SIMULATION APPROACHES FOR THE EFFICIENT PROBABILISTIC RELIABILITY ASSESSMENT OF A CONCRETE STRUCTURE BY THE SBRA METHOD

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We describe simulation approaches suitable for the full probabilistic reliability assessment of a concrete beam by the finite element method. We compare the direct Monte Carlo method with a variance reduction technique based on Importance Sampling. A material model for the concrete beam is presented. The obtained results of the reliability assessment are discussed.

Keywords: *simulation, reliability, concrete structure, SBRA*

1. Introduction

Detection of rare physical events (for example failures), which usually occurs with low probability, play key role in the probabilistic reliability assessment of engineering structures. The Monte Carlo simulation technique became popular tool for the probabilistic reliability assessment of general systems thanks its great robustness [2, 3, 7, 8].

Unfortunately, when direct Monte Carlo simulation technique is applied for probabilistic reliability assessment of engineering structures, sufficient large number of simulation trials (number of samples) must be computed for detection and statistically evaluation of rare events. This is very unwelcome fact, especially when each simulation trial includes still time-consuming finite element analysis and processing [6, 10]. Besides, an insufficient number of simulation trials bring to the designer inaccurate results even a failure may not be detected at all (!). It is well known fact, that variance reduction techniques, such as Importance Sampling and/or Stratified Sampling can significantly reduced the number of required simulation for a same accuracy in the estimation. Furthermore, some additional information about structural behavior of the problem can again increase the efficiency of the simulation process [1, 2].

The research is motivated by the requirement to use some variance reduction technique as a tool, which should be applied automatically in designer's every-day work [11]. For this reason, a variance reduction technique, which will be used by designers, must works correctly as a 'black box' tool as in the case where it seems that no additional information about the structural behavior of a simulated problem is usually available.

Our experiments indicate that a variance reduction technique, which is based on Importance Sampling, is suitable for estimation of very small failure probabilities. The paper

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is organized in a following way. In Section 2, we will introduce shortly the probabilistic reliability assessment of structures. In Section 3, a description of the assumed model is briefly presented. Preprocessing, processing and postprocessing of the probabilistic reliability assessment task are discussed in Sections 4–6. Finally, Section 7 contains conclusions and future work.

2. Probabilistic reliability assessment of structures

Let the resistance of the structure is expressed by the variable R and load effect by variable S. Let the safety of the structure is expressed using the safety function Z in the following way:

$$Z = R - S \ . \tag{1}$$

The situations where $Z < 0$ represents a failure in the structure, whereas situations $Z > 0$ are safe, see for instance [6], [7] and [8]. Both variables R and S are random by nature and the equation (1) can be rewritten as

$$Z = g(X_1, X_2, \ldots, X_n) \ . \tag{2}$$

Here symbols $X_1, X_2, \ldots, X_n$ denotes random variables, which express a rule geometrical and material characteristics, loadings and optionally effects of other factors and the symbol g denotes the performance function of the structure. For more details see for instance [6], [7] and [8]. Than probability of the failure of the structure can be formulated by the form

$$P_{\mathrm{f}} = P(Z < 0) = P(g(X_1, X_2, \ldots, X_n) < 0) \ . \tag{3}$$

The aim of the probabilistic reliability assessment leads to the reliability check expressed by

$$P_{\mathrm{f}} < P_{\mathrm{d}} \ , \tag{4}$$

where the symbol P_{f} denotes the calculated probability of failure and the symbol P_{d} denotes the target design probability P_{d} given in (expert) codes, see for instance [6], [7] and [8].

The equation (2) can be calculated approximately by FORM and SORM methods, see for instance [6] or directly by the simulation approach, see for instance [7] and [2] and [6].

3. Model description

This example was derived from the Calfem home page [4], see 'CALFEM/Pre user interface tutorial', where it is possible to find the finite element model and its solution via Calfem toolbox, too. In this paper, we extend the original deterministic model by the case where all loads are assumed to be random variables. Moreover, the probabilistic reliability assessment of the structure will be estimated by simulation approach using direct Monte Carlo method and Importance Sampling method.

Consider the concrete frame subjected to a uniformly distributed loads F_1, $F_2, \ldots, F_6$ as shown in Fig. 1. The model has the following deterministic parameters: Young's modulus $E = 10.5\,\mathrm{GPa}$, Poisson's ratio $\nu = 0.15$ and thickness $t = 0.20\,\mathrm{m}$. All loads are assumed to be normal random variables with parameters as shown in Tab. 1.

The frame is discretized using the finite element code CALFEM. We assumed the finite element mesh denoted as Geometry5, see Fig. 2.

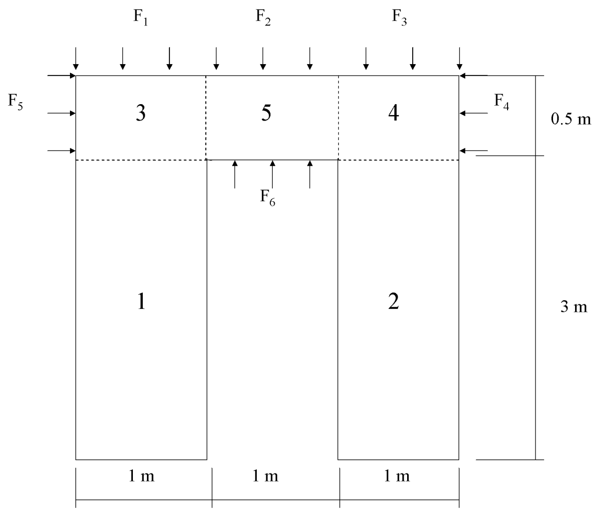

Fig.1: The model problem for the probabilistic reliability assessment; the geometry of the frame contains 5 sub-domains denoted by symbols 1, 2, . . . , 5

Variable name	Mean value	Standard deviation
F_1	15 kN	5 kN
F_2	15 kN	5 kN
F_3	15 kN	5 kN
F_4	4 kN	4 kN
F_5	4 kN	4 kN
F_6	0 kN	4 kN

Tab.1: Parameters of the random loads

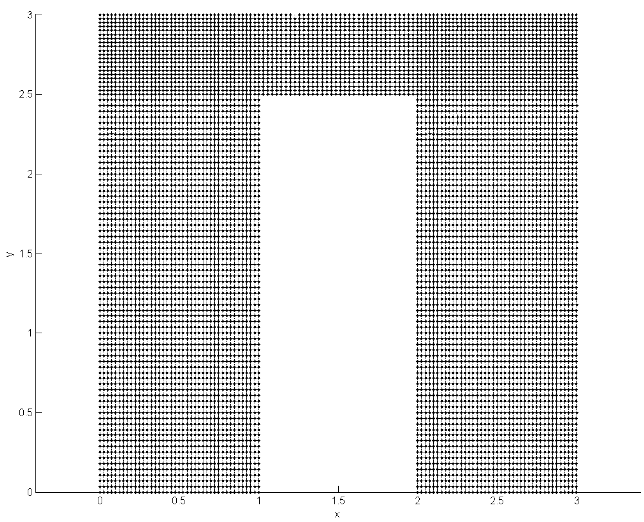

Fig.2: The finite element mesh of the frame using CALFEM (Problem name: FEMFrame2_Geometry5)

In our model, the safety function (1) was expressed in the following way. The R denoted concrete tensile strength described by normal random variable with parameters $R = 1 \pm 0.1\,\mathrm{MPa}$. The S denoted the maximum value of the main principal stress of an element of the structure. For evaluating of deterministic values of S we used modified deterministic Calfem finite element model taken from [4]. The computation of probability of failure by (3) was powered by the SBRA method, see for instance [7], [8] and [11].

4. Preprocessing

In order to detect low probability events on tail areas, we used the variance reduction technique based on Importance Sampling. With the current implementation, the importance sampling density function is set as the uniform distribution on the same domain as the original distribution. Numerical experiments indicated advantages of this selection when no additional information about structural behavior is available. For implementation details see [9, 10]. Moreover, the same probabilistic reliability assessment problem was solved also by the direct Monte Carlo simulation.

Because of the fact that we assume stochastic character of loads in our model, the stochastic contribution of random loads will influence only the right hand side vectors of the linear system of equations.

5. Processing

When the FEM mesh was applied, 1 000 simulation steps were computed, so the corresponding multiple system of linear equations had 1 000 right hand sides and the total number of unknowns was $16\,188 \times 1\,000$.

As the solver of this multiple linear system of equations we used the fast SBCG algorithm [10]. To solve all 1 000 linear systems of equations, only 446 matric-vector operations were needed. Let us notice that the solution of *one* linear system by the classical PCG required 205 matrix-vector operations [10].

6. Postprocessing

The aim of this example was to find a distribution of main principal stresses of elements in the structure. For computation of element stresses $Es = (\sigma_{\mathrm{x}}, \sigma_{\mathrm{y}}, \tau_{\mathrm{xy}})$ from the element displacement vector we used the Calfem call 'planrs'.

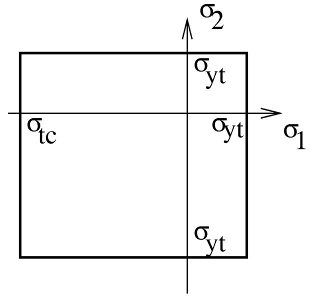

Fig.3: The Rankine–type failure condition

Value	D1Max	D2Max	D3Max	D4Max	D5Max	GlobMax	SF
0	1	1	1	1	1	1	0.911
900000	0.001	0.152	0	0.002	0.045	0.152	0.005
1E+6	0	0.073	0	0	0.013	0.073	0
1.1E+6	0	0.024	0	0	0.005	0.024	0
1.2E+6	0	0.008	0	0	0	0.008	0
1.3E+6	0	0.002	0	0	0	0.002	0
1.4E+6	0	0	0	0	0	0	0
1.5E+6	0	0	0	0	0	0	0
1.6E+6	0	0	0	0	0	0	0
1.7E+6	0	0	0	0	0	0	0
1.8E+6	0	0	0	0	0	0	0
1.9E+6	0	0	0	0	0	0	0
2E+6	0	0	0	0	0	0	0

Tab.2: Probability to exceed selected values of the maximal principal stresses of each domains; direct Monte Carlo results of the probabilistic reliability assessment (MC 1 000 steps)

Value	D1Max	D2Max	D3Max	D4Max	D5Max	GlobMax	SF
0	1	1	1	1	1	1	0.961
9e+5	0.00221	0.224	2.39e−7	0.00408	0.0137	0.226	0.00764
1e+6	0.00203	0.134	3.5e−11	0.00128	0.00739	0.136	0.000208
1.1e+6	6.02e−5	0.0129	0	8.09e−6	0.0041	0.013	1.25e−5
1.2e+6	2.46e−6	0.00452	0	2.98e−7	0.00145	0.00452	5.26e−6
1.3e+6	2.39e−7	0.00401	0	4.27e−9	0.000331	0.00401	0
1.4e+6	1.59e−9	0.00131	0	4.27e−9	5.76e−6	0.00131	0
1.5e+6	3.5e−11	4.08e−5	0	0	3.14e−7	4.08e−5	0
1.6e+6	0	2.67e−6	0	0	4.27e−9	2.67e−6	0
1.7e+6	0	2.45e−6	0	0	4.27e−9	2.45e−6	0
1.8e+6	0	4.42e−9	0	0	0	4.42e−9	0
1.9e+6	0	4.27e−9	0	0	0	4.27e−9	0
2e+6	0	4.27e−9	0	0	0	4.27e−9	0

Tab.3: Probability to exceed selected values of the maximal principal stresses of each domains; importance Sampling results of the probabilistic reliability assessment (IS 1 000 steps)

The postprocessing procedures was based on a simple material model for concrete. The concrete is a material that has different behavior under different loading conditions. The material strength of material in compression is noticeable higher than the strength in tension. The behavior of the material was assumed to be linear elastic and a Rankine–type failure condition was used (Figure 3). This type of failure condition uses principal stresses σ_1, σ_2 that are compared with limit stresses for compression σ_{yc} and for tension σ_{yt}. We calculated for each element in domain the maximum and the minimum values (variables denoted here as σ_1, σ_2) of the main principal stress in the following way:

Algorithm Myprincs [Compute principal stresses]

$$\sigma_1 = 0.5\left(\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}} + \sqrt{(\sigma_{\mathrm{x}} - \sigma_{\mathrm{y}})^2 + 4\,\tau_{\mathrm{xy}}^2}\right) ,$$
$$\sigma_2 = 0.5\left(\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}} - \sqrt{(\sigma_{\mathrm{x}} - \sigma_{\mathrm{y}})^2 + 4\,\tau_{\mathrm{xy}}^2}\right) .$$

The algorithm Myprinc was run in each simulation. These simulation results were subsequently statistically processed, see Tab. 2, Tab. 3. In order to obtain information about the distribution of the maximum value of the main principal stresses in geometry, the reliability analysis was computed in five sub-domains of the structure separately. Tables contain results of the variable σ_1 taken from the algorithm Myprincs. For instance, the column denoted as $D5max$ contains probabilities of exceeding values of the first column of the table in the geometry domain no. 5.

The minimum observed value of σ_2 was -1.25×10^6 for Importance Sampling method. This observed value is very far from the critical value -20 MPa, so results of σ_2 were not printed.

Analyzing results of Tab. 2 we can see that the direct Monte Carlo method did not detect extrem events in which the variable σ_1 was greater than 1.4×10^6 at all. On the other hand, Importance Sampling method detected low probability cases $\sigma_1 > 2\times10^6$.

The row of Tabs. 2 and 3 denotes as 'SF' contain results of the safety function (1). The probability of a failure was estimated by direct Monte Carlo method as $P_\mathrm{f} = 1 - 0.911 = = 0.0890$. When Importance Sampling was applied, the probability of failure was estimated as $P_\mathrm{f} = 1 - 0.961 = 0.0390$.

Let us accent that the Importance Sampling approach benefits the detection of low probability (critical) events.

7. Conclusions and future work

In this paper, we present results of the probabilistic reliability assessment of a concrete beam by the direct Monte Carlo method with a variance reduction technique based on Importance Sampling. These simulation methods were compared on a linear elasticity model of the structure. The behavior of the structure was assumed to be linear elastic and a Rankine-type failure condition was used. Our experiments indicate that the Importance Sampling method is suitable for estimation of very small failure probabilities. In future work we would like to solve effectively real 3D large large scale reliability problems.

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References

[1] Beranger M., Laurent B.: Quantification of rare accidental events on nuclear power plant, using Monte Carlo simulation, In ESREL 2001: European Safety & Reliability International Conference, pages 871–878, Torino, Italy, 2001

[2] Bucher C., Macke M.: Time-variant reliability – Computational approaches based on FORM and importance sampling, In AMAS Course on Reliability-Based Optimization RBO'02, pages 9–34, Warsaw, September 23–25, 2002

[3] Briš R. and Praks P.: Simulation Approach for Modeling of Dynamic Reliability using Time Dependent Acyclic Graph, Special Issue of the International Journal of Polish Academy of Sciences 'Maintenance and Reliability' Nr 2(30)/2006, Warsaw, Ed. I. B. Frenkel, A. Lisnianski, pg. 26–28, ISSN 1507-2711, http://darmaz.pollub.pl/ein/fultext/30.pdf (as of March 13th, 2008)

[4] CALFEM: Project Web Hosting – Open Source Software, http://calfem.sf.net
[5] Feng Y.T., Owen D.R.J., Peric D.: A block conjugate gradient method applied to linear system with multiple right hand sides, Comput. Methods in Appl. Mech. Eng., 127 1995, pp. 203–215
[6] Haldar A., Mahadevan S.: Reliability Assessment using Stochastic Finite Element Analysis, Willey, New-York, ISBN 0-471-36961-6, 2000
[7] Marek P., Guštar M., Anagnos T.: Simulation Based Reliability Assessment, CRC Press Inc., Boca Raton, Florida, 1995
[8] Marek P., Brozzetti J., Guštar M. (eds.): Probabilistic assessment of structures using Monte Carlo simulation: background, exercises and software, CRC Press, Inc., Boca Raton, Florida, U.S.A., ISBN 80-86246-08-6, 2001
[9] Praks P., Konečný P.: Direct Monte Carlo Method vs. improved methods considering applications in designers every day work, Chapter 23 in book: Marek P., Brozzetti J., Guštar M., Tikalsky P. (eds.): Probabilistic Assessment of Structures using Monte Carlo Simulation. Basics, Exercises, Software (Second edition), Published by ITAM – Academy of Sciences of the Czech Republic, 2003 (10 pages, CD-ROM), ISBN: 80-86246-19-1
[10] Praks P.: Sensitivity analysis using iterative solvers, Ph.D. Thesis, in Czech. VŠB-Technical University Ostrava, 2006
[11] Simulation based reliability assessment method SBRA, http://www.itam.cas.cz/sbra

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