BIFURCATION AND CHAOS IN DRIVE SYSTEMS

Lubomír Houfek, Ctirad Kratochvíl, Martin Houfek*

The paper is focused on analysis of dynamic properties of drive systems. It describes the possible ways of stability analysis and possible ways of analysis of bifurcation of steady states and possible occurrence of chaotic behavior.

Key words: *drive systems, bifurcation, chaotic behavior*

1. Introduction

Stability analysis cannot be omitted when examining the dynamic properties of drive systems. In case of nonlinear systems and its models one can also expect occurrence of chaotic movements. The approach towards the analysis of its occurrence possibilities will be different when analyzing models with one or a few degrees of freedom or models of real technical systems. Those problems are addressed in the contribution.

2. Occurrence of chaos in drive systems and its modelling

Dissipative dynamic system can be characterized as systems whose behaviour with increasing time asymptotically approaches steady states if there is no energy added from the outside. Such system description is possible with relatively simple nonlinear equations of motion. For certain values of parameters of those equations the solution does not converge towards expected values, but chaotically oscillates. Strong dependency on small changes of initial conditions occurs as well. When analyzing such phenomena its mathematical essence can be connected with existence of 'strange attractor' in phase plane. Possible creation of chaos can be seen in repeated bifurcation of solution, with so called cumulation point behind which the strange attractor is generated. Phase diagram of system solution then transfers from stable set of trajectories towards new, unstable and chaotic set. Creating the global trajectory diagrams is of essential importance. When succesfull, the asymptotic behavior of systems model is described.

3. Global behaviour of model of dissipative system

The problems of drive systems analysis belong into the most discussed problem of current engineering practice. There is still a number of conceptual and methodical questions which require further and mainly deeper understanding and explanation. One of those is the existence of deterministic chaos. And mainly the problem of deterministic chaos, its identification (and expectation) are described in this contribution.

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Such system when nonlinear can be extremely sensitive on small changes of initial conditions or certain design parameters. A typical example would be a clearance in drive systems. In the chaotic case the solution becomes unstable and new solution appears, unstable and diametrically different. In the case of the presentation of the computational results in phase plane the transition appears from stable set of trajectories towards new, unstable set (chaotic trajectories). It is clear that it is extremely important to understand the parameters areas in which the chaotic phenomena can occur mainly with controlled systems and/or with systems containing subsystems of different physical nature.

The changes in number of solutions of corresponding equation of motion depending on the change of a single variable (when all other variables remain constant) is called bifurcation. The importance of bifurcation diagrams is in the possibility to show the area of dynamic system parameter for which the system behavior is chaotic. One of the possible approaches of bifurcation diagram construction is in following steps:

- select one parameter which is considered bifurcation from the set of technical parameters characterizing the properties and behavior and determine the interval of such parameter change
- perform the solution of system of equations which describe the system under consideration (or its model) depending of the particular value of bifurcation parameter
- change the value of bifurcation parameter for sufficiently low value
- repeat the above described steps until the full interval of bifurcation parameter is fulfilled
- draw bifurcation diagram where each partial calculation corresponds to a single point.

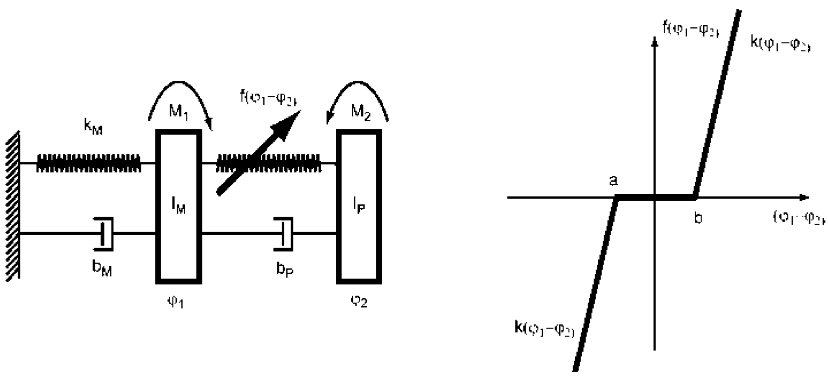

Fig.1: Model of drive system

4. Sample model of drive system

We selected a simple drive system for verification of theoretical conclusions from last chapter. The model of the system is on figure 1. This system represents a machine unit consisting of drive element and a work element coupled to each other by stiff clutch. This clutch does not have linear characteristic. The characteristic is nonlinear and its diagram is on figure 1 too.

System equations of motion can be written in form:

$$\begin{aligned} I_{\mathrm{M}}\,\ddot{\varphi}_1 + k_{\mathrm{M}}\,\varphi_1 + f(\varphi_1,\varphi_2) + b_{\mathrm{M}}\,\dot{\varphi}_1 + b_{\mathrm{P}}\,(\dot{\varphi}_1-\dot{\varphi}_2) &= M_{\mathrm{M}} + M_1^{\mathrm{M}}\,\cos(\omega\,t)\;, \\ I_{\mathrm{P}}\,\ddot{\varphi}_2 - f(\varphi_1,\varphi_2) + b_{\mathrm{P}}\,(\dot{\varphi}_1-\dot{\varphi}_2) &= M_{\mathrm{P}} - M_1^{\mathrm{P}}\,\cos(\omega\,t-\nu)\;, \end{aligned} \tag{1}$$

where nonlinear function $f(\varphi_1,\varphi_2)$ is defined by

$$f(\varphi_1,\varphi_2) = \begin{cases} k\,(\varphi_1-\varphi_2) & \text{for} \quad (\varphi_1-\varphi_2) < a \ , \\ 0 & \text{for} \quad a < (\varphi_1-\varphi_2) < b \ , \\ k\,(\varphi_1-\varphi_2) & \text{for} \quad (\varphi_1-\varphi_2) > b \ . \end{cases} \tag{2}$$

Lets analyze the case of symmetrical clearance, when $|a| = |b| = a$. Then for $|\varphi_1 - \varphi_2| \le a$ following equations hold

$$\begin{aligned} I_\mathrm{M}\,\ddot\varphi_1 + k_1\,\varphi_1 + b_\mathrm{M}\,\dot\varphi_1 + b_\mathrm{P}\,(\dot\varphi_1-\dot\varphi_2) &= M_\mathrm{M} + M_1^\mathrm{M}\,\cos(\omega\,t)\ , \\ I_\mathrm{P}\,\ddot\varphi_2 + b_\mathrm{P}\,(\dot\varphi_1-\dot\varphi_2) &= M_\mathrm{P} - M_1^\mathrm{P}\,\cos(\omega\,t-\nu) \end{aligned} \tag{3}$$

and for $|\varphi_1 - \varphi_2| > a$ the following equations hold

$$\begin{aligned} I_\mathrm{M}\,\ddot\varphi_1 + k_1\,\varphi_1 + k\,(\varphi_1-\varphi_2) + b_\mathrm{M}\,\dot\varphi_1 + b_\mathrm{P}\,(\dot\varphi_1-\dot\varphi_2) &= M_\mathrm{M} + M_1^\mathrm{M}\,\cos(\omega\,t)\ , \\ I_\mathrm{P}\,\ddot\varphi_2 + k\,(\varphi_1-\varphi_2) + b_\mathrm{P}\,(\dot\varphi_1-\dot\varphi_2) &= M_\mathrm{P} - M_1^\mathrm{P}\,\cos(\omega\,t-\nu)\ . \end{aligned} \tag{4}$$

We can do a set of computational simulation with this system. The aim of this simulation is to recognize a possibility of occurrence of chaotic behaviour or to find a bifurcation states as we describe it in chapter 3. MATLAB – Simulink system was selected for simulation, because implementation of the equations (1) to (4) is quite simple and algorithms for simulation of dynamic systems in Matlab are robust and sophisticated. Good and long experience with this simulation system is another reason for selection of this system.

5. Results

The next parameters was selected for system on figure 1 and described by equations (1) to (4) – see table 1.

These parameters were obtained in MAXON MOTOR catalogue. DC motors represent both drive and work element.

Value	Quantity	Format	Description
I_M	1.5	$\mathrm{kg\,m^2}$	Moment of inertia of drive element
I_P	1.25	$\mathrm{kg\,m^2}$	Moment of inertia of work element
M_1^M	25	N m	Dynamic moment of drive element
M_1^P	55	N m	Dynamic moment of work element
M_M	1519	N m	Static moment of drive element
M_P	565	N m	Static moment of work element
b_P	1.73	N/m s	Damping of drive element
b_M	1.43	N/m s	Damping of work element
k_P	15000	N/m	Stiffness of drive element
k_M	1000	N/m	Stiffness of work element
ν	45	$^\circ$	Phase angle between excitation effects of drive element and work element

Tab.1: Parameters of drive system

The numerical simulations were performed for these parameters. Angular velocity of excitation was selected as parameter to change, in interval from 1 rad/s to 200 rad/s. The angular displacement and angular velocity of the system was calculated for phase plane and attractors projection. The excitation in the clutch must be calculated as well for attractors projection.

The phase plane for both angular displacement and for relative angular displacement is drawn. The Poincarre map and attractor for relative angular displacement were solved and drawn too. Another possibility of system analysis is to transfer angular displacements into frequency domain and perform the analysis in this domain as described in chapter 3. The FFT from relative angular displacement was calculated and analyzed.

Next pictures show all of earlier discussed schemes for excitation angular velocity $\omega = 18.93\,\mathrm{rad/s}$, $\omega = 110\,\mathrm{rad/s}$, $\omega = 130\,\mathrm{rad/s}$, $\omega = 140\,\mathrm{rad/s}$ and $\omega = 200\,\mathrm{rad/s}$. These angular velocities were chosen as representative's values. The changes in dynamic system evolution depend on chosen parameter changes. On the spectrum of relative angular displacement you can see it very well. Although the structure of the system does not change and one parameter changes only, in the spectrum you can see many natural frequencies. These frequencies appear and disappear with the changes of angular velocity ω. Phase planes also have different shapes. Poincarre maps are interesting for analysis as well. Here we can discover typical diagrams known from theoretic studies, as stable focus for $\omega = 110\,\mathrm{rad/s}$, see Fig. 11. The system is closest to the chaotic behavior for $\omega = 18.93\,\mathrm{rad/s}$. The high dumping in the system protects the creation of fully chaotic behavior, instead relaxation oscillation occurs. The global information about system is shown on bifurcation diagram.

6. Conclusion

Chaos and other statements of nonlinear systems became phenomenon in variety of engineering problems in last years. Therefore we focused on it also in analysis of drive systems. Based on performed analysis we can state following recommendations:

- when evaluating the properties and behavior of dynamic system it is useful to define such parameters of models, which can influence the occurrence of parasitic motion including chaotic one (fluctuation of initial conditions, links gaps, control parameters),

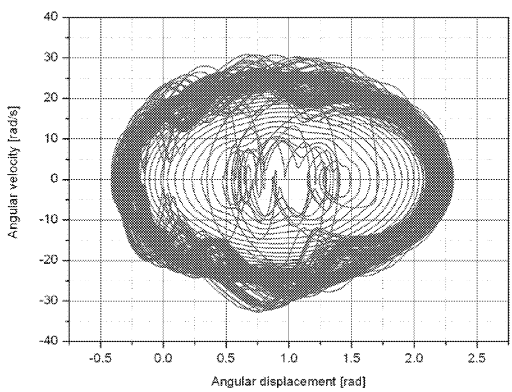

Fig.2: Phase plane for angular displacement φ_1 for $\omega = 18.93\,rad/s$

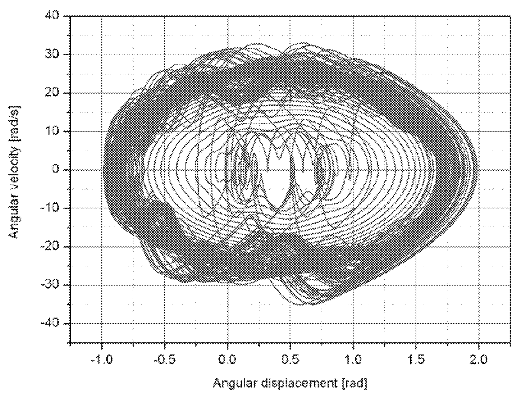

Fig.3: Phase plane for angular displacement φ_2 for $\omega = 18.93\,rad/s$

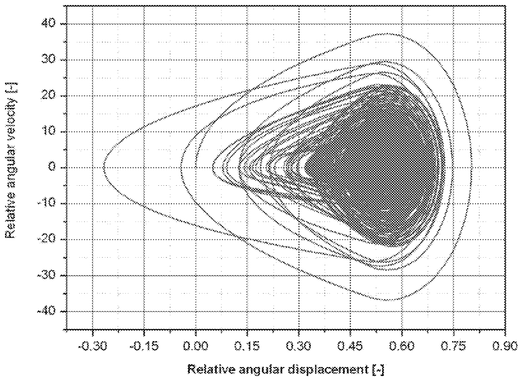

Fig.4: Phase plane for relative angular displacement for $\omega = 18.93$ rad/s

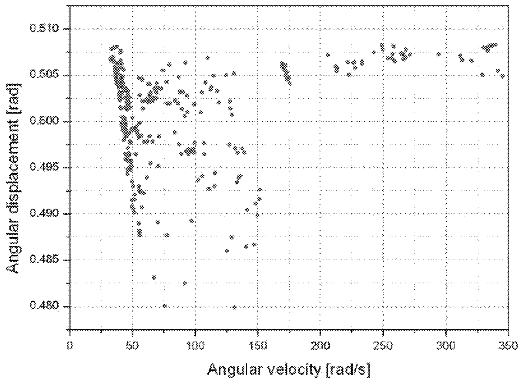

Fig.5: Poincare map for relative angular displacement for $\omega = 18.93$ rad/s

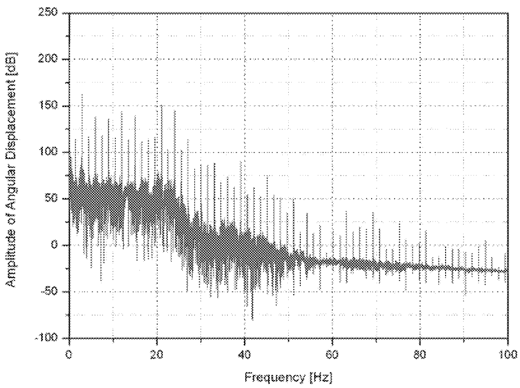

Fig.6: Spectrum of relative angular displacement for $\omega = 18.93$ rad/s

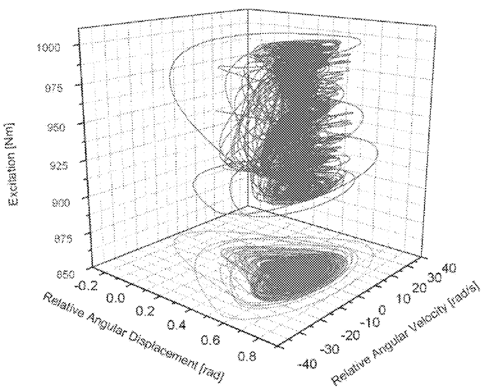

Fig.7: Attractor for relative angular displacement for $\omega = 18.93$ rad/s

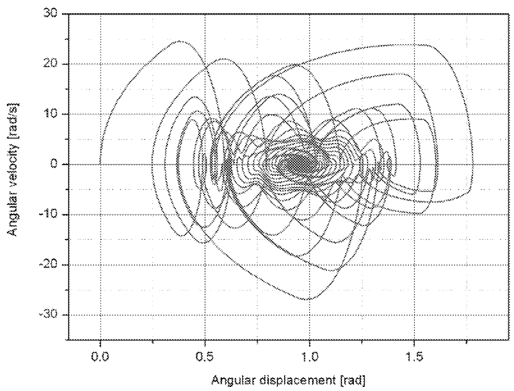

Fig.8: Phase plane for angular displacement φ_1 for $\omega = 110$ rad/s

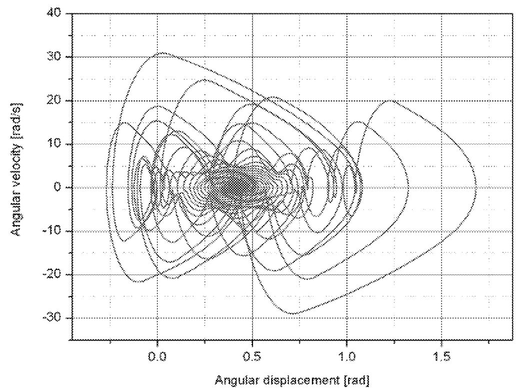

Fig.9: Phase plane for angular displacement φ_2 for $\omega = 110$ rad/s

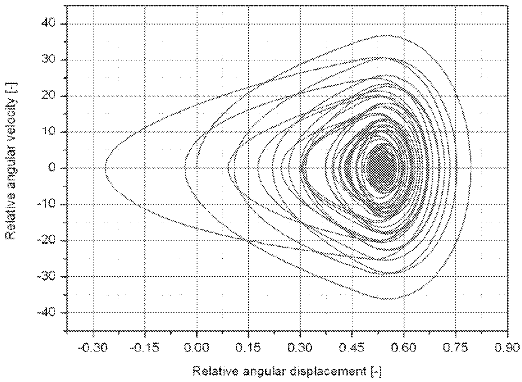

Fig.10: Phase plane for relative angular displacement for $\omega = 110$ rad/s

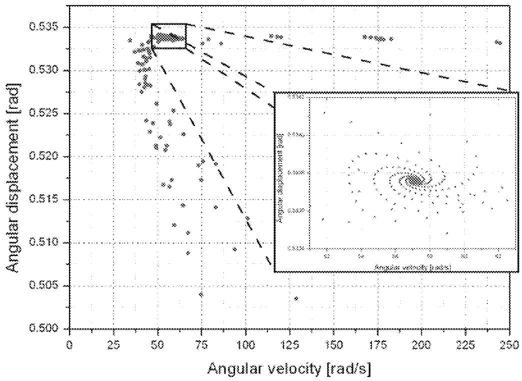

Fig.11: Poincare map for relative angular displacement for $\omega = 110$ rad/s

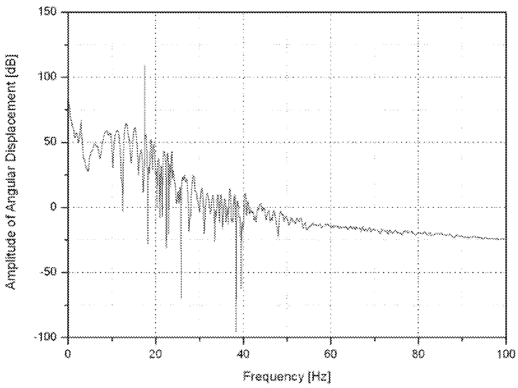

Fig.12: Spectrum of relative angular displacement for $\omega = 110$ rad/s

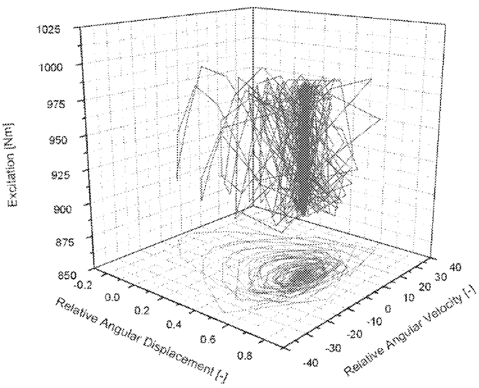

Fig.13: Attractor for relative angular displacement for $\omega = 110$ rad/s

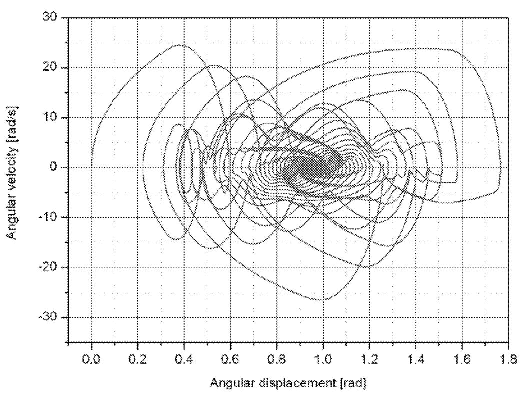

Fig.14: Phase plane for angular displacement φ_1 for $\omega = 130$ rad/s

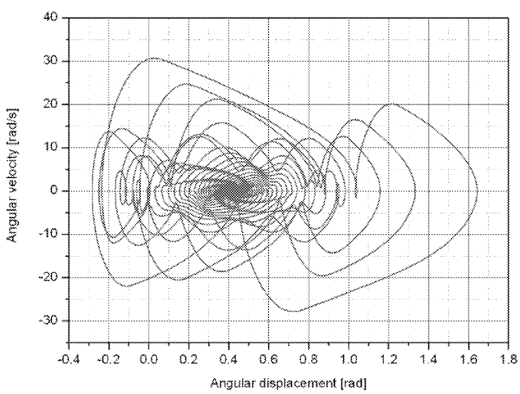

Fig.15: Phase plane for angular displacement φ_2 for $\omega = 130$ rad/s

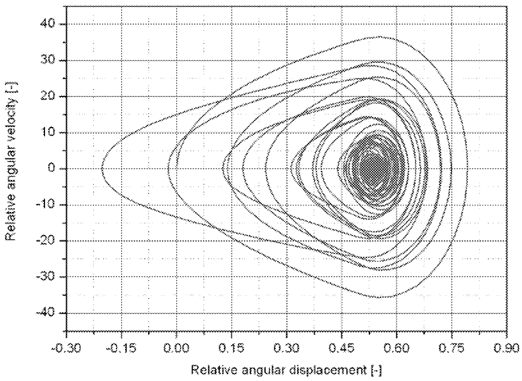

Fig.16: Phase plane for relative angular displacement for $\omega = 130\,\mathrm{rad/s}$

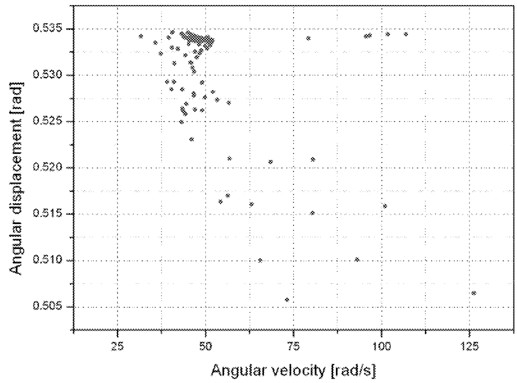

Fig.17: Poincare map for relative angular displacement for $\omega = 130\,\mathrm{rad/s}$

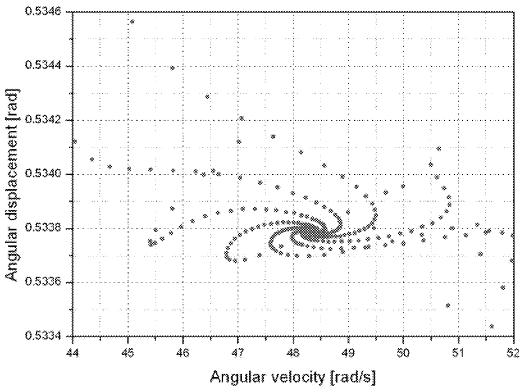

Fig.18: Poincare map for relative angular displacement for $\omega = 130\,\mathrm{rad/s}$ – zoom

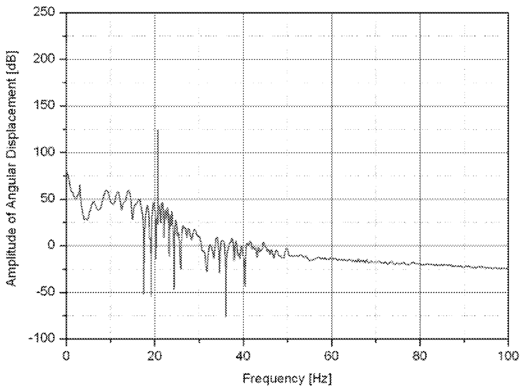

Fig.19: Spectrum of relative angular displacement for $\omega = 130\,\mathrm{rad/s}$

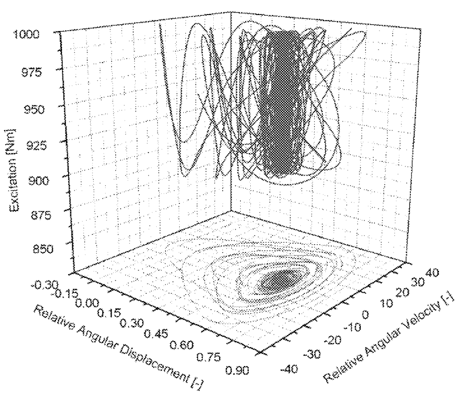

Fig.20: Attractor for relative angular displacement for $\omega = 130\,\mathrm{rad/s}$

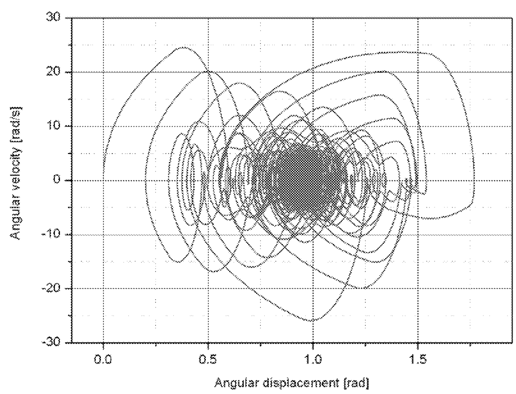

Fig.21: Phase plane for angular displacement φ_1 for $\omega = 140\,\mathrm{rad/s}$

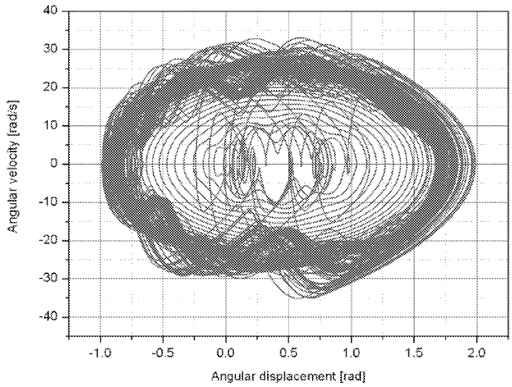

Fig.22: Phase plane for angular displacement φ_2 for $\omega = 140$ rad/s

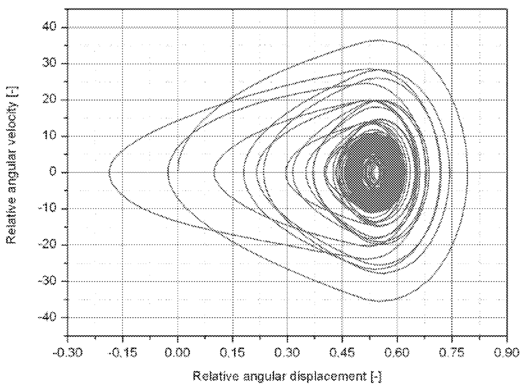

Fig.23: Phase plane for relative angular displacement for $\omega = 140$ rad/s

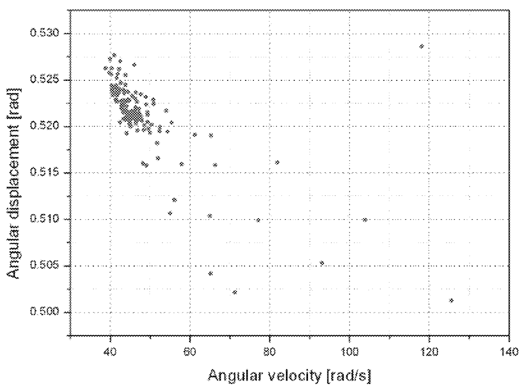

Fig.24: Poincare map for relative angular displacement for $\omega = 140$ rad/s

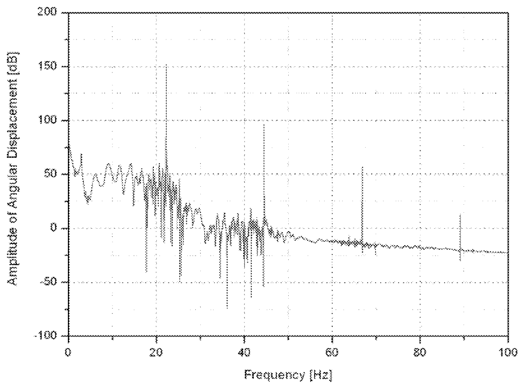

Fig.25: Spectrum of relative angular displacement for $\omega = 140$ rad/s

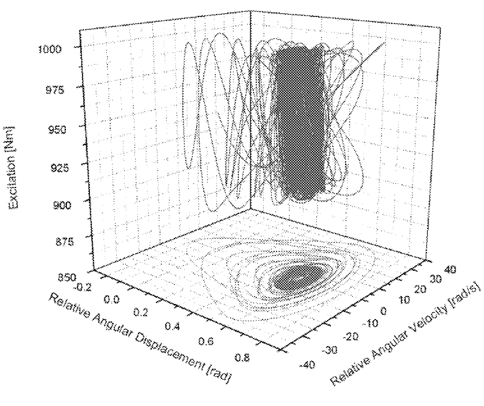

Fig.26: Attractor for relative angular displacement for $\omega = 140$ rad/s

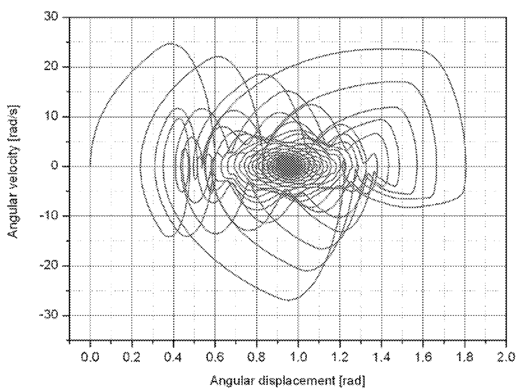

Fig.27: Phase plane for angular displacement φ_1 for $\omega = 200$ rad/s

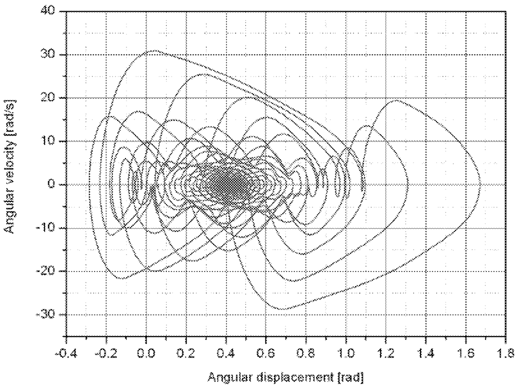

Fig.28: Phase plane for angular displacement φ_2 for $\omega = 200\,\mathrm{rad/s}$

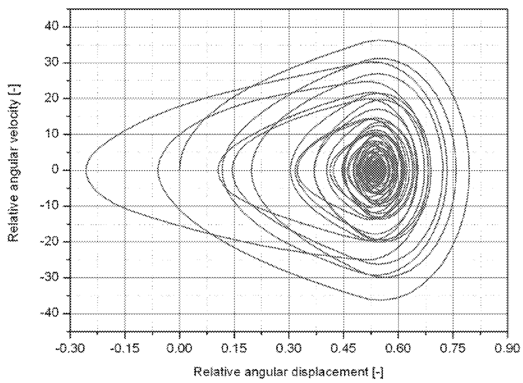

Fig.29: Phase plane for relative angular displacement for $\omega = 200\,\mathrm{rad/s}$

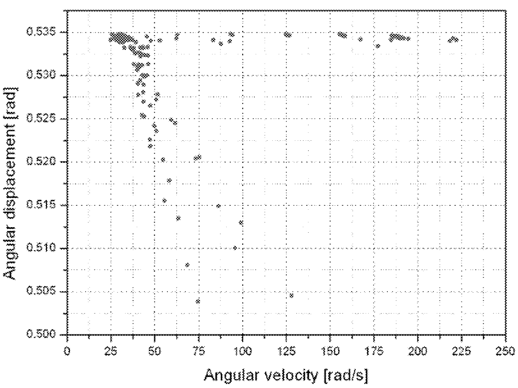

Fig.30: Poincare map for relative angular displacement for $\omega = 200\,\mathrm{rad/s}$

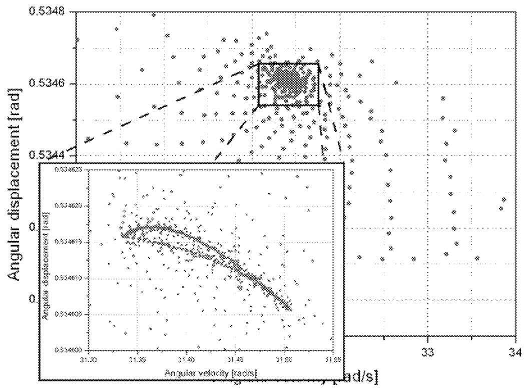

Fig.31: Poincare map for relative angular displacement for $\omega = 200\,\mathrm{rad/s}$ – zoom

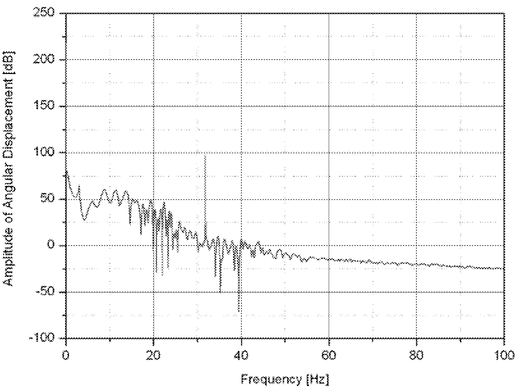

Fig.32: Spectrum of relative angular displacement for $\omega = 200\,\mathrm{rad/s}$

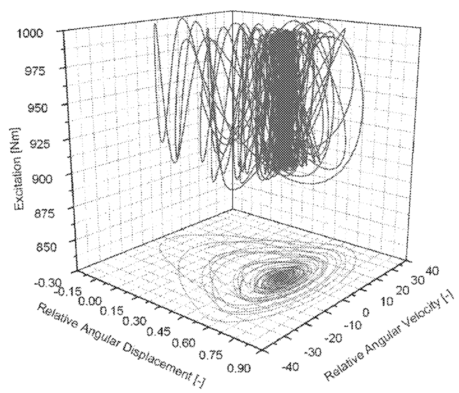

Fig.33: Attractor for relative angular displacement for $\omega = 200\,\mathrm{rad/s}$

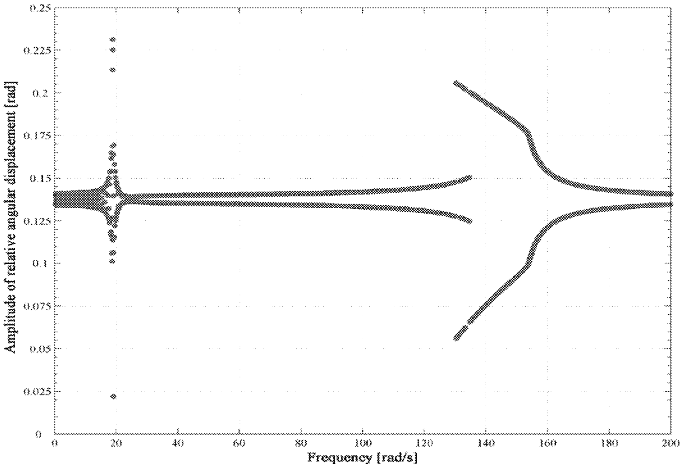

Fig.34: Bifurcation diagram

– to observe the evolution of responses in phase planes based on changes of selected parameters and to identify typical chaos effects,
– if such effect occur then evaluate Fourier spectrum of responses. Chaotic motion corresponds to broadband spectra, even when exciting spectra is narrowband.

With respecting given recommendations it is not difficult to identify the areas of possible occurrence of chaos in technical systems using mathematical modelling. However, we do not want to disvalue the analytical approaches with above described alternative approach.

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