APPLICATION OF THE GENERALIZED DIFFERENTIAL QUADRATURE METHOD TO THE FREE VIBRATIONS OF DELAMINATED BEAM PLATES

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Differential Quadrature Method (DQM) is a powerful tool in the treatment of the structural and dynamical systems. In this paper the free vibration analysis of a beam plate having single across the width delamination has been done successfully using the GDQM for the first time. The problem was formulated using a one dimensional model. The results agrees well with the previous work done by analytical and finite element methods. Accurate results has been obtained without the use of the δ technique [12] and with no restrictions of the sub-domain size [11]. Various examples were introduced to represent the accuracy of the solution and high accuracy results have been obtained.

Key words : *delaminated beam plates, vibration, generalized differential quadrature method*

1. Introduction

The method of differential quadrature discretizes any derivative at a point by a weighted linear sum of functional values at its neighbouring points.The basic procedure in the DQM is the determination of weighting coefficients. Richard Bellman 1972 introduced the DQM based on the idea of integral quadrature [1] and from this time the DQM continued to evolve. Bellman used two procedures to obtain the weighting coefficients. The first procedure used a simple functions as test functions but unfortunately when the sampling points are relatively large (say 13) the coefficient matrix become ill conditioned. The second procedure is similar to the first one with the exception that the coordinates of grid points should be chosen as the roots of the Nth order Legendre polynomial. C. Shu [2] introduced a series of papers to generalize the idea of the DQ and to make it easy to deal with the boundary conditions. Recently T. Y. Wu introduced the generalization of the differential quadrature rule [10]. In this paper we will use the Generalized Differential Quadrature (GDQ) introduced by the last author.

1.1. Treatment of boundary conditions

All the boundary value problems contains a number of boundary conditions which must be satisfied. When there is one boundary condition at each boundary point there is no problems, but if there is more than one boundary condition at each boundary point, four counted methods for the treatment of boundary conditions in the DQM. The first method is the so called δ-technique proposed by Bert et al. [3], which restricts the boundary condition at boundary points and derivative boundary conditions at the δ points, which have very

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small distance δ away from the boundary. And so this approach can not satisfy derivative boundary conditions exactly at boundary points. The solution accuracy depends on the proper choice of δ. If the value of δ is small enough, the approach produce good results in some situations such as clamped condition, however, failed to work well in the other situations such as simply-supported and free edges. On the other hand, too small values for δ will cause ill-conditioned matrices.

The second method which introduced by Wang and Bert [4] can be referred as Modifying Weighing Coefficient Matrices (MWCM). This method depends on imposing the derivative boundary conditions into the weighing coefficient matrices. MWCM gives the most accurate results in treating simply supported boundary conditions but it fails to treat other boundary conditions such as clamped and free.

The third method introduced by Shu and Du [5] can be referred as Substituting Boundary conditions into Governing Equations (SBCGE). This method substitutes the boundary conditions directly into the governing equation and the derivative boundary conditions can be coupled to provide the values of the function values at to points optimally adjacent to the first and last points. This method works well at any type of boundary conditions, and so this method treats the drawbacks of the previous methods. Although SBCGE is an efficient method in treating boundary conditions, it suffers some lack of generality and consumes much time and effort in preparing the modified matrices.

The fourth method introduced by Shu and Du [6] can be referred as Coupling Boundary Conditions with Governing Equations (CBCGE). By this method the vibration problem can be transformed into a general boundary value problem. This method works efficiently with any combinations of boundary conditions, but it gives less accurate results than those obtained by (SBCGE).

2. Differential quadrature method

The main idea of the DQM is that the derivative of a function at a sample point can be approximated as a weighted linear summation of the value of the function at all of the sample points in the domain. Using this approximation, the differential equation is reduced to a set of algebraic equations. The number of equations depends on the selected number of sample points.

$$\frac{\mathrm{d}^m}{\mathrm{d}x^m}\begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{bmatrix} \approx [C_{ij}^m]\begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{bmatrix} , \qquad i,j = 1,2,\ldots,N \ , \tag{1}$$

where $f(x_i)$ is the value of the function at the sample point x_i, and C_{ij}^m are the weighting coefficients of the m^{th}-order differentiation attached to these functional values. Quan et al. [7] introduced a Lagrangian interpolation polynomial to overcome the numerical ill-conditions in determining the weighting coefficients C_{ij}^m

$$f(x) = \sum_{i=1}^{N} \frac{M(x)}{(x-x_i)\, M_1(x_i)}\, f(x_i) \ , \tag{2}$$

where

$$M(x) = \prod_{j=1}^{N} (x - x_j) \ ,$$

$$M_1(x_i) = \prod_{j=1}^{N} (x_i - x_j) \qquad \text{for} \quad i = 1, 2, \dots, N \ ,$$

substituting equation (2) into equation (1) yields,

$$C^1_{ij} = \frac{M_1(x_i)}{(x_j - x_i)\, M_1(x_j)} \qquad \text{for} \quad i, j = 1, 2, \dots, N \quad \text{and} \quad i \neq j \ , \tag{3}$$

$$C^1_{ii} = -\sum_{j=1,\, j\neq i}^{N} C^1_{ij} \qquad \text{for} \quad i = 1, 2, \dots, N \ . \tag{4}$$

After the sample points have been selected, the coefficients of the first order weighting matrix can be obtained from equations (3) and (4). The number of the test functions must exceed the highest order of the derivative in the governing equations; that is $N > m$. Higher order coefficient matrices can be obtained from the first order weighting matrix as follows,

$$C^2_{ij} = \sum_{k=1}^{N} C^1_{ik}\, C^1_{kj} \qquad \text{for} \quad i, j = 1, 2, \dots, N \ , \tag{5}$$

$$C^3_{ij} = \sum_{k=1}^{N} C^1_{ik}\, C^2_{kj} \qquad \text{for} \quad i, j = 1, 2, \dots, N \ , \tag{6}$$

$$C^4_{ij} = \sum_{k=1}^{N} C^1_{ik}\, C^3_{kj} \qquad \text{for} \quad i, j = 1, 2, \dots, N \ . \tag{7}$$

The GDQ introduced by T. Y. Wu [10] considers the general case. A one dimensional field variable is described by a differential equation in a field domain $a = x_1 \leq x \leq x_N = b$ and may be restricted by a set of conditions a given points. In this rule the authors use the Hermite interpolation shape function as a field function.

The reason of using the Hermite interpolation shape function is its ability to prescribe at each support abscissa not only the value but also the first $n_i - 1$ derivative of the desired polynomial. For more details about the GDQ refer to [10].

3. Problem formulation

The one dimensional model in Fig. 1 represents the delaminated beam plate as a four interconnected Euler-Bernoulli beams. The material is assumed to be linear, elastic, and isotropic.

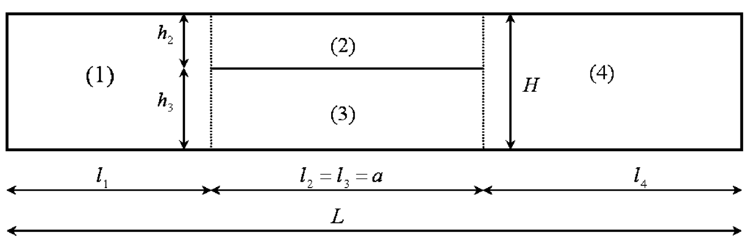

Fig.1: Delaminated beam plate model

The governing equations of each sub-laminate can be represented as,

$$D_i \frac{\partial^4 \bar{w}_i}{\partial \bar{x}^4} - m_i \frac{\partial^2 \bar{w}_i}{\partial \bar{x}^2} = 0 , \qquad i = 1,2,3,4 , \tag{8}$$

where $\bar{w}_i$, m_i are transverse deflection and mass per unit length of each sub-laminate. The flexural rigidity of each sub-laminate is defined as

$$D_i = \frac{E\, h_i^3}{12\,(1-\nu^2)} \tag{9}$$

and E is the modulus of elasticity and ν is Poisson's ratio.

For free vibrations

$$\bar{w}_i(\bar{x}_i, t) = \bar{W}_i(\bar{x}_i)\, \sin(\omega\, t) . \tag{10}$$

Substituting equation (10) into equation (8) to eliminate the time terms,

$$D_i \frac{\partial^4 \bar{W}_i}{\partial \bar{x}^4} - m_i\, \omega^2\, \bar{W}_i = 0 , \qquad i = 1,2,3,4 , \tag{11}$$

ω is the natural frequency of the free vibrations.

Introducing the non-dimensional variables,

$$x_i = \frac{\bar{x}_i}{l} , \qquad W_i = \frac{\bar{W}_i}{H} \tag{12}$$

equation (9) becomes

$$S_i \frac{\partial^4 W_i}{\partial x^4} - \Omega^2\, W_i = 0 , \qquad S_i = \left(\frac{t_i}{H}\right)^2 \left(\frac{L}{l_i}\right)^4 , \qquad i = 1,2,3,4 , \tag{13}$$

where L is the total length of the base beam-plate and $\Omega^2 = m_1\, L^4\, \omega^2 / D_1$ is the non-dimensional frequency. In this work we will use the same number N of sampling points in each sub-domain. We have two junctions ($x = l_1$ and $x = l_1 + a$) and two supports ($x = 0$ and $x = L$), at each one there are two conditions ($n = 2$) to be satisfied and only one condition at the rest of the domain points ($n_j = 1$). Thus for each sub-domain $M = \sum_{j=1}^{N} n_j = (N-2) \times 1 + 2 \times 2 = N + 2$. The total number of independent variable in the whole domain is, $4\,(N-2) + 4 \times 2 = 4\,N$ independent variable as follows,

$$\begin{aligned}
\mathbf{U}^{[1]} &= [U_1, U_2, U_3, \ldots, U_{N+2}] = \\
&= [w_1, w_1^{(1)}, w_2, \ldots, w_{N-1}, w_N, w_N^{(1)}] , \\
\mathbf{U}^{[2]} &= [U_{N+1}, U_{N+2}, \ldots, U_{2N+1}, U_{2N+2}] = \\
&= [w_N, w_N^{(1)}, w_{N+1}, \ldots, w_{2N-2}, w_{2N-1}, w_{2N-1}^{(1)}] , \\
\mathbf{U}^{[3]} &= [U_{N+1}, U_{N+2}, U_{2N+3}, U_{2N+4}, \ldots, U_{3N}, U_{2N+1}, U_{2N+2}] = \\
&= [w_N, w_N^{(1)}, w_{2N}, w_{2N+1}, \ldots, w_{3N-3}, w_{2N-1}, w_{2N-1}^{(1)}] , \\
\mathbf{U}^{[4]} &= [U_{2N+1}, U_{2N+2}, U_{3N+1}, U_{3N+2}, \ldots, U_{4N-2}, U_{4N-1}, U_{4N}] = \\
&= [w_{2N-1}, w_{2N-1}^{(1)}, w_{3N-2}, w_{3N-1}, \ldots, w_{4N-4}, w_{4N-4}^{(1)}] ,
\end{aligned} \tag{14}$$

where

$$w^{(1)} = \frac{\mathrm{d}w}{\mathrm{d}x} \ .$$

Applying the Differential Quadrature discretization to the non-dimensional governing equation (13),

$$S_i \sum_{j=1}^{N+2} E_{ij}^{(4)} U_j^{[i]} - \Omega^2 U_i^{[i]} = 0 \ , \qquad i = 1,2,3,4 \ , \tag{15}$$

where $E_{ij}^{[4]}$ is the weighting coefficients matrix of the fourth order derivative in the Generalized Differential Quadrature. It must be noted that the matrix $[E_{ij}^{(4)}]$ is of order $N{\times}M$.

To complete the formulation we have to discuss the boundary conditions. The solution will be obtained for the case of clamped-clamped.

The clamped boundary conditions at $x = 0$

$$W_1(0) = \frac{\partial W_1}{\partial x}(0) = 0 \ , \tag{16}$$

the clamped boundary conditions at $x = L$

$$W_4(L) = \frac{\partial W_4}{\partial x}(L) = 0 \ , \tag{17}$$

the balance of moment at the junction at $x = l_1$

$$M_1 - M_2 - M_3 + P_2 \left(\frac{h_3}{2}\right) - P_3 \left(\frac{h_2}{2}\right) = 0 \ , \tag{18}$$

the balance of moment at the junction at $x = l_1 + a$

$$M_4 - M_2 - M_3 + P_2 \left(\frac{h_3}{2}\right) - P_3 \left(\frac{h_2}{2}\right) = 0 \ , \tag{19}$$

where P_2 and P_3 are the axial forces in regions 2 and 3 during vibration.

The balance of shear at the junction at $x = l_1$

$$Q_1 - Q_2 - Q_3 = 0 \ , \tag{20}$$

the balance of shear at the junction at $x = l_1 + a$

$$Q_4 - Q_2 - Q_3 = 0 \ . \tag{21}$$

To eliminate the non-homogenous terms in the boundary conditions (18) and (19) we have to apply the compatibility of axial shortening between the upper and lower sub-laminate, for symmetric delamination,

$$\frac{1}{2}\int_0^a \left(\frac{\mathrm{d}w_2}{\mathrm{d}x}\right)^2 \mathrm{d}x + \frac{(1-\nu^2)\, P_2\, a}{E\, h_2} = \frac{1}{2}\int_0^a \left(\frac{\mathrm{d}w_3}{\mathrm{d}x}\right)^2 \mathrm{d}x + \frac{(1-\nu^2)\, P_3\, a}{E\, h_3} + H\, \frac{\mathrm{d}w_1(l_1)}{\mathrm{d}x} \ , \tag{22}$$

for nonsymmetric delamination,

$$\frac{1}{2}\int_0^a \left(\frac{\mathrm{d}W_2}{\mathrm{d}x}\right)^2 \mathrm{d}x + \frac{(1-\nu^2)\,P_2\,a}{E\,h_2} = \frac{1}{2}\int_0^a \left(\frac{\mathrm{d}W_3}{\mathrm{d}x}\right)^2 \mathrm{d}x + \\ + \frac{(1-\nu^2)\,P_3\,a}{E\,h_3} + H\left(\frac{\mathrm{d}W_1(l_1)}{\mathrm{d}x} + \frac{\mathrm{d}W_1(l_1+a)}{\mathrm{d}x}\right) . \tag{23}$$

Simplifying equations (22) and (23) by eliminating the insignificant higher order terms we can get the following expressions, for symmetric delamination

$$P_2\left(\frac{h_3}{2}\right) - P_3\left(\frac{h_2}{2}\right) = \frac{H\,h_2\,h_3\,E}{2\,a\,(1-\nu^2)}\,\frac{\mathrm{d}W}{\mathrm{d}x}(l_1) \ ,$$

for non-symmetric delamination

$$P_2\left(\frac{h_3}{2}\right) - P_3\left(\frac{h_2}{2}\right) = \frac{H\,h_2\,h_3\,E}{2\,a\,(1-\nu^2)}\left[\frac{\mathrm{d}W}{\mathrm{d}x}(l_1) + \frac{\mathrm{d}W}{\mathrm{d}x}(l_1+a)\right] .$$

Transforming the boundary conditions into the DQ form we get, the clamped boundary conditions at $x=0$ and $x=L$

$$U_1 = 0 \ , \tag{24}$$

$$U_2 = 0 \ , \tag{25}$$

$$U_{4N-1} = 0 \ , \tag{26}$$

$$U_{4N} = 0 \ , \tag{27}$$

the balance of moment at the junction at $x = l_1$

$$\frac{H^3}{l_1^2}\sum_{j=1}^{N+2} E^2_{Nj}\,U^{[1]}_j - \frac{h_2^3}{l_2^2}\sum_{j=1}^{N+2} E^2_{1j}\,U^{[2]}_j - \frac{h_3^3}{l_3^2}\sum_{j=1}^{N+2} E^2_{1j}\,U^{[3]}_j + \frac{6\,H\,h_2\,h_3}{l_1\,a}\,U^{[1]}_{N+2} = 0 \ , \tag{28}$$

the balance of moment at the junction at $x = l_1 + a$

$$\frac{H^3}{l_4^2}\sum_{j=1}^{N+2} E^2_{1j}\,U^{[4]}_j - \frac{h_2^3}{l_2^2}\sum_{j=1}^{N+2} E^2_{Nj}\,U^{[2]}_j - \frac{h_3^3}{l_3^2}\sum_{j=1}^{N+2} E^2_{Nj}\,U^{[3]}_j + \frac{6\,H\,h_2\,h_3}{l_4\,a}\,U^{[4]}_{2N+2} = 0 \ , \tag{29}$$

the balance of shear at the junction at $x = l_1$

$$\frac{H^3}{l_1^3}\sum_{j=1}^{N+2} E^3_{Nj}\,U^{[1]}_j - \frac{h_2^3}{l_2^3}\sum_{j=1}^{N+2} E^3_{1j}\,U^{[2]}_j - \frac{h_3^3}{l_3^3}\sum_{j=1}^{N+2} E^3_{1j}\,U^{[3]}_j = 0 \ , \tag{30}$$

the balance of shear at the junction at $x = l_1 + a$

$$\frac{H^3}{l_4^3}\sum_{j=1}^{N+2} E^3_{1j}\,U^{[4]}_j - \frac{h_2^3}{l_2^3}\sum_{j=1}^{N+2} E^3_{Nj}\,U^{[2]}_j - \frac{h_3^3}{l_3^3}\sum_{j=1}^{N+2} E^3_{Nj}\,U^{[3]}_j = 0 \ . \tag{31}$$

Now we have $4\,(N-2)$ equations from the governing equation (15) and eight equations from the boundary conditions which gives a total number of $4\,N$ equations which is exactly the same number of the independent variables.

Now we decompose the function values in the whole domain into two portions. One is based on the interior points $\mathbf{U}_{\mathrm{i}}$, and the other is based on the boundary points $\mathbf{U}_{\mathrm{b}}$. We put the coefficients of the interior points located in the governing equations in the matrix $\mathbf{A}_{\mathrm{ii}}$, and the coefficients of the boundary points located in the governing equation in the matrix $\mathbf{A}_{\mathrm{ib}}$. Also we must decompose the boundary conditions to get the matrix $\mathbf{A}_{\mathrm{bi}}$ which contains the coefficients of the interior points located in the boundary conditions, and the matrix $\mathbf{A}_{\mathrm{bb}}$ that contains the coefficients of the boundary points located in the boundary conditions.

This system of equations can be written as,

$$\mathbf{A}_{\mathrm{ib}}\,\mathbf{U}_{\mathrm{b}} + \mathbf{A}_{\mathrm{ii}}\,\mathbf{U}_{\mathrm{i}} = \Omega^2\,\mathbf{U}_{\mathrm{i}}\ , \tag{32}$$

$$\mathbf{A}_{\mathrm{bb}}\,\mathbf{U}_{\mathrm{b}} + \mathbf{A}_{\mathrm{bi}}\,\mathbf{U}_{\mathrm{i}} = \mathbf{0}. \tag{33}$$

$\mathbf{U}_{\mathrm{i}}$ and $\mathbf{U}_{\mathrm{b}}$ are defined by equation (14) in the global coordinate system.

From the above equations we can write,

$$\mathbf{U}_{\mathrm{b}} = -\mathbf{A}_{\mathrm{bb}}^{-1}\,\mathbf{A}_{bi}\,\mathbf{U}_{\mathrm{i}}\ . \tag{34}$$

By substituting equation (34) into equation (32) we get the generalized egenvalue problem,

$$\mathbf{A}^*\,\mathbf{U}_{\mathrm{i}} = \Omega^2\,\mathbf{U}_{\mathrm{i}} \tag{35}$$

and

$$\mathbf{A}^* = \mathbf{A}_{\mathrm{ii}} - \mathbf{A}_{\mathrm{ib}}\,\mathbf{A}_{\mathrm{bb}}^{-1}\,\mathbf{A}_{\mathrm{bi}}\ .$$

A Matlab program has been written to solve the generalized eigen value problem and get the normalized frequencies Ω and the corresponding mode shapes.

4. Results and discussion

In this section we will verify the method of solution by applying it to some numerical examples and compare the results with those obtained by another methods of solution.

4.1. Choice of sampling points

Choosing the number and type of sampling points has a great effect on the accuracy of the DQM results. Equal spacing sampling points used in earlier papers gives some inaccurate results. It is found that the optimal selection of the sampling points in the vibration problems is the normalized Chebyshev-Gauss-Lobatto points,

$$x_i = \frac{1}{2}\left[1-\cos\left(\frac{i-1}{N-1}\,\pi\right)\right]\ , \qquad i = 1,2,3,4\ . \tag{36}$$

But there are various types of grid distribution which gives an acceptable results, for more details see ref. [2]. In order to represent the results some examples have been solved and the results were compared with those in the literature. The first example is the case of a delaminated beam-plate with a single midplane symmetric delamination of various length ratios namely (a/L). Tables from 1 to 3 shows the results of the first three non-dimensional natural frequencies using 8 and 12 sampling points in each region. The results in tables from 1 to 3 were compared to the analytical results in reference [8] and shows excellent accuaracy. The absolute relative error typed in tables 1 to 3 is to represent the accuracy of the DQ method and this relative error is equal to (Present − Reference)/Reference × 100. Examining the three tables we can say that the maximum absolute error in all cases is negligible except for some cases in the second and third frequencies and these errors decreases rapidly if we increase the number of sampling points from 8 to 12 sampling points in each region.

a/L	Analytical [8]	Present (8 sampling points)	Absolute Relative error %	Present (12 sampling points)	Absolute Relative error %
0.00	22.39	22.37	0.08	22.37	0.08
0.10	22.37	22.37	0.00	22.37	0.00
0.20	22.35	22.35	0.00	22.35	0.00
0.30	22.23	22.24	0.04	22.24	0.04
0.40	21.83	21.83	0.00	21.83	0.00
0.50	20.88	20.89	0.04	20.89	0.04
0.60	19.29	19.29	0.00	19.29	0.00
0.70	17.23	17.23	0.00	17.23	0.00
0.80	15.05	15.05	0.00	15.05	0.00
0.90	13.00	13.00	0.00	13.00	0.00

Tab.1: Normailzed fundamental frequency Ω for clamped-clamped beam-plate with mid-plane central delamination

a/L	Analytical [8]	Present (8 sampling points)	Absolute Relative error %	Present (12 sampling points)	Absolute Relative error %
0.00	61.67	61.67	0.00	61.67	0.00
0.10	60.76	60.21	0.90	60.21	0.90
0.20	55.97	55.67	0.53	55.67	0.53
0.30	49.00	49.02	0.04	49.02	0.04
0.40	43.87	43.13	1.68	43.14	1.66
0.50	41.45	41.14	0.74	41.14	0.74
0.60	40.93	40.05	2.15	40.06	2.12
0.70	40.72	40.01	1.74	40.03	1.69
0.80	39.01	39.96	2.43	39.96	2.43
0.90	35.38	35.53	0.42	35.56	0.50

Tab.2: Normailzed second frequency Ω for clamped-clamped beam-plate with mid-plane central delamination

Mode shapes are an important aspects of the vibration analysis problems. The first two mode shapes for some combinations of length and depth ratios are presented here. In Fig. 2a the fundamental normalized frequency of a beam plate having a symmetric thin delamination $(h_2/H = 0.2)$. It is clear that the crack is fully opened and the thinner part (the upper delamination) vibrates more than other parts. The same opening crack phenomenon happens in the second vibration mode shape Fig. 2b.

a/L	Analytical [8]	Present (8 sampling points)	Absolute Relative error %	Present (12 sampling points)	Absolute Relative error %
0.00	120.91	120.91	0.00	120.91	0.00
0.10	120.81	120.86	0.04	120.83	0.01
0.20	118.76	118.88	0.10	118.88	0.10
0.30	109.04	109.16	0.11	109.16	0.11
0.40	93.57	93.62	0.05	93.59	0.02
0.50	82.29	82.41	0.14	82.29	0.00
0.60	77.64	77.96	0.41	77.69	0.06
0.70	77.05	77.78	0.94	77.18	0.16
0.80	75.33	76.67	1.77	75.43	0.13
0.90	69.17	71.19	2.92	69.19	0.02

Tab.3: Normailzed third frequency Ω for clamped-clamped beam-plate with mid-plane central delamination

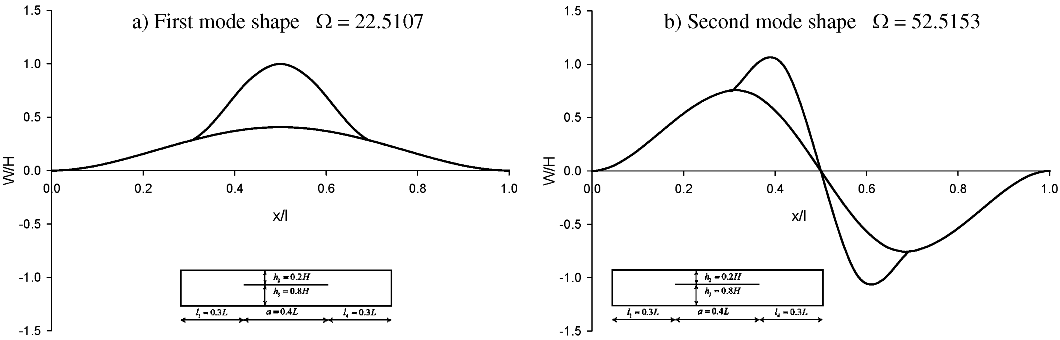

Fig.2: Normalized mode shapes of a symmetric medium delamination

As the delamination goes deep into the depth, the crack tends to be closed. To show this particular aspect the first two mode shapes for a delaminated beam plate having a central mid-plane delamination ($h_2/H = 0.5$), It is clear the crack is fully closed although we are using a free model and there is no contact between the upper and lower delaminations.

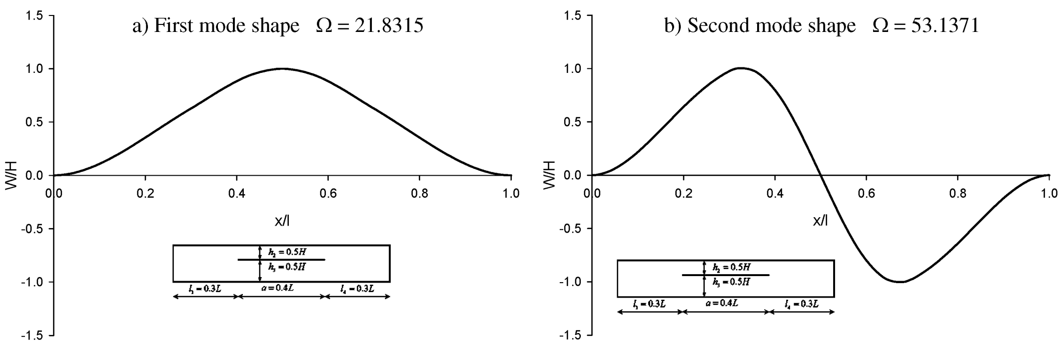

Fig.3: Normalized mode shapes of a symmetric central delamination

In order to test the method in solving non-symmetric delamination problems, the two mode shapes of a delaminated beam plate having a non-symmetric delamination is presented in Fig. 4. The same opening crack phenomenon can be realized here as the upper delamination is thin enough to make the crack open.

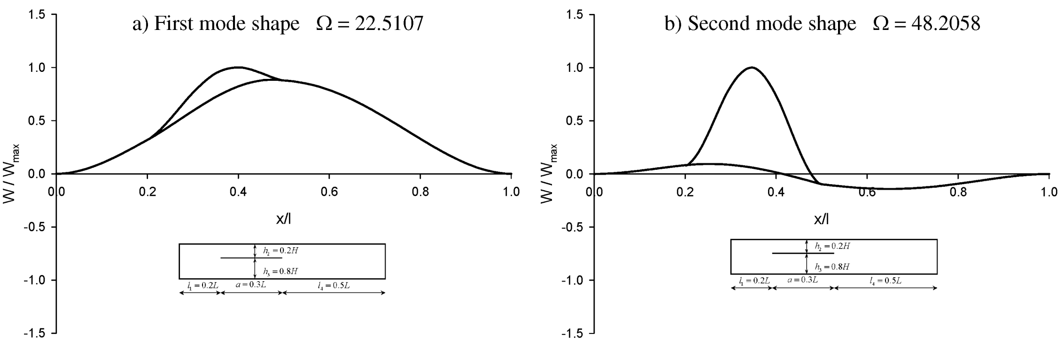

Fig.4: Normalized mode shapes of a non-symmetric delamination

From the above discussion it is clear that the Generalized Differential Quadrature Method is a powerful method in solving domain decomposition problems and especially the delaminated beam plate problems. The obtained results above does not affected by the length of each sub-laminate as reported by T. Y. Wu [11] as the continuity of slope between each sub-laminate is corrected. For example when we equate the slope of regions 1 and 2 we write $U^{[1]}_{N+2} = U^{[2]}_{N+2}$ which must be transformed in the formulation to become $U^{[1]}_{N+2}/l_1 = U^{[2]}_{N+2}/l_2$ due to the normalization process. It is clear that this little change in the formulation vanishes if the two regions have the same length ($l_1 = l_2$). This is why an accurate results were obtained for equal length regions.This also may be the reason that some researchers that uses the GDQ method can't get accurate results in the domain decomposition problems.

5. Conclusion

It can be demonstrated that the GDQ method is an efficient method in solving the vibration problems of the delaminated beam-plates. The effort in solving this problem in [8] or [9] is much greater than the computational effort by the present method. Excellent accuracy is obtained by a very few grid points in each sub-domain (8 points in our case). The power of the DQ method is its ease in treating the various combinations of boundary conditions.

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