PROBABILISTIC MODEL FOR MASONRY STRENGTH OF EXISTING STRUCTURES

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In the Czech Republic numerous existing structures are made of different types of masonry. Decisions concerning upgrades of these structures should be preferably based on the reliability assessment, taking into account actual material properties. Due to inherent variability of masonry, information on its mechanical properties has to be obtained from tests. Estimation of masonry strength from measurements may be one of key issues in the assessment of existing structures. The standard technique provided in the Eurocode EN 1996-1-1 is used to develop the probabilistic model of masonry strength taking into account uncertainties in basic variables. In a numerical example characteristic and design values of the masonry strength derived using principles of the Eurocode are compared with corresponding fractiles of a proposed probabilistic model. It appears that the characteristic value based on the probabilistic model is lower than that obtained by the standard technique. To the contrary, the partial factor for masonry recommended in EN 1996-1-1 seems to be rather conservative.

Keywords : probabilistic model, masonry strength, statistical methods, existing structures tures

1. Introduction

Existing structures including those registered as cultural heritage are often affected by numerous environmental influences that may yield deterioration and gradual loss of their durability and reliability. Hence upgrades of such structures including design of adequate construction interventions is an important issue for civil engineers. Construction interventions may also become necessary in case of a change in use, concern about faulty building materials or construction methods, discovery of a design/construction error, structural damage following extreme events, complaints from users regarding serviceability etc. [1]. Rehabilitation of these structures is a matter of a great economic significance as more than 50 % of all construction activities apply to existing structures [2]. Decisions about various interventions should be always a part of the complex assessment of a structure, considering relevant input data including information on actual material properties.

In the Czech Republic numerous existing structures are made of different types of masonry. Due to inherent variability of masonry, information on its actual mechanical properties has to be obtained from tests. Estimation of masonry strength from measurements may then be one of key issues of the assessment of an existing structure.

Probabilistic framework for design and assessment of masonry structures has been suggested by Mojsilovic & Faber [3] to allow more consistent representation of the material characteristics, description of uncertainties and more economical designs or decisions about

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repairs. In the present paper the standard technique provided in Eurocode EN 1996-1-1 [4] is used to develop the probabilistic model of masonry compressive strength in the direction perpendicular to the bed joints (the key characteristic of masonry). An example of the assessment of a masonry structure built in the 19th century is used throughout the paper to clarify general concepts. Masonry strength is estimated from a limited number of destructive tests and series of non-destructive tests of its constituents. Probabilistic model for the model variable is based on experimental results reported in the literature. The characteristic and design values of masonry strength derived using principles of Eurocodes are compared with appropriate fractiles of a proposed probabilistic model. The present paper is an extension of the recent contribution to the Eleventh International Conference on Structural Studies, Repairs and Maintenance of Heritage Architecture STREMAH XI [5].

2. Evaluation of tests

Residential house, located in the downtown of Prague, was built in about 1890. Analysis of the six-storey masonry building is based on models for several parts of the structure. The present paper is focused on estimation of compressive strength of unreinforced masonry – the key issue of the assessment.

Mechanical properties of masonry are strongly dependent on properties of its constituents. Commonly, there is a large variability of mechanical properties within a structure due to workmanship and inherent variability of materials as indicated by Lourenco [6] and Stewart and Lawrence [7]. In the present case information about material properties needs to be obtained from tests. Series of non-destructive tests was supplemented by few destructive tests. In addition previous experience on accuracy of applied testing procedures is taken into account in evaluation of test results.

2.1. Strength of masonry units

Non-destructive tests of strength of masonry units by Schmidt hammer were made in 33 selected locations all over the structure. Histogram of the obtained measurements is indicated in Fig. 1. It appears that the sample includes an extreme measurement (maximum) that may result from an error within the measurement procedure. Therefore, the test proposed by Grubbs [8] is used to indicate whether the hypothesis that there is no outlier in the sample can be rejected. At the significance level 0.05 the test indicates that the hypothesis can be rejected and the measurement is deleted from the sample.

Point estimates of the sample characteristics – mean, coefficient of variation and skewness – are then estimated by the classical method of moments described by Ang and Tang [9] for which prior information on the type of an underlying distribution is not needed. The sample characteristics are indicated in Tab. 1.

It appears that the sample coefficient of variation and skewness of the masonry unit strength estimated by the non-destructive tests are low. These characteristics may provide valuable information for the choice of an appropriate statistical distribution to fit the sample data. However, it is emphasized that the sample size may be too small to estimate convincingly the sample skewness.

The sample characteristics in Tab. 1 indicate that the strength of masonry units estimated by the non-destructive tests might be described by a two-parameter lognormal distribution



Fig.1: Histogram of the masonry unit strength obtained by non-destructive tests

Variable	Symbol	Mean	Coefficient of variation	Skewness
Strength of masonry units (non-destructive tests)	$f_{ m b}'$	$43.1\mathrm{MPa}$	0.08	0.15
Conversion factor – masonry units	$\eta_{ m b}$	0.45	0.2	unknown
Strength of mortar (non-destructive tests)	$f_{ m m}'$	$1.26\mathrm{MPa}$	0.41	-0.06
Conversion factor – mortar	$\eta_{ m m}$	1	0.2	unknown
Model variable	K	0.68	0.26	unknown

Tab.1: Statistical characteristics of variables influencing the masonry strength



Fig.2: Histogram of the masonry unit strength obtained by the non-destructive tests without the outlier and the considered theoretical models

having the lower bound at the origin (LN0) or by a more general three-parameter shifted lognormal distribution having the lower bound different from zero (LN). Another possible theoretical model is the popular normal distribution.

Probability density functions of these three theoretical models (considering sample characteristics) and a sample histogram without the outlier are shown in Fig. 2. It follows that, due to the low sample coefficient of variation and skewness, all the considered models describe the sample data similarly. To compare goodness of fit of the considered distributions, Kolmogorov-Smirnov and chi-square tests described by Ang and Tang [9] are further applied. It appears that no distribution should be rejected at the 5% significance level; however, the lognormal distribution LN0 seems to be the most suitable model. Therefore, this distribution is considered hereafter.

The conversion factor $\eta_{\rm b}$ is further taken into account to determine normalised compressive strength of masonry units $f_{\rm b}$:

$$\eta_{\rm b} = \frac{f_{\rm b}}{f_{\rm b}'} , \qquad (1)$$

where $f'_{\rm b}$ denotes strength of masonry units estimated from the non-destructive tests. Previous experience indicates that the coefficient of variation of the conversion factor may be assessed by the value 0.2. Using a limited number of measurements, the mean value of the conversion factor was estimated by the value 0.45.

2.2. Mortar strength

Estimation of mortar strength may be a complicated issue since sufficiently large specimens for destructive tests can hardly be taken. Therefore, a non-destructive testing method based on a relationship between hardness and strength of mortar was developed in the Klokner Institute of the Czech Technical University in Prague.

This method is used in the assessment. Histogram of 29 measurements is indicated in Fig. 3. Point estimates of the sample characteristics given in Tab. 1 are estimated using the method of moments. The sample coefficient of variation of mortar strength is considerably greater than that of the strength of masonry units. The sample distribution seems to be nearly symmetric as the skewness is about zero. This indicates that a normal distribution might be a suitable model. However, normal distribution is not recommended for description of the variables with the coefficient of variation exceeding, say, 0.20 as negative values can be predicted. Due to the zero skewness, a three-parameter lognormal distribution LN0 is assumed hereafter for the mortar strength estimated by the non-destructive tests. Probability density functions of the theoretical models are shown in Fig. 3.



Fig.3: Histogram of the mortar strength obtained by the non-destructive tests and the considered theoretical models

The conversion factor η_m is applied to derive compressive strength of masonry mortar fm from results of the non-destructive tests:

$$\eta_{\rm m} = \frac{f_{\rm m}}{f_{\rm m}'} , \qquad (2)$$

where $f'_{\rm m}$ is the mortar strength estimated from the non-destructive tests. Previous experience indicates that the conversion factor has the unit mean and coefficient of variation 0.2 as indicated in Tab.1.

2.3. Model variable

The EN 1996-1-1 model for the characteristic compressive strength of unreinforced masonry introduces also the model variable K (see eq. (4) bellow). In the present study, the Group 1 of masonry units is assumed and the model variable is 0.55. The probabilistic model of K is assumed to include model uncertainties including lack of experimental evidence, simplifications related to the EN 1996-1-1 model and the probabilistic modelling, and unknown quality of the execution.

Contrary to the models of the strengths of the constituents, it is hardly possible to obtain experimental data on the model variable in the assessment of a specific existing structure. Therefore, available previous experience and reported experimental data need to be used in the development of a probabilistic model.

Evaluation of 20 experimental results [10] reveals that the mean of the model variable is about 1.2-times the characteristic value given in EN 1996-1-1 [4] and the coefficient of variation is 0.2.

Considering information provided in the JCSS background material [11], it is estimated that the mean of the model variable is about 1.3-times the characteristic value and the coefficient of variation is 0.34. The sample size is, however, unknown. Using an engineering experience, it is assumed that this information is relatively weak compared to the previous one [10] and the sample size is assumed to be 10.

Combining these two samples yields:

$$n = n_{1} + n_{2} = 20 + 10 = 30 ,$$

$$m_{K} = \frac{n_{1} m_{1} + n_{2} m_{2}}{n} = \frac{20 \times 1.2 \times 0.55 + 10 \times 1.3 \times 0.55}{30} = 0.68 ,$$

$$s_{K} = \sqrt{\frac{n_{1} s_{1}^{2} + n_{2} s_{2}^{2}}{n} + \frac{n_{1} n_{2}}{n^{2}} (m_{1} - m_{2})^{2}} =$$

$$= \sqrt{\frac{20 \times 0.13^{2} + 10 \times 0.24^{2}}{30} + \frac{20 \times 10}{30^{2}} (0.66 - 0.72)^{2}} = 0.18 .$$
(3)

3. Masonry strength in accordance with present standards

3.1. Characteristic value

According to EN 1996-1-1 [4] the characteristic compressive strength of unreinforced masonry made with general purpose mortar can be estimated as:

$$f_{\rm k} = K f_{\rm b}^{0.7} f_{\rm m}^{0.3} = K (\mu_{\eta_{\rm b}} \, \mu_{f_{\rm b}'})^{0.7} (\mu_{\eta_{\rm m}} \, \mu_{f_{\rm m}'})^{0.3} = = 0.55 \times (0.45 \times 43.1)^{0.7} \times (1 \times 1.26)^{0.3} = 4.7 \,\mathrm{MPa} \,,$$
(4)

where μ denotes the mean value. Note that estimates of the mean values of $f'_{\rm b}$ and $f'_{\rm m}$, based on the coverage method and related to an appropriate confidence level, should rather be used in eq. (4) than the point estimates determined by the method of moments. The difference may become significant particularly for small sample sizes. In the considered case, however, this influence is negligible and the point estimates of the mean values are applied.

It is emphasized that a rather simplified empirical model for the masonry strength considered in EN 1996-1-1 [4] may not fit available experimental data properly. Other theoretical models may then be used to describe the compressive strength of a particular type of masonry. For instance, application of an exponential function similar to that in eq. (4), but with general exponents, may improve estimation of the resulting strength [11]. More advanced models can be found in [12].

3.2. Design value

Design value of the masonry strength is derived from the characteristic value using the partial factor $\gamma_{\rm M}$:

$$f_{\rm d} = \frac{f_{\rm k}}{\gamma_{\rm M}} = \frac{4.7}{2.5} = 1.9 \,{\rm MPa} \;.$$
 (5)

The partial factor is dependent on a category of masonry units and class that may be related to execution control. However, EN 1996-1-1 [4] provides insufficient guidance on classification of masonry into the proposed categories of a quality level. Following recommendations of the Czech National Annex to EN 1996-1-1 [4], the partial factor 2.5 seems to be appropriate in this case. Note that dependence of partial factors for masonry and execution control is thoroughly analysed in the previous study [13].

3.3. Target reliability for existing structures

In the design of new structures, the design value of the masonry strength f_d is the fractile corresponding to the probability, EN 1990 [14]:

$$p_{\rm d} = \Phi \left(-\alpha_R \,\beta \right) = \Phi \left(-0.8 \times 3.8 \right) = 0.0012 \;, \tag{6}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standardised normal distribution, the FORM sensitivity factor α_R is approximated by the value -0.8 recommended for the leading resistance variable and the target reliability index β is 3.8 for a fifty-year reference period. The considered values of the sensitivity factor and reliability index are assumed to be implicitly represented by the partial factor 2.5.

In the assessment of existing structures, the target reliability level can be taken as the level of reliability implied by acceptance criteria defined in proved and accepted design codes. The target level should be stated together with clearly defined limit state functions and specific models of basic variables. For common existing structures, moderate consequences of failure and moderate costs of safety measures may often be assumed. In this case ISO 2394 [15] indicates $\beta = 3.1$.

The target reliability level can also be established taking into account the required performance level of the structure, reference period, cost of upgrades (including potential losses of the cultural and heritage value) and possible consequences of failure or malfunction. Lower target levels may be used if they are justified on the basis of social, cultural, economical, and sustainable considerations [16]. In contrast to new construction, the economic impact of required changes to existing structures to comply with reliability requirements may be very large [1, 17].

For instance a simple model for estimation of the target reliability level has been proposed by Schueremans & Van Gemert [18]. It has been shown in [19] that the target reliability level based on this model varies within a quite broad range, depending on use of a structure, societal and economic consequences and possible warning of failure. The value recommended in ISO 2394 [15] seems to be approximately in the middle of the range and is thus accepted in the following probabilistic analysis. For the assessment of the existing structure, the design value of the masonry strength is then the fractile corresponding to the probability :

$$p_{\rm d} = \Phi \left(-0.8 \times 3.1 \right) = 0.0066 \ . \tag{7}$$

4. Probabilistic analysis

4.1. Probabilistic model

It has been recognised that present standards and professional codes of practice adopt a conservative approach including the partial factor method to take into account various uncertainties. This may be appropriate for new structures where safety can often be easily increased. However, such an approach may fail for existing structures where requirements to improve the strength may lead to demanding repairs. In case of historical structures repairs may additionally yield loss of a cultural and heritage value, ICOMOS [20].

Therefore, probabilistic model for the masonry strength is proposed to estimate the characteristic and design values from the statistical data obtained by the tests and from previous experience and reduce the uncertainties implicitly covered by the model in EN 1996-1-1 [4]. Considering eq. (4), the compressive strength of masonry – random variable f, is given by:

$$f = K \left(\eta_{\rm b} f_{\rm b}'\right)^{0.7} \left(\eta_{\rm m} f_{\rm m}'\right)^{0.3} \,. \tag{8}$$

All the variables in eq. (8) are considered as random variables. Statistical characteristics are provided in Tab. 1.

In the previous section the lognormal distribution LN0 is proposed to describe variability of the strength of the constituents estimated by the non-destructive tests. In the absence of statistical data and considering general experience, the lognormal distribution LN0 is adopted also for the other variables influencing the strength of masonry. However, it is emphasized that if there is any evidence to support another distribution, then such a distribution should be preferably applied.

When all the basic variables included in eq. (8) are described by the lognormal distribution LN0, it can be easily shown that the strength of masonry has also the lognormal distribution LN0, which is in agreement with assumptions of previous studies [7,21]. The natural logarithm of the masonry strength is normally distributed with the mean and standard deviation:

$$\mu_{\ln(f)} = \mu_{\ln(K)} + 0.7 \left[\mu_{\ln(\eta_{\rm b})} + \mu_{\ln(f'_{\rm b})} \right] + 0.3 \left[\mu_{\ln(\eta_{\rm m})} + \mu_{\ln(f'_{\rm m})} \right] ,$$

$$\sigma_{\ln(f)} = \sqrt{\sigma_{\ln(K)}^2 + 0.7^2 \left[\sigma_{\ln(\eta_{\rm b})}^2 + \sigma_{\ln(f'_{\rm b})}^2 \right] + 0.3^2 \left[\sigma_{\ln(\eta_{\rm m})}^2 + \sigma_{\ln(f'_{\rm m})}^2 \right]}$$
(9)

where $\mu_{\ln(X)}$ and $\sigma_{\ln(X)}$ denote the mean and standard deviation of $\ln(X)$:

$$\mu_{\ln(X)} = \mu_X - 0.5 \,\ln[1 + V_X^2] \; ; \qquad \sigma_{\ln(X)} = \sqrt{\ln[1 + V_X^2]} \; , \tag{10}$$

where μ_X and $V_X = \sigma_X / \mu_X$ are the mean and coefficient of a variable X, respectively, given in Tab. 1.

4.2. Results of the probabilistic analysis

From eqs. (9) and (10), the mean 5.7 MPa and coefficient of variation 0.33 of the masonry strength are derived. Probability density function of the masonry strength and the characteristic and design values are indicated in Fig. 4. In accordance with EN 1996-1-1 [4], the characteristic strength of masonry corresponds to the 5% fractile of the assumed statistical distribution. In the present case the fractile of the lognormal distribution 3.2 MPa is more than 30% lower than the characteristic value estimated by eq. (4) that seems to be considerably unconservative. Similar findings have been achieved earlier by Holicky et al. [10].

Considering the target reliability index 3.8, the 1.2 % fractile of the probability distribution is 2.0 MPa and partial factor 1.6. For the lower target reliability index 3.1, the design value (6.6 ‰ fractile) increases to 2.4 MPa and the partial factor reduces to 1.3. Remarkably, the theoretical design value is by about 25 % greater than the design value estimated by eq. (5). Thus significant economic effects may be achieved when the probabilistic model is used. Characteristic and design values and partial factors for the masonry strength are summarised in Tab. 2.



Fig.4: Probability density function of the masonry strength and the characteristic and design values

Model	Characteristic value or 5% fractile in MPa	Design value $(1.2^{\circ}/_{00})$ or $6.6^{\circ}/_{00}$ fractile) in MPa	Partial factor
Deterministic	4.7	1.9	2.5
Probabilistic (target $\beta = 3.8$)	3.2	2.0	1.6
Probabilistic (target $\beta = 3.1$)	3.2	2.4	1.3

Tab.2: Characteristic and design values and partial factors for the masonry strength

Variable	Symbol	FORM sensitivity factor
Strength of masonry units (non-destructive tests)	$f_{ m b}'$	0.17
Conversion factor – masonry units	$\eta_{ m b}$	0.42
Strength of mortar (non-destructive tests)	$f_{ m m}'$	0.36
Conversion factor – mortar	$\eta_{ m m}$	0.18
Model variable	K	0.79

Tab.3: FORM sensitivity factors of the variables influencing the masonry strength

Sensitivity analysis is further conducted to investigate the importance of basic variables on the resulting probabilistic model. FORM sensitivity factors given in Tab. 3 are evaluated by the software package Comrel[®]. It follows that the model variable K is the most influencing variable and the proposed model may be improved particularly by reducing variability of this variable.

5. Concluding remarks

The following conclusions are drawn from the presented assessment of masonry strength of an existing structure:

- Due to inherent variability of masonry, information on its actual mechanical properties has to be obtained from tests and estimation of masonry strength from measurements may be one of key issues in assessment of existing structures.
- Available samples should be verified by an appropriate test of outliers as extreme measurements, possibly due to an error, may significantly affect sample characteristics.
- Appropriate models for basic variables influencing masonry strength should be selected on the basis of the statistical tests, taking into account general experience with distribution of masonry unit strength.
- Lognormal distribution having the lower bound at the origin may be a suitable model for masonry strength.
- 5 % fractile of a proposed probabilistic model for masonry strength is more than 30 % lower than the characteristic value according to EN 1996-1-1.
- For common existing structures, moderate consequences of failure and moderate costs of safety measures may often be assumed and in accordance with ISO 2394 the target reliability index reduces to 3.1.
- The theoretical design value (6.6% fractile corresponding to the reliability index 3.1) is greater by about 25% than the design value estimated in accordance with EN 1996-1-1.
- Significant economic effects may be achieved when the probabilistic model of masonry strength is used.
- The model for masonry strength may be improved particularly by reducing the variability of the model variable.

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