TO THE ANALYSIS OF INTERNAL DYNAMICS OF SPECIAL CASE IN A CLASS OF NON-LINEAR PARAMETRIC PSEUDOPLANETARY SYSTEMS

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Within the frame of the mass discretisation method has been designed the mathematical – physical model of the pseudoplanetary system of seven degrees of freedom for the analysis of the influence of the number of satellites and for free or elastic mounting of the sun wheel as a next phase of the solution of general differential system with branched – split power flow.

Keywords : *non-linear dynamics, parametric systems, planetary systems*

1. Introduction

The present world development of highly powerful and high-speed transmissive systems with the minimum dimensions and the masses leads to a planetary gear systems with branched – split power flows.

This development trend requires with regard to the reliability and the safety of these complicated constructions with many degrees of freedom the sophisticated both analytical, numerical and experimental research. The rising demands on the profundity and accuracy of analysis of dynamics of these systems can be achieved by more accurately mathematical-physical modelling of the non-linear systems with many degrees of freedom as well as more exacting analytical and numerical computational methods including the experimental technics.

This study continues the basic research in the area of non-linear time heteronymous transmissive systems with many degrees of freedom [1], [2], [3] and elaborates the first part of the analysis of the class of non-linear parametric pseudoplanetary system with seven degrees of freedom.

2. Mathematical-physical model

The dynamical force as the primary constructional data constitutes the first stage of non-linear dynamical analysis of the regular and irregular – chaotic motions in the complicated planetary chain-branched differential or pseudoplanetary systems as occur for example in the Wilson's transmissive systems in an automotive technics, in a drive mechanisms of the weighty caterpillar vehicles with the limitary constructional spaces or on the other side in the aeronautical technics for example in the turbo-prop engines, where the revolutions of the turbine presently reach to as many as 90 thousands per minute.

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Such systems can show the various dynamical behaviour, where in the certain sensible areas of the tuning occur the new often even unexpected phenomena as the patterns of the solution ambiguity, the bifurcation, splitting of the amplitude-frequency characteristics, the transitions of the regular motions into irregular up to the deterministic character etc.

We illustrate now the solution of the more complicated system with the cog wheels, i.e. the solution of the mathematical-physical model of the pseudoplanetary gear box (the revolutions of the satellite carrier $\varphi_{n4} = 0$, [1]) with three double planets – satellite gears $j \equiv I = 3$, see Fig. 1, as the special case of the general basic system with $j \equiv I \geq 1$ double satellites, [1], [3].

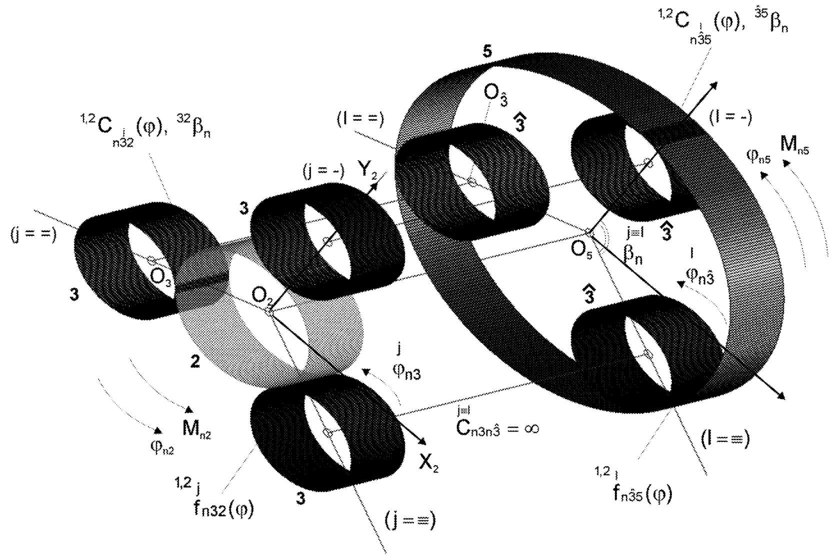

Fig.1: Kinematic scheme of the mathematical-physical model with spur and helical gearing ($^{1,2}C_{\overset{j}{n32}}(\varphi)$, $^{1,2}C_{\overset{I}{n\hat{3}5}}(\varphi)$ ... the resulting stiffness in the gear mesh, $^{3,2}\beta_n$, $^{\hat{3},5}\beta_n$ ... the inclination angle of gearing, $^{1,2}\overset{j}{f}_{n32}$, $^{1,2}\overset{I}{f}_{n\hat{3}5}$ ... the resulting deviation from ideal involute gearing, $\overset{j\equiv I}{C}_{n3n\hat{3}}$ ... the torsional stiffness of the satellite shafts)

The scheme in the Fig. 1 constitutes the kinematic model with a helical (gear meshes 32) and spur (gear meshes $\hat{3}5$) gearing.

The system with seven degrees of freedom here is solved as the isolated non-linear non-conservative system without the influence of adjacent shafts (C_{umn2}, $C_{n5ws} = \infty$), with the torsion-resistant shafts of satellites ($\overset{j\equiv I}{C}_{n3n\hat{3}} = \infty$), with the absolutely stiff supported gear ring 5 and the satellites 3, $\hat{3}$ ($\overset{I}{C}_{n5p}$, $\overset{j}{C}_{n34}$, $\overset{I}{C}_{n\hat{3}4}$, $\overset{j,j+1}{C}_{n4}$), i.e. the solution variant with 'floating' or in the elastic bearing supported sun gear 2. The oscillating system is for the internal dynamics analysis loaded by the constant external torsional moments M_{um} = const., M_{ws} = const. and excited by the internal disturbing sources, i.e. with the time variable stiffness $^{1,2}C_{\overset{j}{n\hat{3}5}}(\varphi)$ in the spur gear mesh ($^{\hat{3},5}\beta_n = 0$) of the gear ring 5 and I – satellites $\hat{3}$ as well as the disturbing function in gear mesh $^{1,2}\overset{j}{f}_{n32}(\varphi)$ which occurs in this example among the satellites 3 and the sun gear 2 in the branch $j = -$ and which we will mark below as the main branch. Whereas the gearing of the sun gear 2 and of satellites 3 is considered as helical with the inclination angle ${}^{3,2}\beta_{\mathrm{n}} \neq 0$, the small variation of the stiffness function ${}^{1,2}C_{\underset{\mathrm{n}32}{j}}(\varphi)$ will be here neglected and the function supposed ${}^{1,2}C_{\underset{\mathrm{n}32}{j}}(\varphi) \approx$ const. Further, in the first stage of the solution, will be performed the solution only in the directions of the co-ordinate axes y, so that the frictions forces $\overset{j}{F}_{\mathrm{Tn}}(\varphi)$ occurred in the orthogonal direction towards the gear mesh line, i.e. in the directions x of the co-ordinate system and analogously the frictions moments $M_{\underset{F_T}{j}}(\varphi)$ that produce the additional, even if not very marked, run inequality and the system efficiency, will be neglected in this solution stage.

In the mathematical-physical model is considered the inverse phase of the mesh at the impact effects in the gear mesh. The solution hence will be contain except the normal gear mesh also the phase of the contact bounce of the tooth faces with the related re-contacts with the impacts both at the normal and the inverse mesh.

For the composition of the motion equations of the solved example we result from the motion equations of the general basic planetary differential system in the form [1]

$$\begin{aligned}&\mathbf{M}\,\mathbf{v}'' + {}_1\mathbf{K}(\beta,\delta_i,H)\,\mathbf{v}' + \sum_{K_1>1} {}_{K_1}\mathbf{K}(D,D_i,H)\,|\mathbf{w}'(\mathbf{v}')|^{K_1}\,\mathrm{sgn}(\mathbf{w}'(\mathbf{v}')) + \\ &\quad + {}_1\mathbf{C}(\varepsilon,\kappa,Y_{\mathrm{n}},U_{\mathrm{n}},V_{\mathrm{n}},H,\tau)\,\mathbf{v} + \sum_{K>1} {}_K\mathbf{C}(\varepsilon,\kappa,I_{\mathrm{n}},H,\tau)\,\mathbf{w}^K(\mathbf{v}) = \mathbf{F}(a_{\mathrm{n}},b_{\mathrm{n}},\bar{\varphi},H,\tau)\ ,\end{aligned} \tag{1}$$

where $\mathbf{v}$ means generally the m-dimensional vector of displacement of system vibration, $\mathbf{w}^K(\mathbf{v})$ K-th power of vector $\mathbf{v}$, which is defined by the expression $\mathbf{w}^K(\mathbf{v}) = \mathbf{D}(\mathbf{w}(\mathbf{v})\,\mathbf{w}^{K-1}(\mathbf{v}))$. $\mathbf{D}(\mathbf{w}(\mathbf{v}))$ means the diagonal matrix, whose elements at the main diagonal are comprised by the elements of vector $\mathbf{w}(\mathbf{v}) \equiv \mathbf{v}$. Furthermore $\mathbf{M}$ is the matrix of the mass and the inertia forces, ${}_1\mathbf{K}$ and ${}_{K_1}\mathbf{K}$ are the matrices of the linear and non-linear damping forces, ${}_1\mathbf{C}$ and ${}_K\mathbf{C}$ are the matrices of the linear and non-linear reversible forces and $\mathbf{F}(\tau)$ is the vector of the non-potential external excitation with the components a_{n}, b_{n} and with the phase angle $\overline{\varphi}$. H is the Heaviside's function, which allows to describe the motions – contact bounces – due to the strongly non-analytical non-linearities, for example due to technological tooth backlash $s(\tau)$. The corresponding linear and non-linear coefficients of the damping are denoted by β, δ_i, D, D_i, the linear parametric stiffness functions by the symbols Y_{n}, U_{n}, V_{n} and the non-linear parametric functions, so-called the parametric non-linearities, by the symbol I_{n}. ε and κ are the coefficients of the mesh duration and the amplitude modulation of the stiffness function ${}_1\mathbf{C}(\tau)$. Derivative by the non-dimensional time τ are denoted by dashes, $\tau = \omega_{\mathrm{c}}\,t$, ω_{c} is the mesh frequency, t is the time.

If we introduce in the general system (1) all above mentioned assumptions and the parameters of the solved example as well as the new dependent variables in the forms of the relative motions $y_{\underset{\mathrm{n}32}{j}}(\varphi)$ and $y_{\underset{\mathrm{n}\hat{3}5}{I}}(\varphi)$ in all gear meshes of kinematic pairs as the measure of the dynamical load of gearing [1], [3], we obtain after the longer arrangements and the omission of all weak non-linearities the system of strong non-linear time heteronymous – parametric differential equations of second order for the relative motions in the particular branches in the form $j(\equiv I)$

$$\begin{aligned}&y''_{\underset{32}{-}}(\tau) + \bar{y}''_{21\mathrm{res}}(\tau) + 2\,D_{\underset{32}{-}}\,{}^{1,2}q_{32}\left[H(y_{\underset{32}{-}}(\tau)) + H(-y_{\underset{32}{-}}(\tau) - \bar{s}_{32}(\tau))\right] y'_{\underset{32}{-}}(\tau) + \\ &\quad + 2\,D_{\underset{32}{=}}\,{}^{1,2}q_{32}\,K_{\mathrm{I}}\left[H(y_{\underset{32}{=}}(\tau)) + H(-y_{\underset{32}{=}}(\tau) - \bar{\bar{s}}_{32}(\tau))\right] y'_{\underset{32}{=}}(\tau) +\end{aligned}$$

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\begin{aligned}
&+ 2\, D_{\bar{\bar{\bar{32}}}}\, {}^{1,2}q_{32}\, K_{\mathrm{I}} \left[H(y_{\bar{\bar{\bar{32}}}}(\tau)) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] y'_{\bar{\bar{\bar{32}}}}(\tau) - \\
&- 2\, D_{\bar{52}}\, {}^{1,2}q_{52}\, K_{\mathrm{II}}\, Z_{\hat{3}} Z_3^{-1} \left[H(y_{\bar{\hat{3}5}}(\tau)) + H(-y_{\bar{\hat{3}5}}(\tau) - \bar{s}_{\hat{3}5}(\tau)) \right] y'_{\bar{\hat{3}5}}(\tau) + \\
&+ {}^{1,2}q^2_{32} \left(1 + K_{\mathrm{III}} \frac{\partial^{1,2} f_{32}(\tau)}{\partial \varphi} \right) \left[H(y_{\bar{32}}(\tau)) + H(-y_{\bar{32}}(\tau) - \bar{s}_{32}(\tau)) \right] y_{\bar{32}}(\tau) + \\
&+ {}^{1,2}q^2_{32}\, K_{\mathrm{I}} \left[H(y_{\bar{\bar{32}}}(\tau)) + H(-y_{\bar{\bar{32}}}(\tau) - \bar{\bar{s}}_{32}(\tau)) \right] y_{\bar{\bar{32}}}(\tau) + \\
&+ {}^{1,2}q^2_{32}\, K_{\mathrm{I}} \left[H(y_{\bar{\bar{\bar{32}}}}(\tau) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] y_{\bar{\bar{\bar{32}}}}(\tau) - \\
&- \left({}^{1,2}q^2_{52}\, K_{\mathrm{II}} + \xi_{25}\, K_{\mathrm{II}}\, c(\hat{\tau}) \right) \left[H(y_{\bar{\hat{3}5}}(\tau)) + H(-y_{\bar{\hat{3}5}}(\tau) - \bar{s}_{\hat{3}5}(\tau)) \right] y_{\bar{\hat{3}5}}(\tau) \\
= {} & K_{\mathrm{IV}}\, M_0^* \left[H(y_{\bar{32}}(\tau)) + H(-y_{\bar{32}}(\tau) - \bar{s}_{32}(\tau)) \right] \Big\{ 1 - \\
&- \frac{2}{3} \left[H(y_{\bar{\bar{32}}}(\tau)) + H(-y_{\bar{\bar{32}}}(\tau) - \bar{\bar{s}}_{32}(\tau)) \right] \left[H(y_{\bar{\bar{\bar{32}}}}(\tau)) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] - \\
&- \frac{1}{2} \left[H(y_{\bar{\bar{32}}}(\tau)) + H(-y_{\bar{\bar{32}}}(\tau) - \bar{\bar{s}}_{32}(\tau)) \right] \left[H(-y_{\bar{\bar{\bar{32}}}}(\tau)) + H(y_{\bar{\bar{\bar{32}}}}(\tau) + \bar{\bar{\bar{s}}}_{32}(\tau)) \right] - \\
&- \frac{1}{2} \left[H(-y_{\bar{\bar{32}}}(\tau)) + H(y_{\bar{\bar{32}}}(\tau) + \bar{\bar{s}}_{32}(\tau)) \right] \left[H(y_{\bar{\bar{\bar{32}}}}(\tau)) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] \Big\} + \\
&\qquad + {}^{1,2}f''_{32}(\tau) \left[H(y_{\bar{32}}(\tau)) + H(-y_{\bar{32}}(\tau) - \bar{s}_{32}(\tau)) \right] \,, \qquad \text{(2a)}
\end{aligned}
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\begin{aligned}
& y''_{\bar{\bar{32}}}(\tau) + \bar{\bar{\bar{y}}}''_{21\mathrm{res}}(\tau) + 2\, D_{\bar{32}}\, {}^{1,2}q_{32}\, K_{\mathrm{I}} \left[H(y_{\bar{32}}(\tau)) + H(-y_{\bar{32}}(\tau) - \bar{s}_{32}(\tau)) \right] y'_{\bar{32}}(\tau) + \\
&+ 2\, D_{\bar{\bar{32}}}\, {}^{1,2}q_{32} \left[H(y_{\bar{\bar{32}}}(\tau)) + H(-y_{\bar{\bar{32}}}(\tau) - \bar{\bar{s}}_{32}(\tau)) \right] y'_{\bar{\bar{32}}}(\tau) + \\
&+ 2\, D_{\bar{\bar{\bar{32}}}}\, {}^{1,2}q_{32}\, K_{\mathrm{I}} \left[H(y_{\bar{\bar{\bar{32}}}}(\tau)) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] y'_{\bar{\bar{\bar{32}}}}(\tau) - \\
&- 2\, D_{\bar{\bar{52}}}\, {}^{1,2}q_{52}\, K_{\mathrm{II}}\, Z_{\hat{3}}\, Z_3^{-1} \left[H(y_{\bar{\bar{\hat{3}5}}}(\tau)) + H(-y_{\bar{\bar{\hat{3}5}}}(\tau) - \bar{\bar{s}}_{\hat{3}5}(\tau)) \right] y'_{\bar{\bar{\hat{3}5}}}(\tau) + \\
&+ {}^{1,2}q^2_{32} \left(K_{\mathrm{I}} + K_{\mathrm{V}} \frac{\partial^{1,2} f_{32}(\tau)}{\partial \varphi} \right) \left[H(y_{\bar{32}}(\tau)) + H(-y_{\bar{32}}(\tau) - \bar{s}_{32}(\tau)) \right] y_{\bar{32}}(\tau) + \\
&+ {}^{1,2}q^2_{32} \left[H(y_{\bar{\bar{32}}}(\tau)) + H(-y_{\bar{\bar{32}}}(\tau) - \bar{\bar{s}}_{32}(\tau)) \right] y_{\bar{\bar{32}}}(\tau) + \\
&+ {}^{1,2}q^2_{32}\, K_{\mathrm{I}} \left[H(y_{\bar{\bar{\bar{32}}}}(\tau)) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] y_{\bar{\bar{\bar{32}}}}(\tau) - \\
&- \left({}^{1,2}q^2_{52}\, K_{\mathrm{II}} + \xi_{52}\, K_{\mathrm{II}}\, c(\hat{\tau}) \right) \left[H(y_{\bar{\bar{\hat{3}5}}}(\tau)) + H(-y_{\bar{\bar{\hat{3}5}}}(\tau) - \bar{\bar{s}}_{\hat{3}5}(\tau)) \right] y_{\bar{\bar{\hat{3}5}}}(\tau) = \\
= {} & K_{\mathrm{IV}}\, M_0^* \left[H(y_{\bar{\bar{32}}}(\tau)) + H(-y_{\bar{\bar{32}}}(\tau) - \bar{\bar{s}}_{32}(\tau)) \right] \Big\{ 1 - \\
&- \frac{2}{3} \left[H(y_{\bar{32}}(\tau)) + H(-y_{\bar{32}}(\tau) - \bar{s}_{32}(\tau)) \right] \left[H(y_{\bar{\bar{\bar{32}}}}(\tau)) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] - \\
&- \frac{1}{2} \left[H(y_{\bar{\bar{\bar{32}}}}(\tau)) + H(-y_{\bar{\bar{\bar{32}}}}(\tau) - \bar{\bar{\bar{s}}}_{32}(\tau)) \right] \left[H(-y_{\bar{32}}(\tau)) + H(y_{\bar{32}}(\tau) + \bar{s}_{32}(\tau)) \right] - \\
&- \frac{1}{2} \left[H(-y_{\bar{\bar{\bar{32}}}}(\tau)) + H(y_{\bar{\bar{\bar{32}}}}(\tau) + \bar{\bar{\bar{s}}}_{32}(\tau)) \right] \left[H(y_{\bar{32}}(\tau)) + H(-y_{\bar{32}}(\tau) - \bar{s}_{32}(\tau)) \right] \Big\} \,, \qquad \text{(2b)}
\end{aligned}
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\begin{aligned}
& y''_{\overset{\equiv}{32}}(\tau) + \overset{\equiv}{y}''_{21\mathrm{res}}(\tau) + 2\, D_{\overset{-}{32}}\; {}^{1,2}q_{32}\, K_{\mathrm{I}} \left[H(y_{\overset{-}{32}}(\tau)) + H(-y_{\overset{-}{32}}(\tau) - \overset{-}{s}_{32}(\tau)) \right] y'_{\overset{-}{32}}(\tau) + \\
& \quad + 2\, D_{\overset{=}{32}}\; {}^{1,2}q_{32}\, K_{\mathrm{I}} + \left[H(y_{\overset{=}{32}}(\tau)) + H(-y_{\overset{=}{32}}(\tau) - \overset{=}{s}_{32}(\tau)) \right] y'_{\overset{=}{32}}(\tau) + \\
& \quad + 2\, D_{\overset{\equiv}{32}}\; {}^{1,2}q_{32} \left[H(y_{\overset{\equiv}{32}}(\tau)) + H(-y_{\overset{\equiv}{32}}(\tau) - \overset{\equiv}{s}_{32}(\tau)) \right] y'_{\overset{\equiv}{32}}(\tau) - \\
& \quad - 2\, D_{\overset{\equiv}{52}}\; {}^{1,2}q_{52}\, K_{\mathrm{II}}\, Z_{\hat{3}}\, Z_3^{-1} \left[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau)) \right] y'_{\overset{\equiv}{\hat{3}5}}(\tau) + \\
& \quad + {}^{1,2}q_{32}^2 \left(K_{\mathrm{I}} + K_{\mathrm{V}} \frac{\partial^{1,2} f_{32}(\tau)}{\partial\varphi} \right) \left[H(y_{\overset{-}{32}}(\tau)) + H(-y_{\overset{-}{32}}(\tau) - \overset{-}{s}_{32}(\tau)) \right] y_{\overset{-}{32}}(\tau) + \\
& \quad + {}^{1,2}q_{32}^2\, K_{\mathrm{I}} \left[H(y_{\overset{=}{32}}(\tau)) + H(-y_{\overset{=}{32}}(\tau) - \overset{=}{s}_{32}(\tau)) \right] y_{\overset{=}{32}}(\tau) + \\
& \quad + {}^{1,2}q_{32}^2 \left[H(y_{\overset{\equiv}{32}}(\tau) + H(-y_{\overset{\equiv}{32}}(\tau) - \overset{\equiv}{s}_{32}(\tau)) \right] y_{\overset{\equiv}{32}}(\tau) - \\
& \quad - \left({}^{1,2}q_{52}^2\, K_{\mathrm{II}} + \xi_{52}\, K_{\mathrm{II}}\, c(\hat{\tau}) \right) \left[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau)) \right] y_{\overset{\equiv}{\hat{3}5}}(\tau) = \\
& = K_{\mathrm{IV}}\, M_0^* \left[H(y_{\overset{\equiv}{32}}(\tau)) + H(-y_{\overset{\equiv}{32}}(\tau) - \overset{\equiv}{s}_{32}(\tau)) \right] \Bigg\{ 1 - \\
& \quad - \frac{2}{3} \left[H(y_{\overset{-}{32}}(\tau)) + H(-y_{\overset{-}{32}}(\tau) - \overset{-}{s}_{32}(\tau)) \right] \left[H(y_{\overset{=}{32}}(\tau)) + H(-y_{\overset{=}{32}}(\tau) - \overset{=}{s}_{32}(\tau)) \right] - \\
& \quad - \frac{1}{2} \left[H(y_{\overset{-}{32}}(\tau)) + H(-y_{\overset{-}{32}}(\tau) - \overset{-}{s}_{32}(\tau)) \right] \left[H(-y_{\overset{=}{32}}(\tau)) + H(y_{\overset{=}{32}}(\tau) + \overset{=}{s}_{32}(\tau)) \right] - \\
& \quad - \frac{1}{2} \left[H(-y_{\overset{-}{32}}(\tau)) + H(y_{\overset{-}{32}}(\tau) + \overset{-}{s}_{32}(\tau)) \right] \left[H(y_{\overset{=}{32}}(\tau)) + H(-y_{\overset{=}{32}}(\tau) - \overset{=}{s}_{32}(\tau)) \right] \Bigg\} \;, \qquad \text{(2c)}
\end{aligned}
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\begin{aligned}
& y''_{\overset{-}{\hat{3}5}}(\tau) + 2\, D_{\overset{-}{\hat{3}5}}\; {}^{1,2}q_{\hat{3}5} \left[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau)) \right] y'_{\overset{-}{\hat{3}5}}(\tau) + \\
& \quad + 2\, D_{\overset{=}{\hat{3}5}}\; {}^{1,2}q_{\hat{3}5}\, K_{\mathrm{VI}} \left[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau) - \overset{=}{s}_{\hat{3}5}(\tau)) \right] y'_{\overset{=}{\hat{3}5}}(\tau) + \\
& \quad + 2\, D_{\overset{\equiv}{\hat{3}5}}\; {}^{1,2}q_{\hat{3}5}\, K_{\mathrm{VI}} \left[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau)) \right] y'_{\overset{\equiv}{\hat{3}5}}(\tau) - \\
& \quad - 2\, D_{\overset{-}{25}}\; {}^{1,2}q_{25}\, K_{\mathrm{VII}}\, Z_3\, Z_{\hat{3}}^{-1} \left[H(y_{\overset{-}{32}}(\tau)) + H(-y_{\overset{-}{32}}(\tau) - \overset{-}{s}_{32}(\tau)) \right] y'_{\overset{-}{32}}(\tau) + \\
& \quad + \left({}^{1,2}q_{\hat{3}5}^2 + \xi_{\hat{3}5}\, c(\hat{\tau}) \right) \Bigg\{ \left[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau)) \right] y_{\overset{-}{\hat{3}5}}(\tau) + \\
& \quad + K_{\mathrm{VI}} \left[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau) - \overset{=}{s}_{\hat{3}5}(\tau)) \right] y_{\overset{=}{\hat{3}5}}(\tau) + \\
& \quad + K_{\mathrm{VI}} \left[H(y_{\overset{\equiv}{\hat{3}5}}(\tau) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau)) \right] y_{\overset{\equiv}{\hat{3}5}}(\tau) \Bigg\} - \\
& \quad - \left({}^{1,2}q_{25}^2\, K_{\mathrm{VIII}} \frac{\partial^{1,2} f_{32}(\tau)}{\partial\varphi} + {}^{1,2}q_{25}^2\, K_{\mathrm{IX}} \right) \left[H(y_{\overset{-}{32}}(\tau)) + H(-y_{\overset{-}{32}}(\tau) - \overset{-}{s}_{32}(\tau)) \right] y_{\overset{-}{32}}(\tau) = \\
& = K_{\mathrm{X}}\, M_6^* \left[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau)) \right] \Bigg\{ 1 - \\
& \quad - \frac{2}{3} \left[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau) - \overset{=}{s}_{\hat{3}5}(\tau)) \right] \left[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau)) \right] - \\
& \quad - \frac{1}{2} \left[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau) - \overset{=}{s}_{\hat{3}5}(\tau)) \right] \left[H(-y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(y_{\overset{\equiv}{\hat{3}5}}(\tau) + \overset{\equiv}{s}_{\hat{3}5}(\tau)) \right] -
\end{aligned}
$$

$$- \frac{1}{2}\Big[H(-y_{\overset{=}{\hat{3}5}}(\tau)) + H(y_{\overset{=}{\hat{3}5}}(\tau) + \overset{=}{s}_{\hat{3}5}(\tau))\Big]\Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\Big\}\ , \qquad (2d)$$

$$\begin{aligned}
& y''_{\overset{=}{\hat{3}5}}(\tau) + 2\,D_{\overset{-}{\hat{3}5}}\,{}^{1,2}q_{\hat{3}5}\,K_{\mathrm{VI}}\Big[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau))\Big]\,y'_{\overset{-}{\hat{3}5}}(\tau) + \\
&\quad + 2\,D_{\overset{=}{\hat{3}5}}\,{}^{1,2}q_{\hat{3}5}\Big[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau) - \overset{=}{s}_{\hat{3}5}(\tau))\Big]\,y'_{\overset{=}{\hat{3}5}}(\tau) + \\
&\quad + 2\,D_{\overset{\equiv}{\hat{3}5}}\,{}^{1,2}q_{\hat{3}5}\,K_{\mathrm{VI}}\Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\,y'_{\overset{\equiv}{\hat{3}5}}(\tau) - \\
&\quad - 2\,D_{\overset{=}{25}}\,{}^{1,2}q_{25}\,K_{\mathrm{VII}}\,Z_3\,Z_{\hat{3}}^{-1}\Big[H(y_{\overset{=}{32}}(\tau)) + H(-y_{\overset{=}{32}}(\tau)) - \overset{=}{s}_{32}(\tau)\Big]\,y'_{\overset{=}{32}}(\tau) + \\
&\quad + \Big({}^{1,2}q^2_{\hat{3}5} + \xi_{\hat{3}5}\,c(\hat{\tau})\Big)\Big\{K_{\mathrm{VI}}\Big[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau))\Big]\,y_{\overset{-}{\hat{3}5}}(\tau) + \\
&\quad + \Big[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau)) - \overset{=}{s}_{\hat{3}5}(\tau)\Big]\,y_{\overset{=}{\hat{3}5}}(\tau) + \\
&\quad + K_{\mathrm{VI}}\Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\,y_{\overset{\equiv}{\hat{3}5}}(\tau)\Big\} - \\
&\quad - {}^{1,2}q^2_{25}\,K_{IX}\Big[H(y_{\overset{=}{32}}(\tau)) + H(-y_{\overset{=}{32}}(\tau) - \overset{=}{s}_{32}(\tau))\Big]\,y_{\overset{=}{32}}(\tau) = \\
& = K_{\mathrm{X}}\,M_6^*\Big[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau) - \overset{=}{s}_{\hat{3}5}(\tau))\Big]\Big\{1 - \\
&\quad - \frac{2}{3}\Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\Big[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau))\Big] - \\
&\quad - \frac{1}{2}\Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\Big[H(-y_{\overset{-}{\hat{3}5}}(\tau)) + H(y_{\overset{-}{\hat{3}5}}(\tau) + \overset{-}{s}_{\hat{3}5}(\tau))\Big] - \\
&\quad - \frac{1}{2}\Big[H(-y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(y_{\overset{\equiv}{\hat{3}5}}(\tau) + \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\Big[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau))\Big]\Big\}\ , \qquad (2e)
\end{aligned}$$

$$\begin{aligned}
& y''_{\overset{\equiv}{\hat{3}5}}(\tau) + 2\,D_{\overset{-}{\hat{3}5}}\,{}^{1,2}q_{\hat{3}5}\,K_{\mathrm{VI}}\Big[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau))\Big]\,y'_{\overset{-}{\hat{3}5}}(\tau) + \\
&\quad + 2\,D_{\overset{=}{\hat{3}5}}\,{}^{1,2}q_{\hat{3}5}\,K_{\mathrm{VI}}\Big[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau) - \overset{=}{s}_{\hat{3}5}(\tau))\Big]\,y'_{\overset{=}{\hat{3}5}}(\tau) + \\
&\quad + 2\,D_{\overset{\equiv}{\hat{3}5}}\,{}^{1,2}q_{\hat{3}5}\Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\,y'_{\overset{\equiv}{\hat{3}5}}(\tau) - \\
&\quad - 2\,D_{\overset{\equiv}{25}}\,{}^{1,2}q_{25}\,K_{\mathrm{VII}}\,Z_3\,Z_{\hat{3}}^{-1}\Big[H(y_{\overset{\equiv}{32}}(\tau)) + H(-y_{\overset{\equiv}{32}}(\tau)) - \overset{\equiv}{s}_{32}(\tau)\Big]\,y'_{\overset{\equiv}{32}}(\tau) + \\
&\quad + \Big({}^{1,2}q^2_{\hat{3}5} + \xi_{\hat{3}5}c(\hat{\tau})\Big)\Big\{K_{\mathrm{VI}}\Big[H(y_{\overset{-}{\hat{3}5}}(\tau)) + H(-y_{\overset{-}{\hat{3}5}}(\tau) - \overset{-}{s}_{\hat{3}5}(\tau))\Big]\,y_{\overset{-}{\hat{3}5}}(\tau) + \\
&\quad + K_{\mathrm{VI}}\Big[H(y_{\overset{=}{\hat{3}5}}(\tau)) + H(-y_{\overset{=}{\hat{3}5}}(\tau)) - \overset{=}{s}_{\hat{3}5}(\tau)\Big]\,y_{\overset{=}{\hat{3}5}}(\tau) + \\
&\quad + \Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\,y_{\overset{\equiv}{\hat{3}5}}(\tau)\Big\} - \\
&\quad - {}^{1,2}q^2_{25}\,K_{\mathrm{IX}}\Big[H(y_{\overset{\equiv}{32}}(\tau)) + H(-y_{\overset{\equiv}{32}}(\tau) - \overset{\equiv}{s}_{32}(\tau))\Big]\,y_{\overset{\equiv}{32}}(\tau) = \\
& = K_{\mathrm{X}}\,M_6^*\Big[H(y_{\overset{\equiv}{\hat{3}5}}(\tau)) + H(-y_{\overset{\equiv}{\hat{3}5}}(\tau) - \overset{\equiv}{s}_{\hat{3}5}(\tau))\Big]\Big\{1 -
\end{aligned}$$

$$
\begin{aligned}
&-\frac{2}{3}\left[H(y_{\overset{-}{\hat{3}5}}(\tau))+H(-y_{\overset{-}{\hat{3}5}}(\tau)-\overset{-}{s}_{\hat{3}5}(\tau))\right]\left[H(y_{\overset{=}{\hat{3}5}}(\tau))+H(-y_{\overset{=}{\hat{3}5}}(\tau)-\overset{=}{s}_{\hat{3}5}(\tau))\right]-\\
&-\frac{1}{2}\left[H(y_{\overset{-}{\hat{3}5}}(\tau))+H(-y_{\overset{-}{\hat{3}5}}(\tau)-\overset{-}{s}_{\hat{3}5}(\tau))\right]\left[H(-y_{\overset{=}{\hat{3}5}}(\tau))+H(y_{\overset{=}{\hat{3}5}}(\tau)+\overset{=}{s}_{\hat{3}5}(\tau))\right]-\\
&-\frac{1}{2}\left[H(-y_{\overset{-}{\hat{3}5}}(\tau))+H(y_{\overset{-}{\hat{3}5}}(\tau)+\overset{-}{s}_{\hat{3}5}(\tau))\right]\left[H(y_{\overset{=}{\hat{3}5}}(\tau))+H(-y_{\overset{=}{\hat{3}5}}(\tau)-\overset{=}{s}_{\hat{3}5}(\tau))\right]\Bigg\}\ ,
\end{aligned}
\tag{2f}
$$

$$
\begin{aligned}
&\overset{j}{y}''_{21}(\tau)+2\,\overset{j}{D}_{21}\,q_{\overset{j}{21}}\,\overset{j}{y}'_{21}(\tau)-2\,D_{\overset{j}{322}}\,q_{322}\left[H(y_{\overset{j}{32}}(\tau))+H(-y_{\overset{j}{32}}(\tau)-\overset{j}{s}_{32}(\tau))\right]y'_{\overset{j}{32}}(\tau)+\\
&+q^2_{\overset{j}{21}}\,\overset{j}{y}_{21}(\tau)-q^2_{322}\left[H(y_{\overset{j}{32}}(\tau))+H(-y_{\overset{j}{32}}(\tau)-\overset{j}{s}_{32}(\tau))\right]y_{\overset{j}{32}}(\tau)=0\ ,\quad (j=-,=,\equiv)
\end{aligned}
\tag{2g–i}
$$

This equations system describes the motion about the equilibrium position in all meshes of the kinematics pairs and the motion of the sun gear 2, where $\overset{j}{y}_{21\mathrm{res}}(\tau)$ is the resulting motion of the sun gear centre in the direction of the mesh line of the related branch of the satellite j. In the equations (2a)–(2i) are being the coefficients

$$
\begin{gathered}
K_{\mathrm{I}}=f_{\mathrm{I}}(J_{32\mathrm{red}},J_2^*)\ ,\qquad K_{\mathrm{II}}=f_{\mathrm{II}}(J_{32\mathrm{red}},J_3,J_{\hat{3}},R_{\mathrm{b}3},R_{\mathrm{b}\hat{3}})\ ,\\
K_{\mathrm{III}}=f_{\mathrm{III}}(m_{32\mathrm{red}},R_{\mathrm{b}3},Z_2,Z_3,i_{32},J_2^*,J_3,J_{\hat{3}})\ ,\qquad K_{\mathrm{IV}}=f_{\mathrm{IV}}(R_{\mathrm{b}3},J_2^*,\omega_{\mathrm{c}})\ ,\\
K_{\mathrm{V}}=f_{\mathrm{V}}(m_{32\mathrm{red}},R_{\mathrm{b}3},Z_2,i_{32},J_2^*)\ ,\qquad K_{\mathrm{VI}}=f_{\mathrm{VI}}(J_{\hat{3}5\mathrm{red}},J_5^*)\ ,\\
K_{\mathrm{VII}}=f_{\mathrm{VII}}(J_{\hat{3}5\mathrm{red}},J_3,J_{\hat{3}},R_{\mathrm{b}3},R_{\mathrm{b}\hat{3}})\ ,\qquad K_{\mathrm{VIII}}=f_{\mathrm{VIII}}(m_{\hat{3}5\mathrm{red}},R_{\mathrm{b}3},Z_3,J_3,J_{\hat{3}})\ ,\\
K_{\mathrm{IX}}=f_{\mathrm{IX}}(m_{\hat{3}5\mathrm{red}},R_{\mathrm{b}3},J_3,J_{\hat{3}})\ ,\qquad K_{\mathrm{X}}=f_{\mathrm{X}}(R_{\mathrm{b}3},J_5^*,Z_3,Z_{\hat{3}},\omega_{\mathrm{c}})\ ,\\
\xi=f_\xi(C,\kappa,m_{\mathrm{red}},\omega_{\mathrm{c}},Z_3,Z_{\hat{3}})\ ,
\end{gathered}
$$

further the tuning coefficients for the torsional motion or translational motion

$$
q=f_{\mathrm{q}}(\Omega,\omega_{\mathrm{c}},Z_3,Z_{\hat{3}})\ ,
$$

where $R_{\mathrm{b}3}$, $R_{\mathrm{b}\hat{3}}$ and Z_3, $Z_{\hat{3}}$ are the radii of generating circles and the numbers of gear cogs 3 and $\hat{3}$, i_{32} is the transmission gear ratio between component part 3 and 2, J are the moments of inertia, m_{red} and J_{red} are the reduced masses and moments of inertia, Ω is the related torsional or translational eigenfrequency. With the asterisk marked parameters are the parameters that are transformed on certain part of the system, in our example on part 3. The linear relative damping (here marked with D) is the functions

$$
D=f_{\mathrm{D}}(k,\omega_{\mathrm{c}},m_{\mathrm{red}},Z_3,Z_{\hat{3}})\ ,
$$

where k is the damping coefficient in the relative motion in gear mesh, or in the translational motion in the bearing. The function $c(\hat{\tau})$ is the unitary resulting stiffness function in the gear mesh $\hat{3}5$ and the non-dimensional time is $\hat{\tau}=Z_{\hat{3}}\,Z_3^{-1}\,\omega_{\mathrm{c}}\,t$.

For the frontal angle of the gear mesh ${}^{3,2}\alpha_{\mathrm{XY}}$ and for the three-satellite transmission system are described the resulting motions of the sun gear 2 in the particular branches by the terms

$$
\begin{aligned}
\overset{-}{y}_{21\mathrm{res}}(\tau)=\overset{-}{y}_{21}(\tau)+\Bigg[&\overset{=}{y}_{21}(\tau)\cos\left(\frac{13\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)+\\
&+\overset{\equiv}{y}_{21}(\tau)\cos\left(\frac{17\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)\Bigg]\cos\left(\frac{3\pi}{2}-{}^{32}\alpha_{\mathrm{XY}}\right)+
\end{aligned}
$$

$$+\left[\bar{\bar{y}}_{21}(\tau)\sin\left(\frac{13\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)+\right.$$
$$\left.+\bar{\bar{\bar{y}}}_{21}(\tau)\sin\left(\frac{17\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)\right]\sin\left(\frac{3\pi}{2}-{}^{32}\alpha_{\mathrm{XY}}\right)\ , \tag{3a}$$

$$\bar{\bar{y}}_{21\mathrm{res}}(\tau)=\bar{\bar{y}}_{21}(\tau)+\left[\bar{y}_{21}(\tau)\cos\left(\frac{3\pi}{2}-{}^{32}\alpha_{\mathrm{XY}}\right)+\right.$$
$$\left.+\bar{\bar{\bar{y}}}_{21}(\tau)\cos\left(\frac{17\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)\right]\cos\left(\frac{13\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)+$$
$$+\left[\bar{y}_{21}(\tau)\sin\left(\frac{3\pi}{2}-{}^{32}\alpha_{\mathrm{XY}}\right)+\right.$$
$$\left.+\bar{\bar{\bar{y}}}_{21}(\tau)\sin\left(\frac{17\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)\right]\sin\left(\frac{13\pi}{6}-{}^{32}\alpha_{XY}\right)\ , \tag{3b}$$

$$\bar{\bar{\bar{y}}}_{21\mathrm{res}}(\tau)=\bar{\bar{\bar{y}}}_{21}(\tau)+\left[\bar{y}_{21}(\tau)\cos\left(\frac{3\pi}{2}-{}^{32}\alpha_{\mathrm{XY}}\right)+\right.$$
$$\left.+\bar{\bar{y}}_{21}(\tau)\cos\left(\frac{13\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)\right]\cos\left(\frac{17\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)+$$
$$+\left[\bar{y}_{21}(\tau)\sin\left(\frac{3\pi}{2}-{}^{32}\alpha_{\mathrm{XY}}\right)+\right.$$
$$\left.+\bar{\bar{y}}_{21}(\tau)\sin\left(\frac{13\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)\right]\sin\left(\frac{17\pi}{6}-{}^{32}\alpha_{\mathrm{XY}}\right)\ . \tag{3c}$$

The resulting trajectory of the motion of the sung gear 2 towards the frame 1 we obtain by the vector sum of the motions (3a), (3b) and (3c).

During the mesh of all $j(\equiv I)$ satellites we suppose the equalized running of the torsion moments M^* in the particular satellite branches.

When the contact bounces of the meshing tooth profiles of some kinematic pairs occur, the remaining in the gear mesh finding satellites transmit the whole torsion moment – the loading of these gears increases. These effects are expressed mathematically by the combinations of the products of Heaviside's functions with the related arguments of the relative motions $y(\tau)$ and the tooth backlash $s(\tau)$ in the equations (2a)–(2i).

3. Numerical analysis

The research of the parameters influence on the qualitative and quantitative properties of the motions in the internal dynamics of the system continues in this study the mathematical-physical model of the non-linear parametric pseudoplanetary system by the numerical analysis, i.e. by the simulation methodology in the MATLAB/Simulink. The Fig. 2 pictures the global simulation scheme of the model of solved pseudoplanetary system. This scheme contains 27 basic subsystems that are in the figure represented by the grey rectangles, in them the geometric, physical and dynamical quantities of the non-linear parametric system are solved. For the illustration of the problems complexity are given in the Fig. 3 the simulation of the unitary resulting stiffness function $c(\hat{\tau})$ in the gear mesh $\hat{3}5$ which is modelled by the Fourier's series with the optional number of the terms n [2] and in the Fig. 4(A) is given the illustration of the subsystem structure for the simulation of Heaviside's functions in one branch of the power flow. These Heaviside's functions allow in the compact form the modelling of the motions in the normal and inverse gear mesh including the impact effects of the meshing tooth profiles – contact bounces – due to the strongly non-analytical non-linearities, for example due to the technological tooth backlash $s(\tau)$. In the Fig. 4(B) is given the illustration of the simulation subsystem for the resulting motion $\bar{y}_{21\mathrm{res}}(\tau)$ of the sun gear 2 in one branch, here $j = -$, of the power flow.

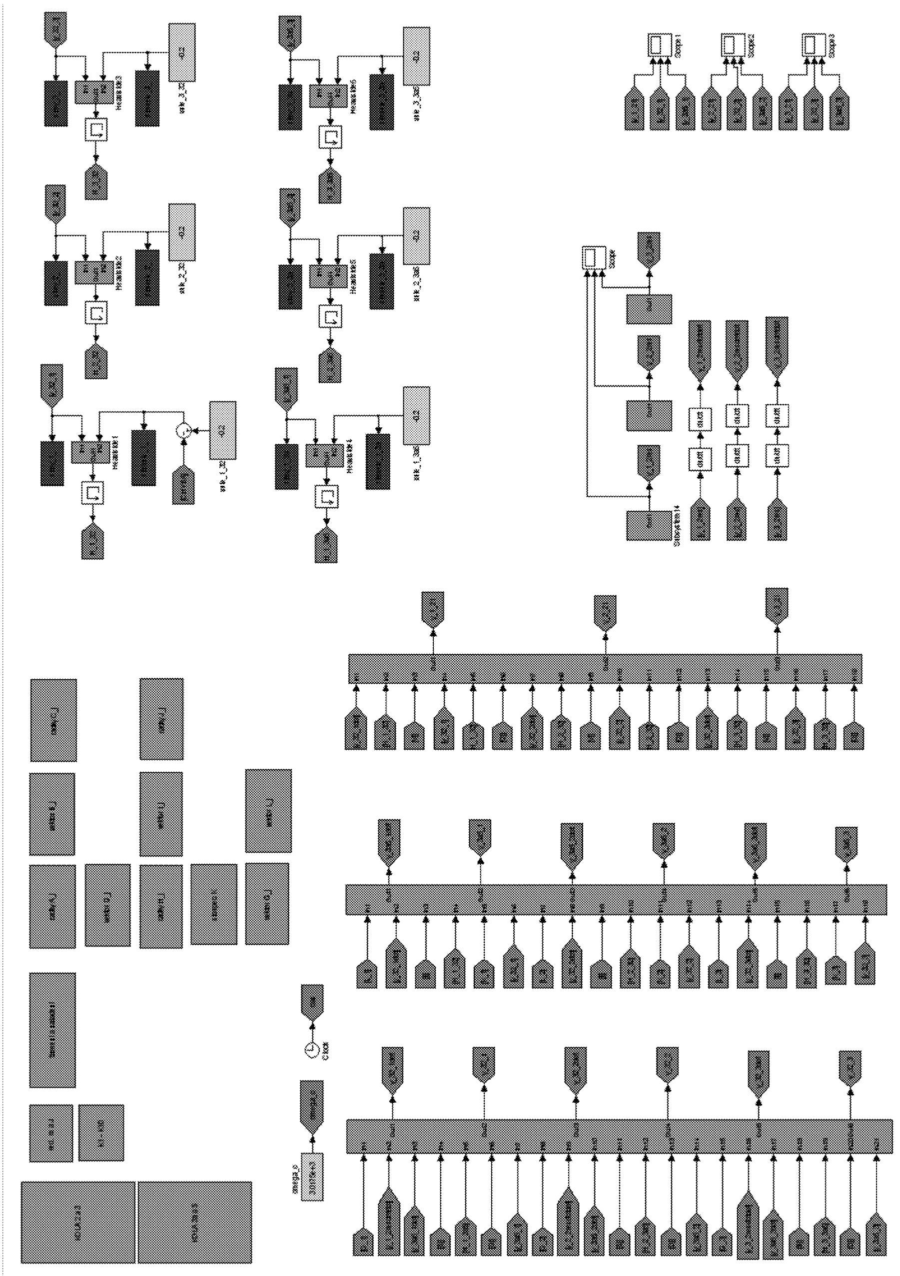

Fig.2: Scheme of the simulation model of the pseudoplanetary system from the Fig. 1

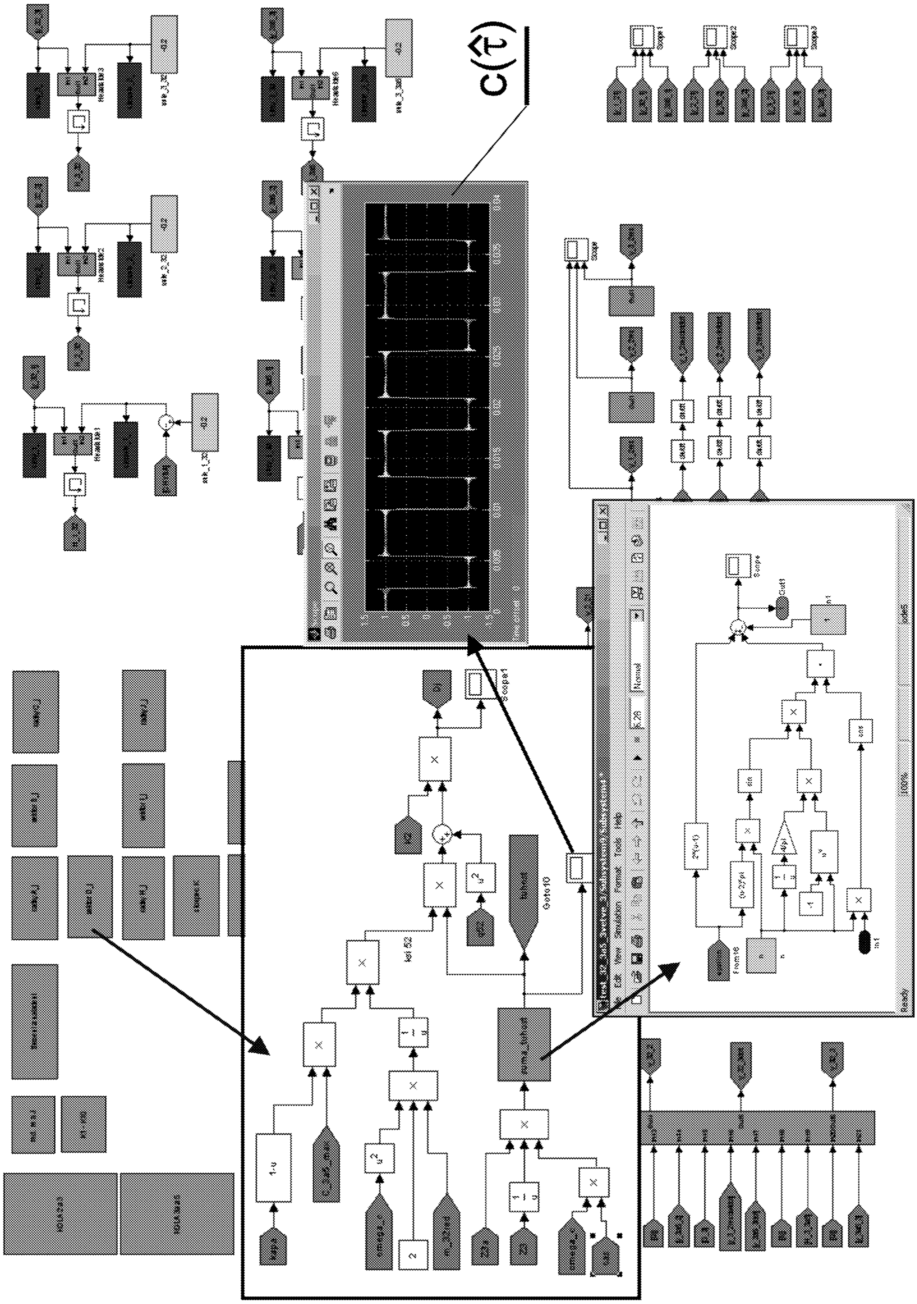

Fig.3: Subsystem for the computation of the unitary resulting stiffness function $c(\hat{\tau})$ in the gear mesh

The studies of the tuning of the numerical methodologies in MATLAB/Simulink as well as own analysis of the influence of the system parameters on the qualitative and qualitative motions properties in the internal dynamics of given pseudoplanetary system will ensue on this stage of the solution.

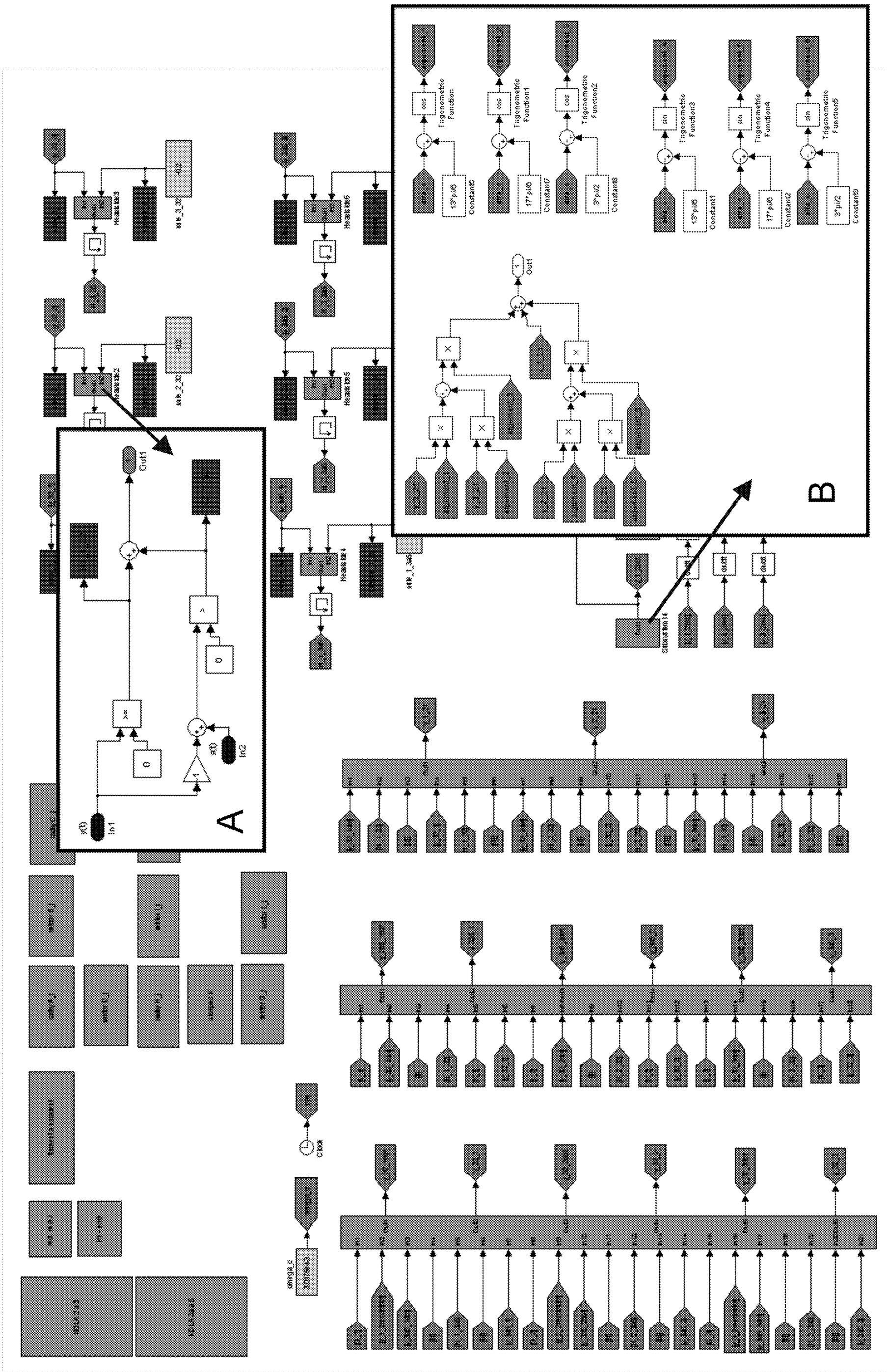

Fig.4: Subsystem for the computation of Heaviside's functions (A) and the subsystem for computation of the resulting motion of the sun gear 2 in the direction y of the co-ordinate system in the branch (j = −) of the power flow (B)

4. Conclusion

Even thought in the Fig. 1 presented substitutive dynamical model of the pseudoplanetary transmission gear system constitutes only one of the possible in the practice incident variation of the constructional arrangement of the planetary transmission systems, this model allows to study, in light of the dynamics, the important and still theoretically live question of the constructional arrangement of the mounting of the sun gear 2 in the stiff or elastic support, or by the number of the satellites $j(\equiv I) \geq 3$ in the absolutely free – floating support. The problem is important also in the term of the operation, reliability as well as the economy of the design and production.

The complexity and the variety of possible transmission system obtained by the series-parallel ordering so-called basic system or its special cases as well as the demandingness on the dynamical analysis of such complicated mathematical-physical non-linear parametric systems with many degrees of freedom requires in the complex both analytical and numerical analysis the automation, i.e. the application of the methodologies of the symbolic computations – manipulations in the project and computational works.

This work contains whole number of an original results and knowledge of the basic research in the area of non-linear time heteronymous transmissive systems with many degrees of freedom, that have the significance both for the science discipline and for the area of so-called scientific design of the class of highly powerful high-speed planetary transmission systems with minimal dimensions and limit exploitation of the constructional materials.

Perspective exploitation of these studies heads for areas of transmission systems applications for example in aeronautics, automotive and defence industry by the heavy belt trucks with limited constructive space for power units.

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