OPTIMISATION OF REINFORCEMENT OF RC FRAMED STRUCTURES

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This paper presents the entire formulation of longitudinal reinforcement minimisation in a concrete structure of known sections and shape under loading by normal force and bending moment. Constraint conditions are given by the conditions of structure reliability in accordance with the relevant codes for ultimate strength and applicability of the sections specified by a designer. Linearization of the non-linear formulation is described, and possibilities of applying linear programming algorithms are discussed. The functioning of the process described is demonstrated on a plane frame structure design.

Keywords: *optimisation, reinforcement, frame*

1. Introduction

The methods for optimising the design of linear elastic structures from the standpoint of minimal cost, shape, or volume of materials are highly developed. The methods used are based on linear methods of mathematical programming and they have been mostly applied to steel structures. Recently, methods of optimisation design have also been developed in the non-linear field [1–4]. These methods are based on the geometrical non-linear behaviour of a structure and on the influence of possible geometrical imperfections.

Unlike in steel structures (especially slender structures), physical non-linear behaviour has an important function in concrete structure design. The influence of geometric non-linearity on the design optimisation of reinforcement is less important, as the minimum slenderness of a pressured member is defined in most codes. Thus, we should formulate reinforcement optimisation design not as minimising the critical load in relation to the parts of a pressured structure, but as the search for minimum reinforcement for the recommended values of load. The method of non-linear mathematical programming can be used to solve this problem [5].

Reinforcement optimisation design of an RC frame structure for constraint conditions based on load-carrying capacity according to Baldur's method of inscribed hyper-spheres is described in [6]. Design of reinforcement for use in columns loaded by eccentric normal force by means of a genetic algorithm is described in [7].

This paper describes an application of the non-linear method of mathematical programming for RC structure reinforcement design with reference to minimising its volume in compliance with the criteria of load-carrying capacity and applicability. We present the complete mathematical formulation of the non-linear problem of optimised reinforcement design for concrete structures. This problem is linearized with the incremental method.

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The simplified method for calculating creep effects is used in order to eliminate the solution of that time-dependent task. The response of a structure under constant load with varying reinforcement is solved by a deformation variant of a FEM that takes physical and geometrical non-linearity into account. The set-up programme enables simultaneous solution of several loading states.

2. Calculation model

The task is defined as follows

a) the target function reaches the extreme

$$\{f(\{A_\mathrm{s}\})\} = \text{extreme}\ , \tag{1a}$$

while keeping the restrictive conditions implicit in the form of

b) the equalities

$$\{h(\{A_\mathrm{s}\})\} = \{0_1\}\ , \tag{1b}$$

c) the inequalities

$$\{g(\{A_\mathrm{s}\})\} \le \{0_2\}\ , \tag{1c}$$

where $\{A_\mathrm{s}\} = \{A_{\mathrm{s}1}, \ldots, A_{\mathrm{s}nt}\}^\mathrm{T}$ are the design variables; $\{f(\{A_\mathrm{s}\})\} = \{f_1(\{A_\mathrm{s}\}), \ldots, f_t(\{A_\mathrm{s}\})\}^\mathrm{T}$ is the vector of the target functions; $f_i(\{A_\mathrm{s}\})$ is the i-th target function; $\{h(\{A_\mathrm{s}\})\}$ is the vector of the restrictive conditions in the form of equations; $\{g(\{A_\mathrm{s}\})\}$ is the vector of the restrictive conditions in the form of inequalities, and $\{0_1\}$ and $\{0_2\}$ are zero vectors of the relevant type.

2.1. Target function

The optimisation problem of the concrete structure may be, e.g. by [9], expressed by the target function:

$$\{f(\{A_\mathrm{s}\})\} = f(\min E_\mathrm{tot},\ \min C_\mathrm{tot},\ \max S_\mathrm{tot})\ , \tag{2}$$

where E_tot is the gross environmental impact, C_tot is the gross cost and S_tot is the gross social-cultural quality.

The multi-criterion problem is possible to convert to the mono-criterion one by the weight implementation among the individual criteria but it is evident that at the different weights given to the individual criteria, obtained optimums can differ. Furthermore, it is not guaranteed that at the searching for optimum by the different method of multi-criterion optimisation (selection strategy), the given method will always converge.

Longitudinal reinforcement optimisation in a plane frame structure is mathematically defined as

$$f(\{A_\mathrm{s}\}) = \sum_{e=1}^{ne} l_e \sum_{i=1}^{ke} A_{\mathrm{s}i}^e = \text{minimal}\ , \tag{3}$$

where $f(\{A_\mathrm{s}\})$ is the target function expressing the overall volume of reinforcement in an RC framed structure; l_e is the length of finite element e; $A_{\mathrm{s}i}$ is the area of reinforcement in the i-th layer of the finite element e; ne is the number of finite elements of which the model of the structure is composed; ke is the number of reinforcing layers in finite element e.

For practical reasons, e.g. to make it possible either to reinforce several members identically or to reduce the number of the optimised quantities, a vector of the reinforcing types $\mathbf{A}_\mathrm{s} = \{A_{\mathrm{s}1}, A_{\mathrm{s}2}, \dots, A_{\mathrm{s}nt}\}^\mathrm{T}$ is introduced by means of which the reinforcement in each layer of each finite member can be defined; nt is the number of reinforcing types in the structure. It is generally valid that $A_{\mathrm{s}k} \neq A_{\mathrm{s}l}$ for $k \neq l$; the components of vector $\mathbf{A}_\mathrm{s}$ are optimised variables.

When we denote $\{A_\mathrm{s}^e\}$ vector of the reinforcing areas of element e, type $(1, ke)$, its components can be determined from the relation

$$\{A_\mathrm{s}^e\} = [B_e]\,\{A_\mathrm{s}\}\ , \tag{4}$$

where $[B_e]$ is the matrix comprising 0 and 1; its elements are defined when a task is assigned. The target function (3) of the optimisation is formulated as

$$f(\{A_\mathrm{s}\}) = \sum_{e=1}^{ne} l_e\,\{i_e\}^\mathrm{T}\,[B_e]\,\{A_\mathrm{s}\}\ , \tag{5}$$

where $\{i_e\}$ is a vector of type $(1, ke)$ comprising only 1.

2.2. Constraint conditions

For an established vector of the design variables, the conditions of equilibrium of the solved structure – with the application of discretion of the solved task via the FEM – can be expressed in the form (1b), where

$$\{h(\{A_\mathrm{s}\})\} = [K(\{A_\mathrm{s}\})]\,\{\Delta\} - \{F\}\ . \tag{6}$$

$[K(\{A_\mathrm{s}\})]$ is the global stiffness matrix of the solved structure; $\{\Delta\}$ is a vector of the nodal parameter of deformation and $\{F\}$ is the loading vector of a structure.

Restrictive conditions in the form of inequalities (1c) are possible to specify

$$g_{\mathrm{d}i}(\{A_\mathrm{s}\}) \le 0 \quad \text{for} \quad i = 1, 2, \dots, nd\ , \tag{7a}$$

$$g_{\mathrm{s}j}(\{A_\mathrm{s}\}) \le 0 \quad \text{for} \quad j = 1, 2, \dots, nc\ , \tag{7b}$$

$$\{A_\mathrm{su}\} \ge \{A_\mathrm{s}\} \ge \{A_\mathrm{sl}\} \ge \{0\}\ , \tag{7c}$$

where the function $g_{\mathrm{d}i}(\{A_\mathrm{s}\})$ expresses the constraint condition for the structural deformations; nd is the overall number of deforming constraint conditions; $g_{\mathrm{s}j}(\{A_\mathrm{s}\})$ is the function expressing the condition of the reinforcement design reliability from the viewpoint of the cross-section load-bearing capacity j (the force constraint condition); nc is the number of sections in which the force condition is controlled.

The conditions (7c) guarantee the minimum reinforcement required by a designer. $\{A_\mathrm{sl}\} = \{A_{\mathrm{sl},1}, \dots, A_{\mathrm{sl},nt}\}^\mathrm{T}$ or $\{A_\mathrm{su}\} = \{A_{\mathrm{su},1}, \dots, A_{\mathrm{su},nt}\}^\mathrm{T}$ is the vector of the minimum/maximum sectional area that restricts the vector components of the reinforcing type.

2.3. Deformation constraint conditions

The codes for concrete structure design or for the technology to be installed in a construction generally require compliance with the conditions for deformations for specified combinations of loads at various time stages of a structural action and in various sections. The i-th condition of deformation is expressed as

$$g_{\mathrm{d}i}(\{A_\mathrm{s}\}) = w_i(F_i, t_{i1}, t_{is}) - w_{i\mathrm{u}} \qquad \text{or} \tag{8a}$$

$$g_{\mathrm{d}i}(\{A_\mathrm{s}\}) = -w_i(F_i, t_{i1}, t_{is}) + w_{i\mathrm{u}} \ , \tag{8b}$$

where $w_i(F_i, t_{i1}, t_{is})$ is the structure deformation at time t_{is} in node (section) s from load F_i, whose action begun at time t_{i1}; $w_{i\mathrm{u}}$ is the specified limit value of deformation. The expression (8a) is valid for positive half-axis and (8b) for negative deformation value.

In order not to consider the problem as time-dependent the influence of creep on deformation w_i can be simplified by a transforming equation (8).

In accordance with the regulations for concrete structure design, it is considered to be valid that

$$w_i(F_i, t_{i1}, t_{is}) = w_{i\mathrm{p}}(F_i, t_{i1}) + \Delta w(F_i, t_{i1}, t_{is}) \ , \tag{9}$$

where $w_{i\mathrm{p}} = w_{i\mathrm{p}}(F_i, t_{i1})$ is the initial deformation at node s at time t_{i1} generated by the load F_i, $\Delta w(F_i, t_{i1}, t_{is})$ is the deformation generated by creep from load F_i acting in the time interval (t_{i1}, t_{is}). The influence of shrinkage in condition (8) can generally be neglected.

In accordance with the regulations for concrete structure design the value of deflection $\Delta w(F_i, t_{i1}, t_{is})$ caused by creep can be expressed as

$$\Delta w(F_i, t_{i1}, t_{is}) = w_{i\mathrm{p}}(F_i, t_{i1})\, \varphi(t_{i1}, t_{is}) \ , \tag{10}$$

where $\varphi(t_{i1}, t_{is})$ is the creep coefficient given by the relevant code – e.g. [1–2].

By inserting (9), (10) into (8a) or (8b), taking into account consideration (7a), the condition of deformation can be adapted as a new condition in the form

$$g_{\mathrm{d}i}(\{A_\mathrm{s}\}) = w_{i\mathrm{p}}(F_i, t_{i1}) - \frac{w_{i\mathrm{u}}}{1 + \varphi(t_{i1}, t_{is})} \ , \tag{11a}$$

$$g_{\mathrm{d}i}(\{A_\mathrm{s}\}) = -w_{i\mathrm{p}}(F_i, t_{i1}) + \frac{w_{i\mathrm{u}}}{1 + \varphi(t_{i1}, t_{is})} \ . \tag{11b}$$

The initial deformation $w_{i\mathrm{p}}$ in a cross-section of local coordinate x_i is calculated according to

$$w_{i\mathrm{p}}(F_i, t_{i1}) = \{w_e(x)\}^\mathrm{T} \{\Delta_e\} = \{w_e(x)\}^\mathrm{T} [L_e] \{\Delta\} \ , \tag{12}$$

where $\{w_e(x)\}$ is a vector of the type $(ndp, 1)$; $\{\Delta_e\} = \{\Delta_e(\{A_\mathrm{s}\})\}$ (or $\{\Delta\} = \{\Delta(\{A_\mathrm{s}\})\}$) is a vector of the nodal deformation parameters of element e (or the structure) deformation loaded by F_i, when it is solved with reinforcement defined by vector $\{A_\mathrm{s}\}$; $[L_e]$ is a transforming matrix comprising 0 and 1 of the type (ndp, nk); $\{\Delta_e\}$ is of the type $(ndp, 1)$, $\{\Delta\}$ is of the type $(nk, 1)$, and ndp (or nk) is the number of nodal parameters of the deformation in element e (or in the whole structure).

2.4. Force constraint conditions

The formulation of force constraint conditions is based on the assumption of RC section behaviour in agreement with the relevant code. Most of the recommendations accept a non-linear model for a section loaded by normal force and bending moment. The model is based on the following preconditions – see Fig. 1:

– Perfect bond between concrete and reinforcement.
– Linear dependence of the strains along the height of the cross-section.
– The stress in the individual materials (steel, concrete) is determined from the stress-strain diagrams defined in the relevant code.

The cross-section is considered to be reliably designed for the given load when the strains of extremely loaded fibres of individual materials are lower than the limit values defined by the design recommendation under fulfilment of these preconditions. Generally, fulfilment of all the conditions is required

a) for concrete fibres under pressure

$$\varepsilon(z_{\mathrm{bc}}) \geq \varepsilon_{\mathrm{bd}} \ , \tag{13a}$$

b) for reinforcement in a layer i

$$\varepsilon_{\mathrm{sc}} \leq \varepsilon(z_{\mathrm{s}i}) \leq \varepsilon_{\mathrm{st}} \ , \tag{13b}$$

where $\varepsilon_{\mathrm{bd}}$ is the limit value of the strains for pressured concrete; $\varepsilon_{\mathrm{sc}}$ (or $\varepsilon_{\mathrm{st}}$) is the limit value of the strains for pressured/tensioned reinforcement; z_{bc} is z-th coordinate of the extremely stressed fibres of the concrete; $z_{\mathrm{s}i}$ is z-th coordinate of the i-th reinforcing layer.

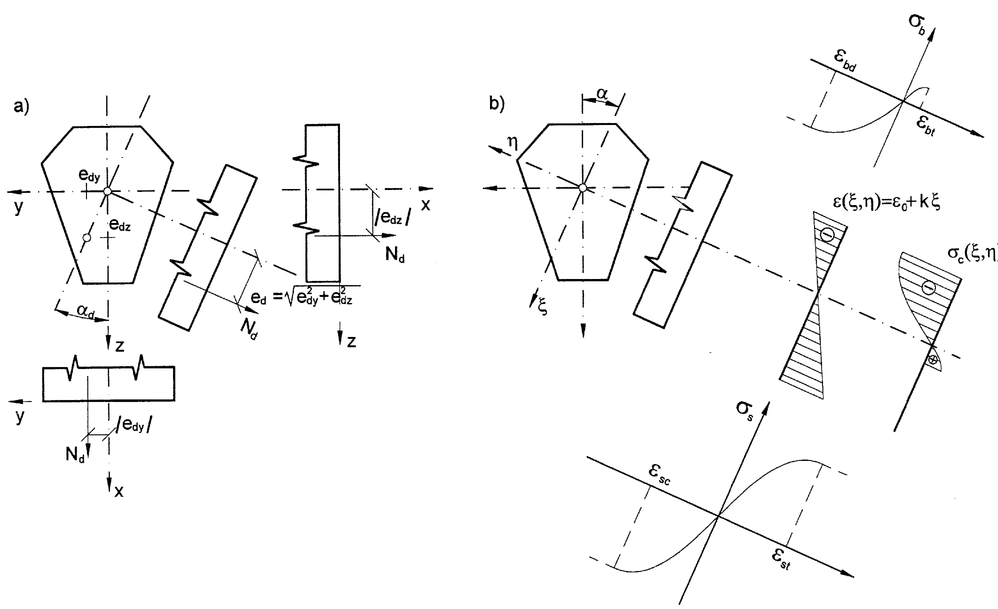

Fig.1: Shape of a cross-section and precondition for calculation of a) a acting loading, b) the course of the strains generated by load N, M; stress-strain diagram of concrete $\sigma_{\mathrm{c}} = \sigma_{\mathrm{c}}(\varepsilon)$; stress-strain diagram of reinforcement $\sigma_{\mathrm{s}} = \sigma_{\mathrm{s}}(\varepsilon)$

In the relations (13) and in Fig. 1 the sign convention is considered to be in accordance with the theory of elasticity (tension > 0, pressure < 0). Some Codes (e.g., Eurocode EC 2, Czech Code 73 1201, German DIN 1045) do not require evaluation of tensile reinforcement as the equation $\varepsilon_{\mathrm{bd}} \geq \varepsilon_{\mathrm{sc}}$ is valid.

The strain $\varepsilon_e(x,z)$ of the fibres of the coordinates z in the cross-section x of finite element e can be expressed as the linear combination of the nodal parameters of the deformation of element e

$$\varepsilon_e(x,z) = \sum_{k=1}^{ndp} n_k(x,z)\,\Delta_{e,k}\ , \tag{14}$$

where $n_k(x,z)$ is a coefficient that can be derived in dependence on coordinates x, z and on the option of approximate functions when formulating the basic relationship for the finite element e, $\Delta_{e,k}$ is the k-th nodal parameter describing the deformation of finite element e.

Equation (14) can be written in matrix form as

$$\varepsilon_e(x,z) = \{N_e(x,z)\}^{\mathrm{T}}\{\Delta\} = \{N_e(x,z)\}^{\mathrm{T}}[L_e]\{\Delta\}\ , \tag{15}$$

where $\{N_e(x,z)\} = \{n_1,\ldots,n_{ndp}\}^{\mathrm{T}}$.

Hence, the conditions of reliability (13) are

$$\{N_e(x,z)\}^{\mathrm{T}}[L_e]\{\Delta\} - \varepsilon_{\mathrm{bd}} \geq 0\ , \qquad z = z_{\mathrm{bc}}\ , \tag{16a}$$

$$\varepsilon_{\mathrm{st}} \geq \{N_e(x,z)\}^{\mathrm{T}}[L_e]\{\Delta\} \geq \varepsilon_{\mathrm{sc}}\ , \qquad z = z_{\mathrm{st}} \quad \text{and} \quad z = z_{\mathrm{sc}}\ , \tag{16b}$$

where z_{st}, z_{sc} is the distance between extremely tensioned or compressed reinforcement and C_{gb}.

3. Solution of optimisation design

The optimisation of reinforcement design into an RC structure is determined by

a) the target function (1a), which is the non-linear function of the vector elements of reinforcement $\{A_{\mathrm{s}}\}$,

b) the constraint conditions that express
   (i) the conditions of equilibrium when a structure is solved by the FEM (6),
   (ii) the conditions of deformation (7a) reducing the displacement of a structure that can be written as (11),
   (iii) the reliability of the cross-sections of the designed structure expressed in view of the code recommendation used that can be written as (7b) or (16),
   (iv) the recommended limits of the reinforcement areas (7c).

The constraint conditions expressed by Eqs. (6), (7a) and (7b), respectively, are a non-linear function of the vector elements $\{A_{\mathrm{s}}\}$, and the conditions (7c) subject to the elements $\{A_{\mathrm{s}}\}$ are linear.

Linearization of the non-linear constraint conditions in terms of using an incremental method is the next step in the theoretical solution. The basic relations of the linear optimisation after linearization and after the introduction of additional variables and auxiliary variables are expressed as

a) target function

$$\{f(\{A_{\mathrm{s}}^k\})\}^{\mathrm{T}}\{\mathrm{d}A_{\mathrm{s}}\} + \sum_{i=1}^{nt}\mathrm{d}A_{\mathrm{s}i}\left\{\frac{\partial f}{\partial A_{\mathrm{s}i}}\right\}^{\mathrm{T}}\{\mathrm{d}A_{\mathrm{s}}\} + \alpha\{e_u\}^{\mathrm{T}}\{u\} = \text{minimal}\ , \tag{17}$$

b) constraint conditions given by

(i) the conditions of equilibrium of a structure (for $j = 1, 2, \ldots, nk$)

$$\sum_{i=1}^{nt} \mathrm{d}A_{\mathrm{s}i} \left[\frac{\partial K}{\partial A_{\mathrm{s}i}}\right] \{\Delta^k\} + [K] \sum_{i=1}^{nt} \mathrm{d}A_{\mathrm{s}i} \left\{\frac{\partial \Delta^k}{\partial A_{\mathrm{s}i}}\right\} + \{u\} = \{0\} \;, \tag{18}$$

where $\{0\}$ is zero vector of type $(1, nk)$,

(ii) the conditions of deformation (1b) for $i = 1, 2, \ldots, nd$

$$\{w_e(x_i)\}^{\mathrm{T}} [L_e] \{\Delta^k\} + \{\nabla_{w_e}(x_i)\}^{\mathrm{T}} \{\mathrm{d}A_{\mathrm{s}}\} - \frac{w_{i\mathrm{u}}}{1+\varphi} + w_i = 0 \qquad \text{or} \tag{19a}$$

$$-\{w_e(x_i)\}^{\mathrm{T}} [L_e] \{\Delta^k\} - \{\nabla_{w_e}(x_i)\}^{\mathrm{T}} \{\mathrm{d}A_{\mathrm{s}}\} + \frac{w_{i\mathrm{u}}}{1+\varphi} + w_i = 0 \tag{19b}$$

(iii) the reliability of the cross-sections in accordance with the concrete design recommendation for $i = 1, 2, \ldots, nc$

– the extremely pressured concrete layer

$$-\{N_e(z_i, z_{\mathrm{bc}}\}^{\mathrm{T}} [L_e] \{\Delta^k\} - \{\nabla_{N_e}(z_i, z_{\mathrm{bc}})\}^{\mathrm{T}} \{\mathrm{d}A_{\mathrm{s}}\} + \varepsilon_{\mathrm{bd}} + c_i = 0 \;, \tag{20a}$$

– the extremely pressured layer of reinforcement (if the relations for the given design recommendation have physical justification)

$$-\{N_e(z_i, z_{\mathrm{sc}}\}^{\mathrm{T}} [L_e] \{\Delta^k\} - \{\nabla_{N_e}(z_i, z_{\mathrm{sc}})\}^{\mathrm{T}} \{\mathrm{d}A_{\mathrm{s}}\} + \varepsilon_{\mathrm{sc}} + s_{\mathrm{c}i} = 0 \;, \tag{20b}$$

– the extremely tensioned layer of reinforcement

$$\{N_e(z_i, z_{\mathrm{st}}\}^{\mathrm{T}} [L_e] \{\Delta^k\} + \{\nabla_{N_e}(z_i, z_{\mathrm{st}})\}^{\mathrm{T}} \{\mathrm{d}A_{\mathrm{s}}\} - \varepsilon_{\mathrm{st}} + s_{\mathrm{t}i} = 0 \;, \tag{20c}$$

(iv) the recommended limits of the reinforcement areas

$$\{A_{\mathrm{sl}}\} - \{A_{\mathrm{s}}^k\} - \{\mathrm{d}A_{\mathrm{s}}\} + \{a_{\mathrm{l}}\} = \{0\} \;, \tag{21a}$$

$$\{A_{\mathrm{s}}^k\} - \{A_{\mathrm{su}}\} + \{\mathrm{d}A_{\mathrm{s}}\} + \{a_{\mathrm{u}}\} = \{0\} \;, \tag{21b}$$

c) additional conditions of non-negativeness for

– the auxiliary variables

$$\{u\} \geq \{0\} \;, \tag{22a}$$

– the additional variables

$$\begin{gathered} \{w\} \geq \{0\} \;, \qquad \{c\} \geq \{0\} \;, \qquad \{s_{\mathrm{t}}\} \geq \{0\} \;, \qquad \{s_{\mathrm{c}}\} \geq \{0\} \;, \\ \{a_{\mathrm{l}}\} \geq \{0\} \;, \qquad \{a_{\mathrm{u}}\} \geq \{0\} \;. \end{gathered} \tag{22b}$$

In equations (17) to (21), $\{A_{\mathrm{s}}^k\}$ expresses the vector of the reinforcement areas in the k-th iterative step, $\{\mathrm{d}A_{\mathrm{s}}\}$ is the vector of the increment of the reinforcement areas by means of which we define the vector of areas in the $(k+1)$-th step

$$\{A_{\mathrm{s}}^{k+1}\} = \{A_{\mathrm{s}}^k\} + \{\mathrm{d}A_{\mathrm{s}}\} \;. \tag{23}$$

$$\{u\} = \{u_1, u_2, \ldots, u_{nk}\}^{\mathrm{T}} \tag{24a}$$

is the vectors of the auxiliary variables; $\{e_{\mathrm{u}}\}$ is the auxiliary vector of the corresponding type comprising units only and α is the coefficient of penalization.

$$\begin{aligned}
\{w\} &= \{w_1, w_2, \ldots, w_{nd}\}^{\mathrm{T}} , \\
\{c\} &= \{c_1, c_2, \ldots, c_{ne}\}^{\mathrm{T}} , \\
\{s_{\mathrm{t}}\} &= \{s_{\mathrm{t}1}, s_{\mathrm{t}2}, \ldots, s_{\mathrm{t},nk}\}^{\mathrm{T}} , \\
\{s_{\mathrm{c}}\} &= \{s_{\mathrm{c}1}, s_{\mathrm{c}2}, \ldots, s_{\mathrm{c},nk}\}^{\mathrm{T}} , \\
\{a_{\mathrm{l}}\} &= \{a_{\mathrm{l}1}, a_{\mathrm{l}2}, \ldots, a_{\mathrm{l},nt}\}^{\mathrm{T}} , \\
\{a_{\mathrm{u}}\} &= \{a_{\mathrm{u}1}, a_{\mathrm{u}2}, \ldots, a_{\mathrm{u},nt}\}^{\mathrm{T}}
\end{aligned} \tag{24b}$$

are the vectors of the additional variables.

Then, it follows

$$\{\nabla_{w_e}(x_i)\}^{\mathrm{T}} = \left\{\frac{\partial}{\partial\{A_{\mathrm{s}}\}}\right\}\left(\{w_e(x_i)\}^{\mathrm{T}}[L_e]\{\Delta^k\}\right) , \tag{25}$$

$$\{\nabla_{N_e}(z_i, z)\}^{\mathrm{T}} = \left\{\frac{\partial}{\partial\{A_{\mathrm{s}}\}}\right\}\left(\{N_e(z_i, z)\}^{\mathrm{T}}[L_e]\{\Delta^k\}\right) , \tag{26}$$

By the mathematical programming we are looking for positive solution. Because our algorithm is started from maximum reinforcement cross section areas it is possible to use following transformation

$$\{\mathrm{d}A_s\} = \{\overline{\mathrm{d}A_{\mathrm{s}}}\} + \{A_{\mathrm{sl}}\} - \{A_{\mathrm{s}}^k\} . \tag{27}$$

4. Example of solution

A plain frame structure of one span and two floors with bilateral cantilevers in the lower floor was the subject of solution. The geometry of the structure and division into finite elements is shown in Fig. 2. With regard to the symmetry, only half of the frame was solved. The columns are rectangular, the primary beams and cantilevers are T-shaped, and the referential axis y of the members is related to a half of the section's depth. Dimensions of the cross-sections are shown in Fig. 4.

The primary beams and cantilever are stressed with uniform dead load g_k^{b} and with uniform fluctuating load q_k^{b}. The columns are stressed with vertical uniform dead load along the entire depth g_k^{col}. In the frame joints the dead load acts by the force G_k and on the cantilever end by the force G_k^{c}. The intensities of the loads are shown in Table 1. The structure was stressed by three loading states:

- LS1 : dead load + full live load,
- LS2 : dead load + live load on the lower primary beam only,
- LS3 : dead load + live load on the upper primary beam and cantilever only.

The loading factors for individual loads are presented in Table 1.

Two reinforcement types were designed for each primary beam and cantilever (Fig. 4b) and one reinforcement type was designed for every column (Fig. 4a). Notation of the reinforcement types is in Fig. 5a. Minimal and maximal (= starting) allowable reinforcement cross section areas for the reinforcement types are presented in Table 2.

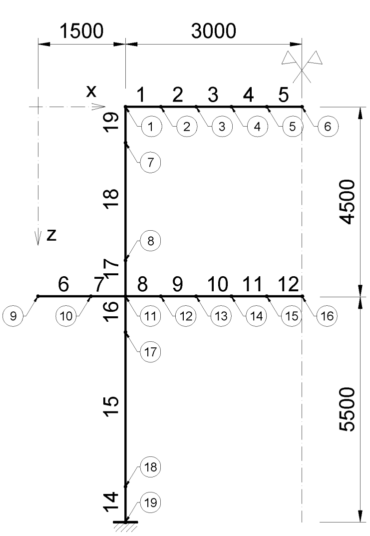

Fig.2: Geometry of the frame structure, division into finite elements

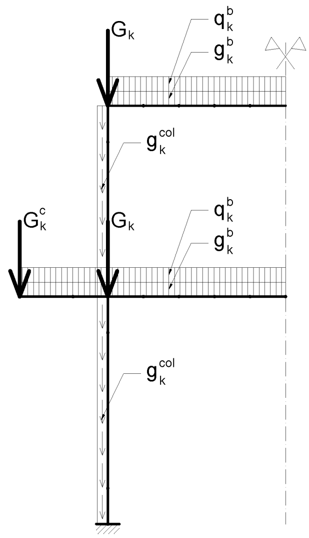

Fig.3: Loading scheme

Load	notation	intensity	γ_f (ČSN)	γ_F (EN)
dead	g_k^b	50 kN/m	1.2	1.35
	g_k^{col}	10 kN/m	1.1	1.35
	G_k	200 kN	1.2	1.35
	G_k^c	50 kN	1.1	1.35
live	q_k^b	50 kN/m	1.3	1.5

Tab.1: Loads and loading factors

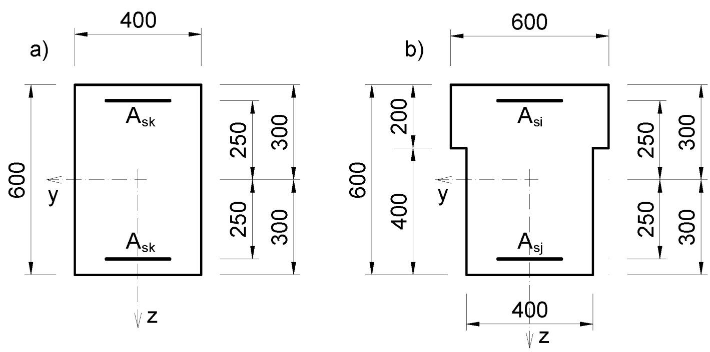

Fig.4: Geometry of the cross-sections a) column, b) primary beam

The calculations were performed in accordance with two standards: ČSN [1] and EN [2]. The concrete used was class B20 (C16/20) and the steel was 10 425 (B400). In the calculation according to the ČSN standard a stress-strain diagram of steel was considered as bilinear with a horizontal plastic line and with calculating values of ultimate tensile strain $\varepsilon_{sd} = 10‰$ of reinforcement. The ultimate compressive strain of concrete is $\varepsilon_{bd} = 2.5‰$ (bilinear stress-strain diagram); concrete under tension did not act. Both stress-strain diagrams corresponded to the principles of the ČSN standard. The calculation according to the EN principles was performed with a bilinear stress-strain diagram for strengthened steel that corresponds to ductile class A. But the ultimate tensile strain of reinforcement was restricted to the value $\varepsilon_{\mathrm{ud}} = 10\,‰$ from a possible $22.5\,‰$. The stress-strain diagram of concrete was designed as bilinear with an ultimate tensile strain $\varepsilon_{\mathrm{cu3}} = 3.5\,‰$.

In the calculations we considered both physical non-linearity (according to the described stress-strain diagrams of materials) and geometrical non-linearity under the premise of small deformations and torsional displacement.

Variant A : a comparative calculation according to both standards under the presumption that in compressed fibres of concrete and in tensioned fibres of reinforcement the values of ultimate strains will not be exceeded, i.e. the requirement of reliability fulfilment from the viewpoint of ULS. These values were verified in 27 cross-sections of the structure. They are defined on the top/end of the members so the area of reinforcement of each reinforcement type was restricted by this condition from below – see Fig. 5a. In the middle of the primary beam the value of ultimate deflection w_{lim} was 20 mm (nodes 6 and 16) and on the end of the cantilever in node 19 it was 7.5 mm. However, these conditions were not of use in the calculation.

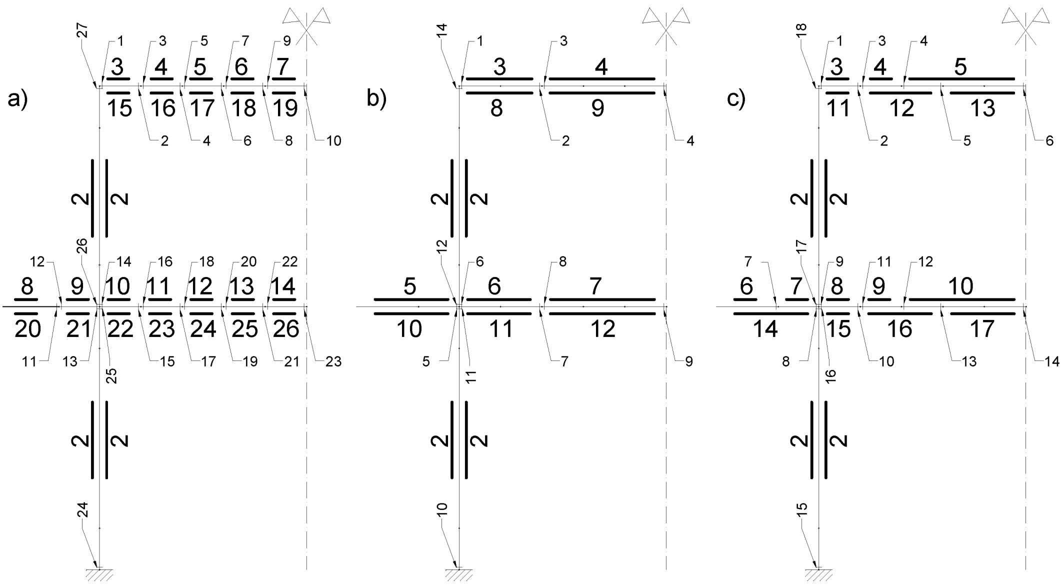

Fig.5: Reinforcement types and cross-sections a) variant A and B, b) variant C1, c) variant C2

In the variant A-ČSN the ultimate strain of reinforcement (or the values close to ultimate strain) was reached in the cross-sections 1 (9.87 ‰), 11 (9.76 ‰), 13 (9.89 ‰), 14 (9.97 ‰), 16 (9.95 ‰), 18 (9.86 ‰), 21 (9.69 ‰) and 27 (5.48 ‰). The ultimate strain of concrete was reached in the cross-sections 1 (−2.18 ‰), 13 (−2.18 ‰), 14 (−2.5 ‰) and 27 (−2.16 ‰), namely at the loading state LS1. In the cross-sections 1, 13, 14 and 27 the optimal failure values were reached (or values close this failure).

In the variant A-EN the ultimate strain of reinforcement in the loading state LS1(or the values close to ultimate strain) was reached in the cross-sections 1 (9.91 ‰), 11 (9.83 ‰), 13 (9.92 ‰), 14 (9.99 ‰), 16 (9.98 ‰) and 18 (9.98 ‰), and the ultimate strain of concrete in the cross-sections 1 (−3.44‰), 13 (−3.45‰), 14 (−3.50‰), 16 (−2.57‰) and 27 (−3.16‰). The loading state LS2 was decisive in the cross-sections 19 (9.72‰), 21 (6.03‰) and 23 (3.79‰).

The resulting areas of reinforcement of different reinforcement types are compared in Table 2.

Reinforcement type			$A_{\mathrm{s,max}}$	$A_{\mathrm{s,min}}$	A-ČSN	A-EN	B-ČSN	B-EN
columns		1	3000	220	347	340	260	269
		2	3000	192	454	673	1516	1674
upper reinforcement	upper primary beam	3	3000	288	1091	1388	2959	2950
		4	3000	288	415	411	328	337
		5	3000	0	129	123	40	49
		6	3000	0	129	124	40	49
		7	3000	0	129	124	1739	1228
	lower primary beam	8	3000	288	505	646	2365	2536
		9	3000	288	1179	1490	2959	2950
		10	3000	288	2325	2720	2959	2950
		11	3000	288	1223	1385	2653	2881
		12	3000	0	381	378	1266	1192
		13	3000	0	130	128	40	49
		14	3000	0	130	136	40	49
lower reinforcement	upper primary beam	15	3000	192	319	386	232	241
		16	3000	192	1094	1041	804	736
		17	3000	192	2154	2176	2959	2950
		18	3000	192	2692	2725	2959	2950
		19	3000	192	2851	2874	2959	2950
	lower primary beam	20	3000	0	127	123	40	49
		21	3000	0	127	216	52	55
		22	3000	192	1254	1215	232	241
		23	3000	192	319	312	232	241
		24	3000	192	323	557	461	773
		25	3000	192	647	1000	1852	1752
		26	3000	192	783	1171	2928	2896
Volume of reinforcement [10^{-2} m^3]			8.25	0.64	2.03895	2.37558	3.71005	3.86596

Tab.2: Areas of the reinforced layers for individual variants of the solution

Variant B: the calculation was identical to variant A, but the conditions of ultimate serviceability were stricter. This means that the values of ultimate deflections w_{lim} were considered so as to restrict the deflections of nodes calculated in variant A – see Table 3, i.e. $w_{\mathrm{lim}}(6) = 6$ mm, $w_{\mathrm{lim}}(16) = 4$ mm and $w_{\mathrm{lim}}(9) = 2$ mm.

The resulting deflections from individual loading states are presented in Table 3. In a column denoted MAX-ČSN and MAX-EN the deflections in nodes during the reinforcing of the structure with reinforcement correspond to the maximal allowable areas of the reinforcement types. In these calculations the ultimate strains of fibres, concrete and reinforcement were not reached in any cross- sections. The restriction of deflection (ULS) was decisive for reinforcement. The ultimate deflection in node 16 was reached in the loading state LS2, and in nodes 6 and 9 in the loading state LS3 (the values are marked by bolts). The loading state LS1 was not limiting from the point of view of the deflection. The areas of reinforcement type are presented in Table 2. The restriction of deflection represented an increase in reinforcement volume in the variant B-ČSN of 82 % and in the variant B-EN of 63 %.

Deflection in nodes [mm]		MAX-ČSN	A-ČSN	B-ČSN	MAX-EN	A-EN	B-EN
LS1	6	1.59	7.67	5.66	4.87	7.21	5.66
	16	2.49	6.15	3.48	2.92	5.09	3.50
	19	1.19	2.71	1.19	1.19	2.36	1.24
LS2	6	2.19	3.39	2.52	2.23	3.19	2.54
	16	3.18	6.59	**3.99**	3.25	5.52	**3.97**
	19	0.41	1.32	0.26	0.45	1.10	0.34
LS3	6	5.00	7.97	**5.98**	5.09	7.52	**5.97**
	16	0.92	2.40	1.03	0.99	1.90	1.09
	19	1.70	3.63	**2.00**	1.73	3.20	**2.00**
Volume of reinforcement [10^{-2} m^3]		8.25	2.03895	3.71005	8.25	2.37558	3.86596

Tab.3: Deflections in the nodes, maximal volume of reinforcement

Variants C1 and C2: the calculation was identical to variant A, but the reinforcement types were designed so that they went through more elements (Fig. 5b and 5c). It means that in these members the same reinforcement (area) was required. The resulting areas of the reinforcement types and the reinforcement volumes are summarised in Table 4. The reinforcement types were designed to correspond as much as possible to the real reinforcement of the structure.

When the areas of reinforcement are changed the rigidity of the members are changed too and with this comes the redistribution of the internal forces in the structure. The internal forces were compared from optimisation calculations according to the EN standard, variant

<table>
<tr><th colspan="3">Reinforcement type (variant A)</th><th>A-ČSN</th><th>C1-ČSN</th><th>C2-ČSN</th><th>A-EN</th><th>C1-EN</th><th>C2-EN</th></tr>
<tr><td colspan="2" rowspan="2">columns</td><td>1</td><td>347</td><td>336</td><td>279</td><td>340</td><td>270</td><td>429</td></tr>
<tr><td>2</td><td>454</td><td>445</td><td>495</td><td>673</td><td>672</td><td>746</td></tr>
<tr><td rowspan="12">upper reinforcement</td><td rowspan="5">upper primary beam</td><td>3</td><td>1091</td><td>1083</td><td rowspan="2">1123</td><td>1388</td><td>1391</td><td rowspan="2">1432</td></tr>
<tr><td>4</td><td>415</td><td>404</td><td>411</td><td>338</td></tr>
<tr><td>5</td><td>129</td><td rowspan="3">117</td><td rowspan="3">59</td><td>123</td><td rowspan="3">51</td><td rowspan="3">258</td></tr>
<tr><td>6</td><td>129</td><td>124</td></tr>
<tr><td>7</td><td>129</td><td>124</td></tr>
<tr><td rowspan="7">lower primary beam</td><td>8</td><td>505</td><td>505</td><td rowspan="2">1183</td><td>646</td><td>645</td><td rowspan="2">1489</td></tr>
<tr><td>9</td><td>1179</td><td>1180</td><td>1490</td><td>1522</td></tr>
<tr><td>10</td><td>2325</td><td>2298</td><td rowspan="2">2291</td><td>2720</td><td>2712</td><td rowspan="2">2751</td></tr>
<tr><td>11</td><td>1223</td><td>1193</td><td>1385</td><td>1372</td></tr>
<tr><td>12</td><td>381</td><td rowspan="3">355</td><td rowspan="3">347</td><td>378</td><td rowspan="3">371</td><td rowspan="3">559</td></tr>
<tr><td>13</td><td>130</td><td>128</td></tr>
<tr><td>14</td><td>130</td><td>136</td></tr>
<tr><td rowspan="12">lower reinforcement</td><td rowspan="5">upper primary beam</td><td>15</td><td>319</td><td>308</td><td rowspan="2">695</td><td>386</td><td>384</td><td rowspan="2">692</td></tr>
<tr><td>16</td><td>1094</td><td rowspan="2">1773</td><td>1041</td><td rowspan="2">1693</td></tr>
<tr><td>17</td><td>2154</td><td rowspan="3">2552</td><td>2176</td><td rowspan="3">2344</td></tr>
<tr><td>18</td><td>2692</td><td rowspan="2">2863</td><td>27251</td><td rowspan="2">2941</td></tr>
<tr><td>19</td><td>2851</td><td>2874</td></tr>
<tr><td rowspan="7">lower primary beam</td><td>20</td><td>127</td><td rowspan="2">116</td><td rowspan="2">58</td><td>123</td><td rowspan="2">50</td><td rowspan="2">225</td></tr>
<tr><td>21</td><td>127</td><td>216</td></tr>
<tr><td>22</td><td>1254</td><td>1216</td><td rowspan="2">1171</td><td>1215</td><td>1210</td><td rowspan="2">1238</td></tr>
<tr><td>23</td><td>319</td><td rowspan="2">331</td><td>312</td><td rowspan="2">566</td></tr>
<tr><td>24</td><td>323</td><td rowspan="3">934</td><td>557</td><td rowspan="3">1144</td></tr>
<tr><td>25</td><td>647</td><td rowspan="2">813</td><td>1000</td><td rowspan="2">1170</td></tr>
<tr><td>26</td><td>783</td><td>1171</td></tr>
<tr><td colspan="3">Volume of reinforcement [10^{-2} m^3]</td><td>2.03895</td><td>2.07379</td><td>2.27380</td><td>2.37558</td><td>2.34515</td><td>2.91107</td></tr>
</table>

Tab.4: Comparison of the resulting areas of reinforcement during submission of other reinforcement types

A-EN and variant B-EN with the calculation of the internal forces in the structure reinforced with the maximal allowable reinforcing areas (variant MAX-EN). (This calculation was also performed under the consideration of the physically non-linear behaviour of materials

Variant		MAX-EN		A-EN		B-EN	
LS	section	M[kNm]	N[kN]	M[kNm]	N[kN]	M[kNm]	N[kN]
LS1	1	−314.7	−104.2	−267.3	−94.7	−305.0	−104.4
	10	327.6	−104.2	375.7	−94.7	337.6	−104.4
	13	−261.6	0.0	−261.6	0.0	−261.5	0.0
	14	−425.5	100.6	−458.2	83.5	−438.2	100.6
	23	215.3	100.6	182.3	83.5	202.3	100.6
	24	11.2	−1811.3	26.1	−1811.3	8.8	−1811.3
	25	−7.5	−1737.0	−38.3	−1737.0	−12.5	−1737.0
	26	154.0	−758.3	158.3	−758.3	164.1	−758.3
	27	−314.7	−697.5	−267.3	−697.5	−305.0	−697.5
LS2	1	−158.7	−66.1	−142.4	−65.8	−156.7	−71.6
	10	145.3	−66.1	161.8	−65.8	147.5	−71.6
	13	−177.2	0.0	−177.2	0.0	−177.2	0.0
	14	−397.7	42.7	−443.9	33.5	−410.1	52.6
	23	243.3	42.7	196.0	33.5	230.6	52.6
	24	46.9	−1473.8	65.3	−1473.8	37.3	−1473.8
	25	−82.1	−1399.5	−113.8	−1399.5	−68.8	−1399.5
	26	138.4	−533.3	152.8	−533.3	165.2	−533.3
	27	−158.7	−472.5	142.4	−472.5	156.7	−472.5
LS3	1	−303.2	−81.8	−251.4	−67.1	−290.6	−74.5
	10	339.0	−81.8	391.1	−67.1	351.7	−74.5
	13	−261	0.0	−261	0.0	−261	0.0
	14	−236.3	105.9	−241.2	86.0	−241.1	92.3
	23	67.4	105.9	62.4	86.0	62.5	92.3
	24	−41.8	−1586.3	−32.6	−1586.3	−32.5	−1586.3
	25	90.1	−1512.0	71.3	−1512.0	55.6	−1512.0
	26	64.9	−758.3	50.0	−758.3	44.4	−758.3
	27	−303.2	−697.5	−251.4	−697.5	−290.6	−697.5

Tab.5: Internal forces in the important cross-sections

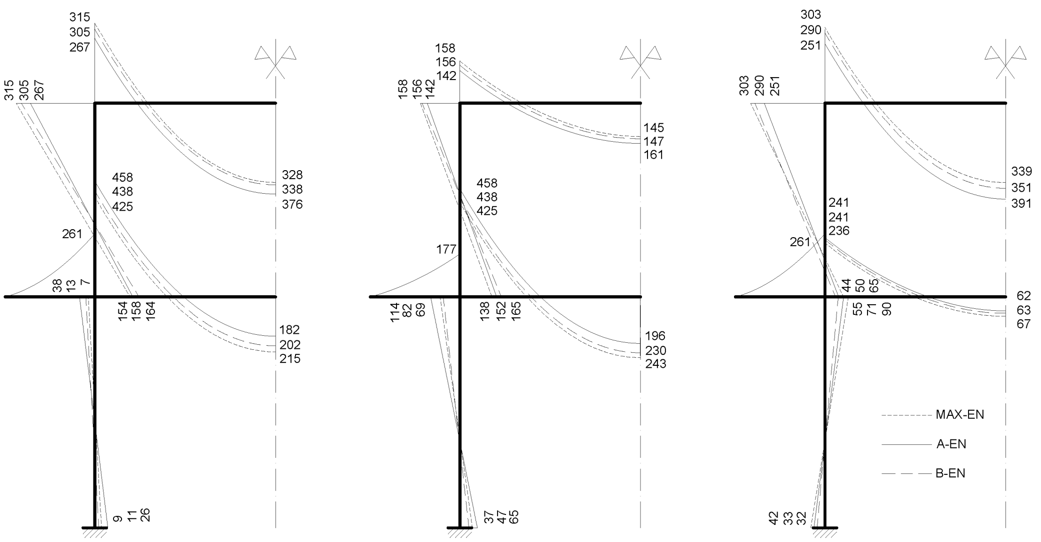

Fig.6: Influence of the change in frame structure rigidity on the internal forces (M)

and geometric non-linearity). Internal forces in the important cross-sections of the frame structure (Fig. 5a) are presented in Table 5 and Fig. 6.

5. Conclusion

In all variants of the solution according to the EN [2] standard a greater volume of reinforcement was reached than in the variants according to the ČSN [1] standard. The reason is the higher values of designed loads. The validity of the obtained results was verified in the solution of the plane structure. By the comparison of the solved examples it was documented that even minor changes by a designer have an influence on the optimal solution. This gives a reason for the utilisation of optimised design, especially for precast structures and members.

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