# ON BUCKLING OF A PLATE WITH MULTIPLE DELAMINATIONS

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Buckling behaviour of a small delaminated plate subjected to compression loading has been studied by means of the finite element analysis. The study shows several trends in the effect of number, orientation and through-the-thickness position of delaminations upon the buckling response. These findings could be used for evaluation of reliability of laminate structures which might be subjected to impact loading, such as aircraft structures.

Keywords: fibre-metal laminate, plates, buckling, delaminations, finite element analysis

# 1. Introduction

Laminated plates which were subjected to impact loading are known to contain multiple delaminations positioned through-the-thickness of the laminate. These delaminations are often oblong and their orientation matches the orientation of the adjacent composite layers which lie further away from the point of impact [1]. It is also known, that delaminations cause reduction of the buckling load of laminated structures [2] and therefore the load carrying capacity of laminate structures which might be subjected to impact loading, such as aircraft structures, could be substantially reduced as well.

It is therefore no surprise that several studies on the buckling of structures with multiple delaminations have been published. However, these studies might be of limited applicability since they utilised computational models which might not be directly applicable to realworld laminate structures. Most of the studies utilised beam-plate models, e.g. [3–8], or 2D continuum models [9–12], which are simple and have helped us to understand many aspects of the effect of delaminations upon the buckling behaviour of delaminated structures but cannot be used to describe behaviour of delaminated plates with embedded, i.e. not throughthe-width, delaminations. Some studies also utilised only a linear buckling analysis approach which cannot account for contact interaction between delaminated sublaminates [13-15], although it has been shown, that it is possible to use sequence of linear buckling analyses to prevent inadmissible overlapping of the delaminated sublaminates [14, 16]. In some studies, the requirement of plate model utilisation and incorporation of contact interactions were fulfilled, but these studies were not focused on the effect of the number, position and orientation of delaminations upon the buckling response [17–19]. Only the studies by Kyuong et al. [20, 21] utilised appropriate non-linear analysis technique and were devoted to analysis of the effect of the number of delaminations upon the buckling response. However, these studies only analysed behaviour of a limited number of variants of a plate with multiple

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through-the-width or circular delaminations of various sizes without greater focus on the position of delaminations, not to mention the effect of orientation of delaminations.

Therefore, in order to get information about the effect of multiple impact induced delaminations upon the buckling response of laminated structures, the present study was designed to study the effect of the number, through-the-thickness position and orientation of delaminations on the buckling load of a plate with multiple circular and/or elliptic delaminations. The study is based on non-linear finite element analyses which take into account contact interactions of delaminated parts of the plate.

#### 2. Analysis

#### 2.1. Problem description

Behaviour of a delaminated square plate subjected to compressive load was studied – see Figure 1. Dimensions of the plate were  $80 \times 80 \times 1.83$  mm. The plate was assumed to be made either of fibre-metal laminate, i.e. of aluminium alloy sheets interleaved with unidirectional carbon fibre/epoxy plies, or the plate was assumed to be made of aluminium alloy only. The structure of the laminate is presented in Table 1.



Fig.1: Mid-plane boundary conditions

Layer	thickness [mm]	ply angle	interface
aluminium	0.4		
composite	0.1575	$\theta$	— A —
composite	0.1575	-θ	
aluminium	0.4		– C –
composite	0.1575	-θ	— D —
composite	0.1575	$\theta$	— Е Е
aluminium	0.4		— г —

Tab.1: Structure of the laminate

The plate contained up to six delaminations. The along-the-thickness positions of delaminations corresponded to the lay-up of the fibre-metal laminate, even if the plate was assumed to be made of metal only. In addition to the six interfaces which exist in the laminate plate, an additional virtual near surface interface Z, 0.2 mm far from the plate surface, was utilised in the case of metal plates in order to study the effect of delamination orientation in the case of local buckling of a delaminated ply.

Orientations and shapes of delaminations were mostly chosen in agreement with the expected orientation of impact induced delaminations. As already mentioned, such delaminations are usually oblong and their orientations match orientations of the adjacent composite layers which lie farther away from the point of impact. In our case, the assumed impacted surface was the bottom one. So when a delamination was assumed to exist at the interface B (see Table 1), the orientation of the delamination matched the orientation of the second outermost layer. When the orientation controlling layer was made of composite, the delamination was assumed to be elliptic with the major axis length of 40 mm and minor axis length of 20 mm. When the orientation controlling layer was made of metal, a circular delamination with the diameter 40 mm was modelled. Sometimes different orientations and shape of delaminations were chosen in order to eliminate the effect of some plate parameters as it will be discussed later. The ply angle (virtual in the case of metal plates),  $\theta$ , was varied in between 0° and 90° with 15° increment.

### 2.2. Finite element mesh

The plate was modelled with layers of 8-node layered continuum shell elements (entitled SC8R in ABAQUS) which employ the first-order shear deformation theory. The number of layers was usually equal to the number of delaminations times two – each delamination was positioned between two layers of elements with the same mesh pattern and these blocks of elements were bonded to the adjacent blocks of elements by the TIE multipoint constraint implemented in ABAQUS. In the case of multiple delaminations with the same shape and orientation, the planar mesh patterns were identical for all the blocks of elements and it was therefore possible to merge nodes and thereby connect the adjacent blocks of elements directly. Sample planar mesh patterns are depicted in Figure 2.

#### 2.3. Boundary conditions

The utilised boundary conditions are depicted in Figure 1. The plate was simply supported along all its edges and one of the edges was uniformly displaced against the opposite



Fig.2: Finite element mesh

edge. Since the 3D continuum shell elements were used to build up the plate, some arrangements had to be made in order to simulate the plate boundary conditions appropriately.

Hence, the nodes on the boundary of the plate with the same in-plane coordinates were constrained to lie on a straight line. Moreover, an extra set of nodes was modelled along the plate edges on the mid-surface of the plate and the movement of the nodes on the top, bottom and mid surfaces was constrained by a set of linear equations

$$u_i^{\text{top}} + u_i^{\text{bottom}} - 2\,u_i^{\text{mid}} = 0 \ . \tag{1}$$

where  $u_i^{\text{top}}$ ,  $u_i^{\text{bottom}}$  and  $u_i^{\text{mid}}$  are the displacements of the nodes on the top, bottom and mid- surface measured along the *i*-direction (x, y, z). All the displacements corresponding to the desired in-plane movement of the plate were then prescribed at the nodes lying in the mid-plane.

#### 2.4. Material

A simple bi-linear elastic-plastic constitutive model with isotropic hardening and von Mises yield condition was used for metal plates and metal layers of the fibre-metal laminate. Corresponding material characteristics are listed in Table 2.

The composite layers of the fibre-metal laminate were modelled as orthotropic linear elastic – properties are listed in Table 3.

2024 T6 aluminium alloy			
$E = 0.725 \mathrm{GPa}$	$\nu = 0.34$		
$\sigma_{y1} = 360 \mathrm{MPa}$	$\varepsilon_{p1} = 0.000$		
$\sigma_{y2} = 521 \mathrm{MPa}$	$\varepsilon_{p2} = 0.077$		

Tab.2: Material properties of the aluminium alloy, true plastic strains  $\varepsilon_{pi}$ and corresponding Cauchy stresses  $\sigma_{yi}$  were used to define the behaviour of the material in the elastic-plastic region

Hexcel unidirectional carbon/epoxy prepreg			
$E_{11} = 126.0 \text{ GPa}$	$E_{22} = 11.0 \text{ GPa}$	$E_{33} = 11.0 \text{ GPa}$	
$\nu_{12} = 0.28$	$\nu_{13} = 0.28$	$\nu_{23} = 0.40$	
$G_{12} = 6.60 \text{ GPa}$	$G_{13}=6.60~\mathrm{GPa}$	$G_{23}=3.93~\mathrm{GPa}$	

Tab.3: Material properties of composite plies [22]

### 2.5. Contact constraints

To prevent unrealistic overlapping of delaminated sublaminates, a surface based frictionless contact interaction was used in between the delaminated sublaminates. The augmented Lagrangian contact algorithm was employed.

In order to ensure buckling of the initially flat plate, it was necessary to introduce an imperfection into the model. This was accomplished by introduction of a small virtual interference between the outermost delaminated sublaminates in the delaminated region. The interference magnitude was chosen to be  $1.10^{-6}$  m, which is less than 1% of the composite ply thickness. This value was found to be sufficiently small not to significantly affect the postbuckling response. Similar result was presented by Tay et al. [23].

#### 2.6. Solution technique

Standard quasi-static non-linear buckling and postbuckling analyses were performed using the Newton-Raphson method. The simulations were performed by the finite element analysis package ABAQUS 6.4. In order to prevent divergence due to buckling of the plate, an artificial damping algorithm implemented in ABAQUS was used.

#### 3. Results

Since the non-linear buckling analyses were utilised to study the buckling behaviour of the delaminated plates, it was necessary to use some special technique to determine the initial buckling load. In this study, the buckling load was defined as the load at which the deflection of the in-plane centre of plate reached value of  $5 \,\mu$ m. This approach was chosen to simplify evaluation of the buckling loads and provided good match with buckling loads identified by the well known Southwell plot technique [24].

First of all, the effect of the orientation of delaminations upon the buckling load values was investigated. Since it was desirable to get rid of the effect of ply anisotropy, the plate was at first assumed to be made of aluminium alloy only. Nevertheless, the number, shape and orientation of delaminations corresponded to a probable pattern of impact-induced delaminations in the laminate plate. Summary of the buckling loads is presented in Figure 3.

Even at the first sight it is evident, that the orientation of delamination did not affect the buckling load as much as the number and position of delaminations. For a plate with a given number, positions and shapes of delaminations, the maximum reduction of the buckling load due to variation of the orientation of delaminations was smaller than 30%. The maximum absolute value of the load reduction was approx. 5 kN, which is not more than 25% of the buckling load of the sound plate.



Fig.3: Metal plates - buckling loads vs. orientation of delaminations

It is interesting, that the relative buckling load reduction was approximately the same for all plates with multiple elliptic delaminations. Only in the case of the plate with one near surface circular and two elliptic delaminations, the dominance of the circular delamination caused smaller relative buckling load reduction due to the orientation of delaminations.



Fig.4: Metal plates – buckling loads vs. orientation of delaminations; alternating and identical orientations



Fig.5: Fibre-metal laminate plates – buckling load vs. orientation of delaminations

However, it is important to note, that this plate with three delaminations exhibited greater buckling load reduction than the plate with just the near surface circular delamination. Hence, the often presented statement, that a plate with a near-surface delamination exhibits the same buckling load as the same plate with additional smaller delaminations positioned closer to the mid-plane of the plate (e.g. [4, 10, 12, 14]) seems to be incorrect and may lead to unconservative buckling load predictions. It should be noted, that plates with one delamination at the B or C interface did not exhibited any buckling load reduction because the plate exhibited standard U-shaped global buckling mode shape unaltered by the presence of a delaminations. Such behaviour is typical for plates with relatively small delaminations which are rather close to the mid-plane [2].

So far, the results were presented for plates with alternating orientation of delaminations. Therefore, there was the question, whether the presence of two or more delaminations of alternating orientations could lead to virtual formation of a 'larger' delamination which



Fig.6: Load vs. in-plane centre deflections; elliptic delaminations positioned at the interfaces B and C (see Table 1)



Fig.7: Fibre-metal laminate plates - limit load vs. orientation of delaminations

would cause greater reduction of the buckling load than in the case of a plate with delaminations with the same orientation. A sort of similar effect was described by Adan et al. [25] who studied behaviour of delaminated beam-plates with two delaminations which had different in-plane and out-of-plane positions. To find out, whether different orientations of delaminations could have such effect, additional two sets of buckling analyses of plates with two delaminations of the same and alternating orientation were performed. The results are presented in Figure 4. It is evident, that the present analysis technique did not predict a significant difference in between the buckling load of the plate with two delaminations which had identical orientations. Nevertheless, it must be kept in mind, that there might have been two different buckling load reduction mechanisms of similar importance, which resulted in approximately the same buckling loads.

When the effect of orientation of delaminations upon the buckling load of the plate made of isotropic material was known, it was possible to proceed to the buckling load analysis of laminate plates. As in the case of metal plates, the number, shape and orientation of delaminations corresponded to impact-induced damage in fibre-metal laminates. Summary of the results is presented in Figure 5. As it can be seen, the situation was not as straightforward as in the case of metal plates. Similar load-orientation dependence can be seen only in the case of the plate with four elliptic delaminations. In the case of the plate with two elliptic delaminations, the characteristic trend can be seen only for the ply orientation greater than  $45^{\circ}$ . For smaller angles, the buckling load remained approximately the same, because of the competing trends to reduce the buckling load due to local buckling of more favourably oriented delamination, as it was demonstrated in the case of metal plates. and to increase the buckling load due to increasing stiffness, as in the fibre-metal laminate without a delamination. See the typical load-deflection curves in Figure 6. Focusing now on the plates with three and six delaminations, it appears, that except for the significant load reduction, the dependence of the buckling load on the ply orientation was nearly the same as in the case of the sound plate. This suggests, that the effect of ply orientation was stronger than the interaction of delaminations. However, the postbuckling behaviour and consequently the load carrying capacity of plates could be still affected by the different number and position of delaminations. This is evident from the summary of loads at the moment of onset of yielding of metal plies as illustrated in Figure 7.

How both the ply orientation and delamination orientation affect the buckling load of a plate with one elliptic delamination can be seen in Figure 8. The results which correspond to so called 'physically realistic' condition of the same ply and delamination orientations are encircled. It is evident, that even though the buckling loads which correspond to the 'physically realistic' condition may not results in a conservative buckling load estimate if the orientations of plies and delamination do not match. However, the level of unconservatism is not high and consequently the models of plates with delaminations which utilise the condition of the equal ply and delamination orientations can be used to estimate the buckling load of plates with any orientation of plies and delaminations. It should be also noticed, that when the ply angle increased from  $75^{\circ}$  to  $90^{\circ}$ , there buckling load often increased as well. This behaviour should be attributed to an interaction in between the ply orientation and delamination orientation, since no similar trend was observed in the case of metal plates.



Fig.8: Fibre-metal laminate plates – buckling load vs. orientation of plies and delaminations



Fig.9: Fibre-metal laminate plates - buckling load vs. number of delaminations

At last, the buckling loads of a fibre-metal laminate plate with up to six circular delaminations at any of the interfaces are presented in Figure 9. All possible variants of a plate with a unique number and position of delaminations were analysed. Again, it is evident, that the position of delaminations had quite significant effect upon the buckling load. The ratio of the lowest to the highest buckling load corresponding to a given number of delaminations was approximately 0.5 for plates with two, three and four delaminations. It is also evident, that with the increasing number of delaminations the minimum buckling load at first decreases and then levels off. In our case, the worst case buckling loads corresponding to plates with four, five and six delaminations were nearly identical. However, due to variation of the buckling load with position of delaminations, it is recommended to model maximum possible number of delaminations if the minimum buckling load of a delaminated structure is to be estimated.

# 4. Conclusions

The results of this study can be summarised as follows:

- It has been shown, that the buckling load is greatly affected by the position and number of delaminations, which means, that it is always necessary to have accurate information about the extent of internal damage in a laminated plate if the buckling load is to be predicted.
- The buckling load of a plate with a single near surface delamination can be further reduced by the presence of additional smaller delaminations positioned closer to the mid-plane of the plate.
- It is possible to identify minimum buckling load for a specific plate configuration and expected size of delaminations if the maximum possible number of delaminations is modelled along the thickness of a plate. The largest delamination should be modelled near the surface.
- As it is well known, the orientation of elliptic delaminations with respect the loading direction affects the buckling load. However, only the absolute value of the angle in between the delamination orientation and loading directions seems to have impact upon the buckling load, whereas the mutual orientation of delaminations does not.
- The variation of the buckling load due to variation of the orientation of delaminations is not as large as due to variation of the number or the out-of-plane position of delaminations.
- For a plate with a given number, position and shape of delaminations, the variation of the orientation of delaminations may result in the reduction of the buckling load by no more than 30%. The variation of the position of delaminations can be responsible for reduction of the buckling load by 50%.
- Buckling loads of laminate plates with delaminations of any orientation could be with reasonable accuracy predicted by models of plates with delaminations which have the same orientation as the adjacent plies.

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