), its strength is reduced to 38 %

6.2. Scope of parametrical calculations and results

Parametrical study is two-dimensional; on the one hand the types of reinforcement are changed, but on the other hand weight coefficients α are varied (the sum of all α-s is always equal to 1). With respect to α coefficients, the results are divided into two groups. The first group, called 'Cost vs. Rigidity' includes only economical and structural criteria, thus $\alpha_\mathrm{B} \in \langle 0,1\rangle$, $\alpha_\mathrm{c} = 1 - \alpha_\mathrm{B}$ and $\alpha_\mathrm{co} = \alpha_\mathrm{so} = \alpha_\mathrm{E} = 0$ (the environmental criteria are omitted). The second group, called 'Eco vs. Rigidity', includes analogically only economical and ecological criteria, thus $\alpha_\mathrm{c} = 0$ a $\alpha_\mathrm{co} = \alpha_\mathrm{so} = \alpha_\mathrm{E} = (1-B)/3$ (the economical criterion is omitted).

The comparison of 'Cost vs. Rigidity' with "Eco vs. Rigidity" has shown a close correspondence, i.e. both criteria minimise a quantity of a building material. The results practically overlap, therefore only the resulting charts for 'Cost vs. Rigidity' are presented hereinafter. The difference lies in the α_B values at which corresponding results were reached. When gradually decreasing α_B (and increasing the rest of the weight coefficients), i.e. descending cross-section rigidities, four phases were detected both in the 'Cost vs. Rigidity' and the 'Eco vs. Rigidity' variants; only α_B boundary values were different.

The phases are distinguished with regard to the 'behaviour' of the optimum results of design variables. Up to values $\alpha_\mathrm{B} = 1$ to $\alpha_\mathrm{B} = \alpha_{\mathrm{B,X\,max}} < 1$ no changes took place, and so the optimal design corresponded with the maximal possible values (phase 1). By a further α_B decrease, the reinforcement amount dropped at the constant maximal cross-sectional heights (thicknesses of walls, ceiling and bottom), up to the value $\alpha_\mathrm{B} = \alpha_\mathrm{B,reinf}$ (phase 2). Phase 3 was characterised by the second stagnation of results, when cross-sectional heights were maximised and reinforcement amounts were constant. This stagnation remained until $\alpha_B = \alpha_\mathrm{B,t+reinf}$; after this value changes occurred to both the cross-sectional reinforcements themselves, and their heights (phase four). The results presented in the charts are arranged uniformly with regards to the number of testing points, not in accordance with real α_B values.

The initial stagnation was caused by a high preference for cross-section rigidities. In phase 2, the cross-section thicknesses remained unchanged (maximal), because the rigidity is primarily influenced by the height of the cross-section. In the second stagnation (phase 3), the reinforcement rate dropped to a level which was regulated by restrictive conditions (cross-section bearing capacity, minimal diameters) and no further drop was possible. In phase 4, ecological and economical criteria dominated and rigidity was no longer critical, thus cross-section heights didn't remain at their maximums.

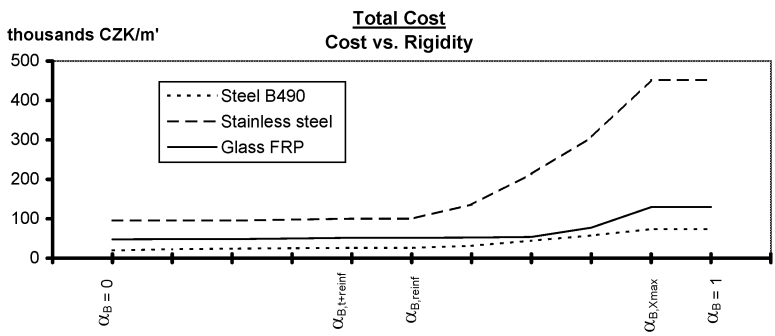

Fig.10: Dependence of total acquisition costs on α_B

7. Conclusions

Every algebraic optimisation task with discrete requirements on design variables can be relaxed, i.e. discrete definition domains are extended by an intermediate region. The relaxed task is solvable with help of standard non-linear solvers and it is the basis for various strategies which were designed to satisfy the original discrete requirements (such as the branch and bound method or the proposed P-to-S method; see Sect. 4.2). In cases when the task cannot be relaxed, it is necessary to use stochastic algorithms or some special method (at worst the combinatory approach). Standard algorithms are fast, but are often trapped by local optima and the final results depend on the initial design. Although the optimisation example of a RC cross-section didn't result in a global optimum determination (verified by a complete combinatorial calculation), the savings considering the achieved solution are practically negligible. As a consequence of the stress-strain diagram modification (relationship $\sigma(\varepsilon)$), various material types can be simulated (as in the case of the collector – metallic and non-metallic materials). On the basis of the proposed cross-section optimisation and choice of critical sections, it is possible to accomplish the optimisation of a structure without significant obstacles; simply the computational requirements are increased.

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