POST-BUCKLING BEHAVIOUR OF IMPERFECT SLENDER WEB

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The stability analysis of slender web loaded in compression is presented. The non-linear FEM equations are derived from the variational principle of minimum of potential energy [1]. To obtain the non-linear equilibrium paths, the Newton-Raphson iteration algorithm is used. Corresponding levels of the total potential energy are defined. The peculiarities of the effects of the initial imperfections are investigated. Special attention is focused on the influence of imperfections on the post-critical buckling mode. The stable and unstable paths of the non-linear solution are separated.

Key words: *stability, post-buckling behaviour, stable and unstable path, geometric non-linear theory, initial imperfection, finite element method*

1. Introduction

The snap-through effect means a sudden modal change in the buckling surface of a slender web. Even in the case when the snap-through of the slender web does not mean the collapse of the structure, we consider it to be a negative phenomenon. In the presented paper we try to explain the behaviour of the snap-through of the slender web loaded in compression. The geometrically non-linear theory represents a basis for the reliable description of the post-buckling behaviour of the slender web. The result of the numerical solution represents a lot of load versus displacement paths. Except the presentation of the different load-displacement paths the level of the total potential energy has been evaluated as well.

The mode of the buckling of the lowest elastic critical load (mode 1) is usually taken as the mode of the initial geometrical imperfections. In such a case we do not have the snap-through effects. To create the snap-through effect, the mode of the initial imperfections has to be taken as the combination of the mode of the lowest elastic critical load (mode 1) and the mode of the second elastic critical load (mode 2).

2. Theory

We assume a rectangular slender web simply supported along the edges (Fig. 1) with the thickness t. The displacements of the point of the neutral surface are denoted $\mathbf{q} = [u, v, w]^{\mathrm{T}}$ and the related load vector is $\mathbf{p} = [p_{\mathrm{x}}, 0, 0]^{\mathrm{T}}$.

We assume the so called von Kármán theory, when the out of plane (plate) displacements (w) are much bigger as in-plane (web) displacements (u, v). Taking into account the non-linear terms we have the strains

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{\mathrm{Lm}} + \boldsymbol{\varepsilon}_{\mathrm{Nm}} - z\,\mathbf{k}\ , \tag{1}$$

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where $\boldsymbol{\varepsilon}_{\rm Lm} = [u_{,\rm x}, v_{,\rm y}, u_{,\rm y} + v_{,\rm x}]^{\rm T}$, $\boldsymbol{\varepsilon}_{\rm Nm} = 1/2\,[w_{,\rm x}^2, w_{,\rm y}^2, 2\,w_{,\rm x}\,w_{,\rm y}]^{\rm T}$, $\mathbf{k} = [w_{,\rm xx}, w_{,\rm yy}, 2\,w_{,\rm xy}]^{\rm T}$, the indexes denote the partial derivations.

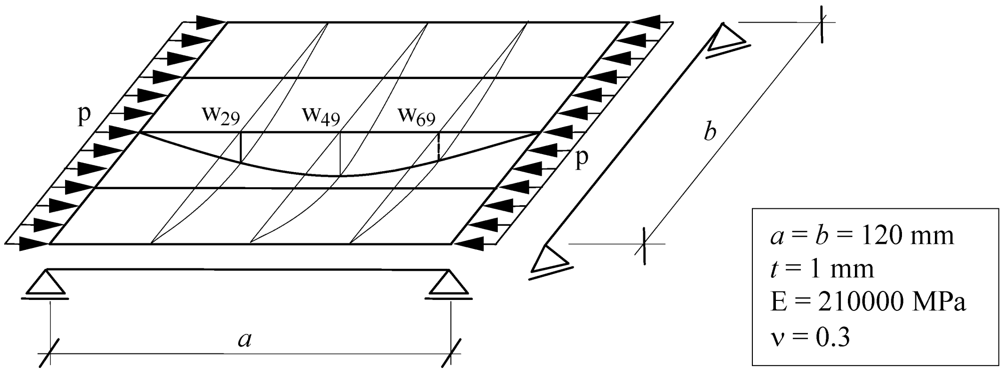

Fig.1: Notations of the quantities of the slender web loaded in compression

The initial displacements will be assumed as the out of plane displacements only and so we have

$$\boldsymbol{\varepsilon}_0 = \boldsymbol{\varepsilon}_{0\rm Nm} - z\,\mathbf{k}_0\ . \tag{2}$$

The specific problem of using the FEM for the solution of non-linear problem of the post-buckling behaviour of the slender web is, that we do not compile the system of the algebraic equations, but even so we use the Newton-Raphson iteration with the combination of the incremental steps.

The increments of the strains are

$$\Delta\boldsymbol{\varepsilon} = \begin{bmatrix} \Delta u_{,\rm x} \\ \Delta v_{,\rm y} \\ \Delta u_{,\rm y} + \Delta v_{,\rm x} \end{bmatrix} + \begin{bmatrix} w_{,\rm x}\,\Delta w_{,\rm x} \\ w_{,\rm y}\,\Delta w_{,\rm y} \\ w_{,\rm x}\,\Delta w_{,\rm y} + w_{,\rm y}\,\Delta w_{,\rm x} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \Delta w_{,\rm x}^2 \\ \Delta w_{,\rm y}^2 \\ 2\,\Delta w_{,\rm x}\,\Delta w_{,\rm y} \end{bmatrix} - z \begin{bmatrix} \Delta w_{,\rm xx} \\ \Delta w_{,\rm yy} \\ 2\,\Delta w_{,\rm xy} \end{bmatrix}\ . \tag{3}$$

The variation is

$$\begin{aligned} \delta\Delta\boldsymbol{\varepsilon} = {} & \begin{bmatrix} \delta\Delta u_{,\rm x} \\ \delta\Delta v_{,\rm y} \\ \delta\Delta u_{,\rm y} + \delta\Delta v_{,\rm x} \end{bmatrix} + \begin{bmatrix} \delta\Delta w_{,\rm x}\,w_{,\rm x} \\ \delta\Delta w_{,\rm y}\,w_{,\rm y} \\ \delta\Delta w_{,\rm x}\,w_{,\rm y} + \delta\Delta w_{,\rm y}\,w_{,\rm x} \end{bmatrix} + \\ & + \begin{bmatrix} \delta\Delta w_{,\rm x}\,\Delta w_{,\rm x} \\ \delta\Delta w_{,\rm y}\,\Delta w_{,\rm y} \\ \delta\Delta w_{,\rm x}\,\Delta w_{,\rm y} + \delta\Delta w_{,\rm y}\,\Delta w_{,\rm x} \end{bmatrix} - z \begin{bmatrix} \delta\Delta w_{,\rm xx} \\ \delta\Delta w_{,\rm yy} \\ 2\,\delta\Delta w_{,\rm xy} \end{bmatrix}\ . \end{aligned} \tag{4}$$

The system of conditional equations we can get form the condition of the minimum of the increment of the total potential energy

$$\delta\Delta U = 0\ . \tag{5}$$

This system can be written as:

$$\mathbf{K}_{\rm inc}\,\Delta\boldsymbol{\alpha} + \mathbf{F}_{\rm int} - \mathbf{F}_{\rm ext} - \Delta\mathbf{F}_{\rm ext} = \mathbf{0}\ , \tag{6}$$

where

$$\mathbf{K}_{\mathrm{int}} = \begin{bmatrix} \mathbf{K}_{\mathrm{inc\,D}} & \mathbf{K}_{\mathrm{inc\,DS}} \\ \mathbf{K}_{\mathrm{inc\,SD}} & \mathbf{K}_{\mathrm{inc\,S}} \end{bmatrix} , \qquad \mathbf{F}_{\mathrm{int}} = \begin{bmatrix} \mathbf{F}_{\mathrm{int\,D}} \\ \mathbf{F}_{\mathrm{int\,S}} \end{bmatrix} ,$$

$$\mathbf{F}_{\mathrm{ext}} = \begin{bmatrix} \mathbf{F}_{\mathrm{ext\,D}} \\ \mathbf{F}_{\mathrm{ext\,S}} \end{bmatrix} , \qquad \Delta\mathbf{F}_{\mathrm{ext}} = \begin{bmatrix} \Delta\mathbf{F}_{\mathrm{ext\,D}} \\ \Delta\mathbf{F}_{\mathrm{ext\,S}} \end{bmatrix} .$$

Matrix of shape functions we can express as

$$\mathbf{q} = \mathbf{B}\,\boldsymbol{\alpha} = \begin{bmatrix} \mathbf{B}_{\mathrm{D}} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\mathrm{S}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_{\mathrm{D}} \\ \boldsymbol{\alpha}_{\mathrm{S}} \end{bmatrix} , \qquad \Delta\mathbf{q} = \mathbf{B}\,\Delta\boldsymbol{\alpha} .$$

We have noted

$$\mathbf{K}_{\mathrm{inc\,D}} = \mathbf{K}_{\mathrm{inc\,DL}} + \mathbf{K}_{\mathrm{inc\,DG}} ,$$

$$\mathbf{K}_{\mathrm{inc\,DL}} = \int_{\Gamma} \mathbf{B}_{\mathrm{Dl}}^{\mathrm{T}}\,\mathbf{D}_{\mathrm{b}}\,\mathbf{B}_{\mathrm{Dl}}\,\mathrm{d}\Gamma \tag{7}$$

is the linear stiffness matrix of the plate,

$$\mathbf{K}_{\mathrm{inc\,DG}} = \int_{\Gamma} \mathbf{B}_{\mathrm{X}}^{\mathrm{T}}\,\mathbf{D}_{\mathrm{inc\,D}}\,\mathbf{B}_{\mathrm{X}}\,\mathrm{d}\Gamma \tag{8}$$

is the nonlinear part of the incremental stiffness matrix of the plate,

$$\begin{bmatrix} w_{,\mathrm{x}} \\ w_{,\mathrm{y}} \end{bmatrix} = \mathbf{B}_{\mathrm{X}}\,\boldsymbol{\alpha}_{\mathrm{D}} , \qquad \mathbf{D}_{\mathrm{inc\,D}} = \frac{E\,t}{1-\nu^2} \begin{bmatrix} A & C \\ C & B \end{bmatrix} ,$$

where

$$A = \frac{3}{2}\,w_{,\mathrm{x}}^2 - \frac{1}{2}\,w_{0,\mathrm{x}}^2 + \frac{1}{2}\,w_{,\mathrm{y}}^2 - \frac{1}{2}\,\upsilon\,w_{0,\mathrm{y}}^2 + u_{,\mathrm{x}} + \upsilon\,v_{,\mathrm{y}} + \frac{t}{\frac{E\,t}{1-\nu^2}}\,\sigma_{\mathrm{xw}} ,$$

$$B = \frac{3}{2}\,w_{,\mathrm{y}}^2 - \frac{1}{2}\,w_{0,\mathrm{y}}^2 + \frac{1}{2}\,w_{,\mathrm{x}}^2 - \frac{1}{2}\,\upsilon\,w_{0,\mathrm{x}}^2 + v_{,\mathrm{y}} + \upsilon\,u_{,\mathrm{x}} + \frac{t}{\frac{E\,t}{1-\nu^2}}\,\sigma_{\mathrm{yw}} ,$$

$$C = w_{,\mathrm{x}}\,w_{,\mathrm{y}} - \frac{1-\nu}{2}\,w_{0,\mathrm{x}}\,w_{0,\mathrm{y}} + \frac{1-\nu}{2}\,(u_{,\mathrm{y}} + v_{,\mathrm{x}}) + \frac{t}{\frac{E\,t}{1-\nu^2}}\,\tau_{\mathrm{w}} .$$

$$\mathbf{K}_{\mathrm{inc\,DS}} = \int_{\Gamma} \mathbf{B}_{\mathrm{X}}^{\mathrm{T}}\,\mathbf{D}_{\mathrm{inc\,SD}}\,\mathbf{B}_{\mathrm{Sl}}\,\mathrm{d}\Gamma \tag{9}$$

is the incremental stiffness matrix of the interaction of the plate-web displacements, where

$$\begin{bmatrix} \Delta u_{,\mathrm{x}} \\ \Delta u_{,\mathrm{y}} \\ \Delta v_{,\mathrm{x}} \\ \Delta v_{,\mathrm{y}} \end{bmatrix} = \mathbf{B}_{\mathrm{Sl}}\,\boldsymbol{\alpha}_{\mathrm{S}} , \qquad \mathbf{D}_{\mathrm{inc\,DS}} = \frac{E\,t}{1-\nu^2} \begin{bmatrix} w_{,\mathrm{x}} & \frac{1-\nu}{2}\,w_{,\mathrm{y}} & \frac{1-\nu}{2}\,w_{,\mathrm{y}} & \upsilon\,w_{,\mathrm{x}} \\ \upsilon\,w_{,\mathrm{y}} & \frac{1-\nu}{2}\,w_{,\mathrm{x}} & \frac{1-\nu}{2}\,w_{,\mathrm{x}} & w_{,\mathrm{y}} \end{bmatrix} .$$

$$\mathbf{F}_{\mathrm{int\,D}} = \mathbf{F}_{\mathrm{int\,DL}} + \mathbf{F}_{\mathrm{int\,DG}} + \mathbf{F}_{\mathrm{int\,DW}} \tag{10}$$

is the vector of the internal forces, where

$$\mathbf{F}_{\mathrm{int\,DL}} = \int_\Gamma \mathbf{B}_{\mathrm{Dl}}^{\mathrm{T}}\,\mathbf{F}_{\mathrm{DL}}\,\mathrm{d}\Gamma\ , \qquad \mathbf{F}_{\mathrm{DL}} = \frac{E\,t^3}{12\,(1-\nu^2)} \begin{bmatrix} w_{,\mathrm{xx}} - w_{0,\mathrm{xx}} + v\,w_{,\mathrm{yy}} - v\,w_{0,yy} \\ w_{,\mathrm{yy}} - w_{0,\mathrm{yy}} + v\,w_{,\mathrm{xx}} - v\,w_{0,xx} \\ (1-\nu)\,(w_{,\mathrm{xy}} + w_{0,\mathrm{xy}}) \end{bmatrix}\ ,$$

$$\mathbf{F}_{\mathrm{int\,DG}} = \int_\Gamma \mathbf{B}_{\mathrm{DlX}}^{\mathrm{T}}\,\mathbf{F}_{\mathrm{Gl}}\,\mathrm{d}\Gamma\ , \qquad \mathbf{F}_{\mathrm{Gl}} = \frac{E\,t}{1-\nu^2}\,\frac{1}{2} \begin{bmatrix} A \\ B \end{bmatrix}\ ,$$

$$A = w_{,\mathrm{x}}^3 + w_{,\mathrm{x}}\,(2\,u_{,\mathrm{x}} + 2\,v_{,\mathrm{y}} - w_{0,\mathrm{x}}^2 - v\,w_{0,\mathrm{y}}^2 + w_{,\mathrm{y}}^2) + w_{,\mathrm{y}}\,(1-\nu)\,(u_{,\mathrm{y}} + v_{,\mathrm{x}} - w_{0,\mathrm{x}}\,w_{0,\mathrm{y}})\ ,$$
$$B = w_{,\mathrm{y}}^3 + w_{,\mathrm{y}}\,(2\,u_{,\mathrm{x}} + 2\,v_{,\mathrm{y}} - w_{0,\mathrm{x}}^2 - v\,w_{0,\mathrm{x}}^2 + w_{,\mathrm{x}}^2) + w_{,\mathrm{x}}\,(1-\nu)\,(u_{,\mathrm{y}} + v_{,\mathrm{x}} - w_{0,\mathrm{x}}\,w_{0,\mathrm{y}})\ ,$$

$$\mathbf{F}_{\mathrm{int\,DW}} = \int_\Gamma \mathbf{B}_{\mathrm{X}}^{\mathrm{T}}\,\mathbf{F}_{\mathrm{DW}}\,\mathrm{d}\Gamma\ , \qquad \mathbf{F}_{\mathrm{DW}} = t \begin{bmatrix} w_{,\mathrm{x}}\,\sigma_{\mathrm{xw}} + w_{,\mathrm{y}}\,\tau_{\mathrm{w}} \\ w_{,\mathrm{y}}\,\sigma_{\mathrm{yw}} + w_{,\mathrm{x}}\,\tau_{\mathrm{w}} \end{bmatrix}\ .$$

Note: We assume the constant distribution of the residual stresses $(\sigma_{\mathrm{xw}}, \sigma_{\mathrm{yw}}, \tau_{\mathrm{w}})$ over the thickness.

$$\mathbf{F}_{\mathrm{ext\,D}} = \int_\Gamma \mathbf{B}_{\mathrm{D}}^{\mathrm{T}}\,\mathbf{p}_{\mathrm{D}}\,\mathrm{d}\Gamma \tag{11}$$

is the vector of the external load of the plate,

$$\Delta\mathbf{F}_{\mathrm{ext\,D}} = \int_\Gamma \mathbf{B}_{\mathrm{DX}}^{\mathrm{T}}\,\Delta\mathbf{p}_{\mathrm{D}}\,\mathrm{d}\Gamma \tag{12}$$

is the increment of the external load of the plate,

$$\mathbf{K}_{\mathrm{inc\,S}} = \int_\Gamma \mathbf{B}_{\mathrm{S}}^{\mathrm{T}}\,\mathbf{D}_{\mathrm{b}}\,\mathbf{B}_{\mathrm{S}}\,\mathrm{d}\Gamma \tag{13}$$

is incremental stiffness matrix of the web,

$$\mathbf{K}_{\mathrm{inc\,SD}} = \mathbf{K}_{\mathrm{inc\,DS}}^{\mathrm{T}} \tag{14}$$

is the incremental stiffness matrix of the web-plate displacements,

$$\mathbf{F}_{\mathrm{int\,S}} = \mathbf{F}_{\mathrm{int\,SL}} + \mathbf{F}_{\mathrm{int\,SG}} + \mathbf{F}_{\mathrm{int\,SW}} \tag{15}$$

is the vector if the internal forces of the web, where

$$\mathbf{F}_{\mathrm{int\,SL}} = \int_\Gamma \mathbf{B}_{\mathrm{SlX}}^{\mathrm{T}}\,\mathbf{F}_{\mathrm{SL}}\,\mathrm{d}\Gamma\ , \qquad \mathbf{F}_{\mathrm{SL}} = \frac{E\,t}{1-\nu^2} \begin{bmatrix} u_{,\mathrm{x}} + v\,v_{,\mathrm{y}} \\ \frac{1-\nu}{2}\,(u_{,\mathrm{y}} + v_{,\mathrm{x}}) \\ \frac{1-\nu}{2}\,(u_{,\mathrm{y}} + v_{,\mathrm{x}}) \\ v_{,\mathrm{y}} + v\,u_{,\mathrm{x}} \end{bmatrix}\ ,$$

$$\mathbf{F}_{\mathrm{int\,SG}} = \int_\Gamma \mathbf{B}_{\mathrm{SlX}}^{\mathrm{T}}\,\mathbf{F}_{\mathrm{SG}}\,\mathrm{d}\Gamma\ , \qquad \mathbf{F}_{\mathrm{SG}} = \frac{E\,t}{1-\nu^2}\,\frac{1}{2} \begin{bmatrix} w_{,\mathrm{x}}^2 + v\,w_{,\mathrm{y}}^2 - w_{0,\mathrm{x}}^2 - v\,w_{0,\mathrm{y}}^2 \\ (1-\nu)\,(w_{,\mathrm{x}}\,w_{,\mathrm{y}} - w_{0,\mathrm{x}}\,w_{0,\mathrm{y}}) \\ (1-\nu)\,(w_{,\mathrm{x}}\,w_{,\mathrm{y}} - w_{0,\mathrm{x}}\,w_{0,\mathrm{y}}) \\ w_{,\mathrm{y}}^2 + v\,w_{,\mathrm{x}}^2 - w_{0,\mathrm{y}}^2 - v\,w_{0,\mathrm{x}}^2 \end{bmatrix}\ ,$$

$$\mathbf{F}_{\mathrm{int\,SW}} = \int_\Gamma \mathbf{B}_{\mathrm{SlX}}^{\mathrm{T}}\,\mathbf{F}_{\mathrm{SW}}\,\mathrm{d}\Gamma\ , \qquad \mathbf{F}_{\mathrm{SW}} = t \begin{bmatrix} \sigma_{\mathrm{xw}} \\ \tau_{\mathrm{w}} \\ \tau_{\mathrm{w}} \\ \sigma_{\mathrm{yw}} \end{bmatrix}\ .$$

$$\mathbf{F}_{\mathrm{ext\,S}} = \int_{\Gamma} \mathbf{B}_{\mathrm{Sl}}^{\mathrm{T}}\,\mathbf{p}_{\mathrm{S}}\,\mathrm{d}\Gamma \tag{16}$$

is the vector of the external load of the web,

$$\Delta\mathbf{F}_{\mathrm{ext\,S}} = \int_{\Gamma} \mathbf{B}_{\mathrm{Sl}}^{\mathrm{T}}\,\Delta\mathbf{p}_{\mathrm{S}}\,\mathrm{d}\Gamma \tag{17}$$

is the increment of the external load of the web.

In the case of the structure in equilibrium $\mathbf{F}_{\mathrm{int}} - \mathbf{F}_{\mathrm{ext}} = \mathbf{0}$, we can do the incremental step $\mathbf{K}_{\mathrm{inc}}\,\Delta\boldsymbol{\alpha} = \Delta\mathbf{F}_{\mathrm{ext}} \Rightarrow \Delta\alpha = \mathbf{K}_{\mathrm{inc}}^{-1}\,\Delta\mathbf{F}_{\mathrm{ext}}$ and $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta\boldsymbol{\alpha}$.

The Newton-Raphson iteration can be arranged in the following way:
We suppose that $\boldsymbol{\alpha}^{i}$ does not represent the exact solution and the residua are $\mathbf{F}_{\mathrm{int}}^{i} - \mathbf{F}_{\mathrm{ext}}^{i} = \mathbf{r}^{i}$.
The corrected parameters are $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta\boldsymbol{\alpha}^{i}$, where $\Delta\boldsymbol{\alpha}^{i} = -\mathbf{K}_{\mathrm{inc}}^{-1}\,\mathbf{r}^{i}$.
We have used the identity of the incremental stiffness matrix with the Jacobbian of the system of the non-linear algebraic equation $\mathbf{J} \equiv \mathbf{K}_{\mathrm{inc}}$.

To be able to evaluate the different paths of the solution, the pivot term of the Newton-Raphson iteration has to be changed during the solution.

For the stable path the determinant of the incremental stiffness matrix must be positive $D = \det \mathbf{K}_{\mathrm{inc}} > 0$, all the main minors must by positive as well $D_{\mathrm{k}} > 0$ and the load must be taken as the pivot term.

3. Illustrative examples

The FEM computer program using a 48 D.O.F. element has been used [2]. The primary path of the solution starts from the zero load level and from the initial displacement. It means that the nodal displacement parameters of the initial displacements and the small value of the load parameter have been taken as the first approximation for the iterative process. To get another paths of the solution we have used random combinations of the parameters as the first approximation. After 'catching' the one point of the path we were able to 'follow' this path and we were able to distinguish the stable and unstable part of this path. Even so, this way of the solution does not guarantee getting all paths.

The presented non-linear solutions of the post-buckling behaviour of the slender web (Fig. 2 and 3) are divided into two parts. On the left side we have load versus nodal displacement parameters relationship, on the right side the relevant level of the total potential energy is drawn. (Unloaded web represents a zero total potential energy level.) Due to the mode of the initial imperfection the nodal displacements $w29$, $w69$ have been taken as the reference nodes (see Fig. 1). The thick line represents the stable path and the thin line represents the unstable path of the solution. More details about the solution of the equilibrium paths are mentioned in [4].

In this paper we shall try to give an answer to the problem of the ability of collapse of the slender web loaded in compression in the second mode of buckling. Fig. 2 shows the solution for the initial displacement $\alpha_{01} = 0.01$ and $\alpha_{02} = 0.15$. We can see that the primary path is in the post-buckling phase in mode 1 ($v1$ – the thick line). The lowest value of the total potential energy is related to the path $v3$ (mode 2). The energy barrier protects the snap from the path $v1$ to the path $v3$. When we increase the mode 2 in the mode of the initial

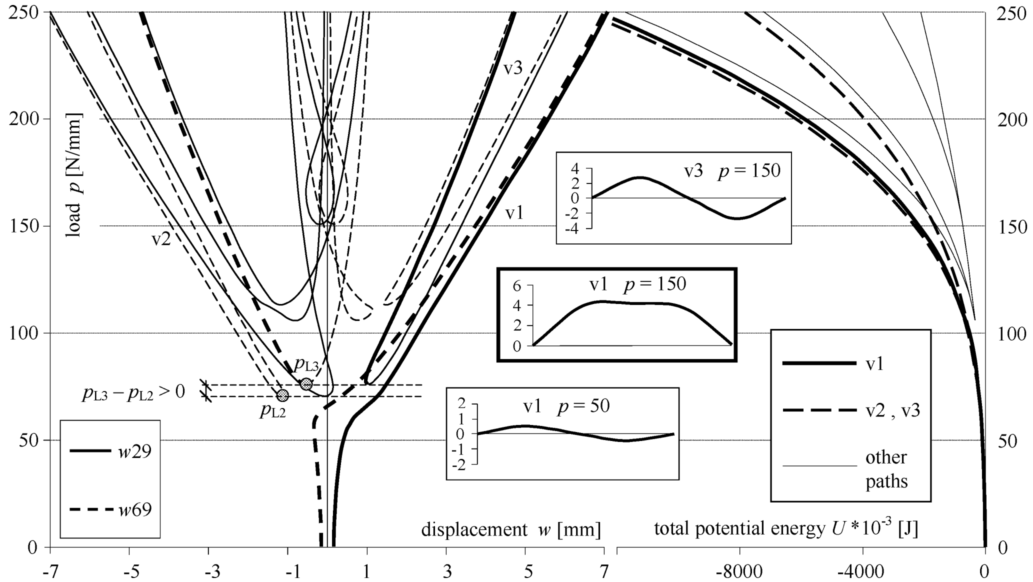

Fig.2: The post-buckling behaviour of the slender web with the initial displacement $w_0 = 0.01\, \sin(\pi\, x/a)\, \sin(\pi\, y/b) + 0.15\, \sin(2\, \pi\, x/a)\, \sin(\pi\, y/b)$

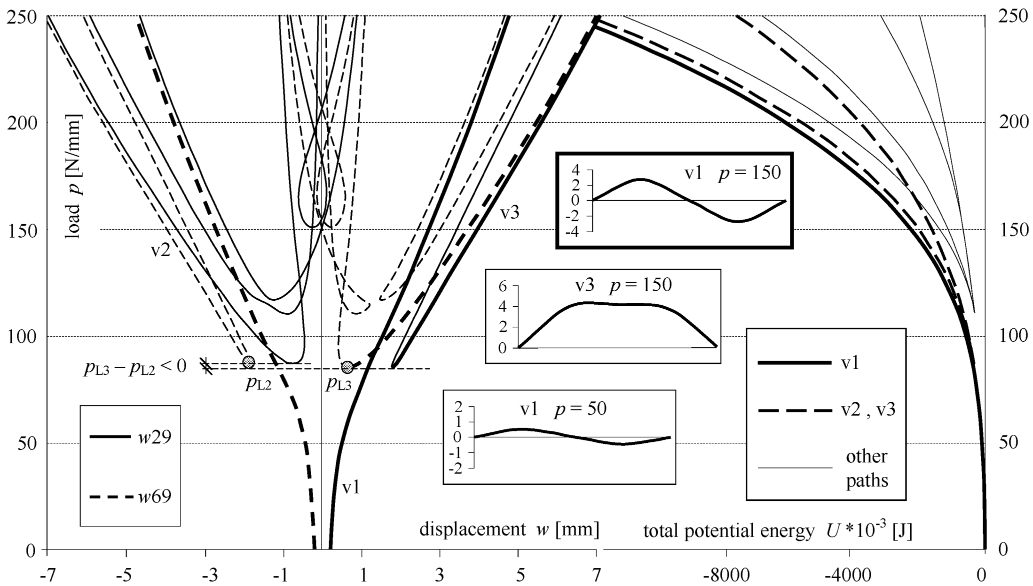

Fig.3: The post-buckling behaviour of the slender web with the initial displacement $w_0 = 0.01\, \sin(\pi\, x/a)\, \sin(\pi\, y/b) + 0.2\, \sin(2\, \pi\, x/a)\, \sin(\pi\, y/b)$

displacement ($\alpha_{01} = 0.01$ and $\alpha_{02} = 0.2$) the post-buckling mode of the slender web is the mode 2 (Fig. 3).

Let we find the connection between the load-deflection path and corresponding level of the total potential energy. From Fig. 2 and 3 we can see, that relative position of limit points in p–w diagram mentions on magnitude of energetic barrier. The increase of the parameter α_{02} is related to decrease of parameter p_{L3}. This is a value of load at limit point of the lowest energy path. If p_{L3} is the lowest limit point in p–w diagram, energetic barrier is eliminated and solution will continue in post-buckling phase in the most convenient way, i.e. in the lowest energy path. The mode of buckling is coincident with the mode of initial imperfection. The benefit of the described procedure is, that we are able to predict a post-buckling behaviour of the web from p–U path diagram only.

4. Conclusion

The influence of the value of the amplitude and the mode of the initial geometrical imperfections for the post-buckling behaviour of the slender web is presented. As the important result we can note, that the level of the total potential energy of the primary stable path can be higher as the total potential energy of the secondary stable path. This is the assumption for the change in the buckling mode of the slender web. This phenomenon is focused here.

The evaluation of the level of the total potential energy for all paths of the non-linear solution is a small contribution in the investigation of the post buckling behaviour of the slender web. Even so we are not able to put a full answer for the mechanism of the snap-trough.

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