CRACK INITIATION CRITERIA FOR SINGULAR STRESS CONCENTRATIONS
Part I: A Universal Assessment of Singular Stress Concentrations

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In the contribution the limits of the validity of standard crack linear elastic fracture mechanics are extended to problems connected with failure of structures caused by general non-crack-like singular stress concentrators. In the present part of the contribution a universal approach to assessment of general singular stress concentrators in terms of linear elastic fracture mechanics is formulated. The approach suggested in this part of the paper facilitates the answer to the question whether or not a crack forms in the vicinity of the stress concentrator and what the parameters controlling crack initiation are. The presented approach follows the basic idea of linear elastic fracture mechanics, i.e. the validity of small scale yielding conditions is assumed, and has a phenomenological character.

Key words : *fracture mechanics, stability criteria, singular stress concentrations, crack initiation, critical stress*

1. Introduction

Modern material technologies require the use of components with complicated geometry and are made up of combinations of different materials subjected to monotonic and cyclic loading conditions in service. The ability to predict fracture of such components is of fundamental importance for assessment of the reliability of engineering structures. The presence of geometrically complicated parts composed of different materials in engineering structures is generally connected with existence of singular stress concentration and represents a weak point for many applications. Owing to a high stress level fracture usually starts at these places. Typically, these kinds of singular stress fields are generated in the vicinity of geometrical and material discontinuities such as free edges in bi-material laminate, bi-material wedges, cracks with their tips at an interface between two different materials and so on, see Fig. 1.

The vicinity of regions with different elastic moduli as well as the presence of an interface between different materials have a pronounced influence on the behaviour of a crack and modelling crack initiation and propagation in such configurations continues to be a problem of significant interest. Numerous studies have been focused on the cases where a stress concentration corresponds to cracks. However, in many cases the stress distribution is singular, but the type of the singularity differs from those of a crack and the approach of standard fracture mechanics cannot be directly applied. Typically, much less attention has

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been paid to such configurations. While the expressions of the singular stress distribution which refer to plane problems corresponding to configurations leading to singular stress distribution have been in most cases well-known for many years, the stability criteria showing conditions for crack initiation in the region near singular stress concentrations are still not known fully.

The aim of the contribution is to extend the limits of the validity of standard crack linear elastic fracture mechanics to problems connected with failure of structures caused by non-crack-like singular stress concentrators. The contribution is divided into four parts. In the first one a universal approach to assessment of general singular stress concentrators in terms of linear elastic fracture mechanics is formulated. Tentative criteria of stability suggested in this part are then used to answer the question whether or not a crack forms in the vicinity of the stress concentrator and what the parameters controlling crack initiation are. The presented approach follows the basic idea of linear elastic fracture mechanics, i.e. the validity of small scale yielding conditions is assumed, and has a phenomenological character.

The second part of the contribution is devoted to applications of the suggested approach to problems connected with fracture mechanics of bi-material wedges. The third part analyses the behaviour of cracks terminating at an interface between different elastic materials. The last part of the contribution is devoted to failure of coated structures.

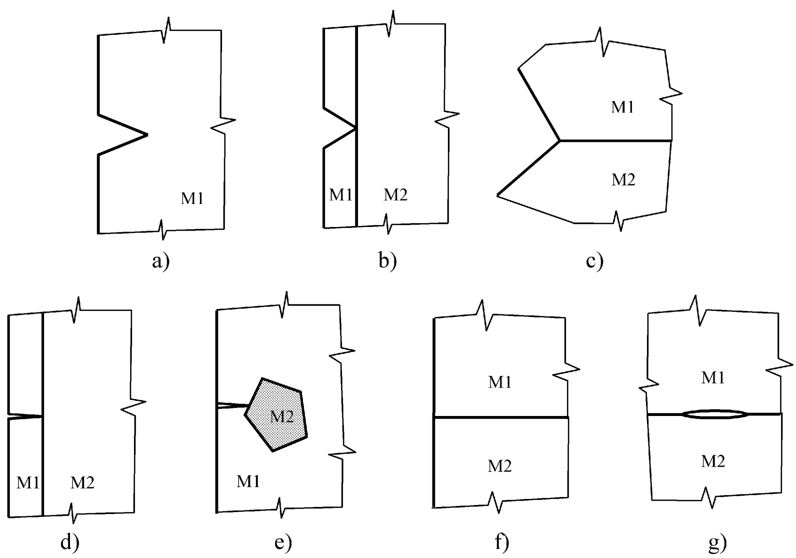

Fig.1: Typical examples of configurations leading to general singular stress distribution: a) V-notch in homogeneous material, b) V-notch with the tip at an interface between two different materials, c) General bi-material notch, d) Crack terminating at an interface, e) Crack terminating at an inclusion surface, f) Free edge singularity, g) Interfacial crack between two materials

2. General singular stress concentrator

In the following the plane elasticity is considered. Fracture mechanics of a general singular stress concentrator (GSSC) is based on an asymptotic analysis of the stress and strain fields.

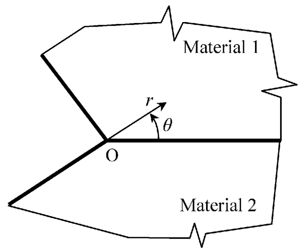

Fig.2: Bi-material notch with the used coordinate systems; the origin of the polar and Cartesian coordinate systems O corresponds to the notch tip

The stress distribution in the vicinity of a general singular stress concentrator differs from the stress field of a crack in a homogeneous material. Considering singular terms only, the stress distribution in the vicinity of a GSSC can generally be expressed in the form (r, θ are polar coordinates with the origin at the tip of a concentrator, see Fig. 2)

$$\sigma_{ij} = \sum_{k=1}^{n} \frac{H_k}{\sqrt{2\pi}}\, r^{-p_k}\, F_{ijk}(\theta, \mathrm{geom}, M, \dots) \tag{1}$$

where n is a number of corresponding singular terms. H_k ($k = 1, 2, \dots, n$) are the generalized stress intensity factors (GSIF). The indices (i, j) represent the polar coordinates (r, θ). The functions F_{ijk} follow from a limiting analytical solution of the problem. The stress singularity exponents p_k may be determined on the basis of the corresponding boundary conditions. They depend on the configuration and the structure of a stress concentrator and the used materials M and are considered within the interval $(0, 1)$. The value of p_k may generally be complex, then $(0 < \mathrm{Re}(p_k) < 1)$. In that case the corresponding generalized stress intensity factor H_k would have complex values as well and the relations for stress distribution would have a different form. A typical example of such a configuration is a crack lying at an interface between two different elastic materials, Fig. 1g, see e.g. [1]. In the following only cases with real values of the stress singularity exponent (and therefore also those of the GSIF) are considered.

A comprehensive theoretical treatment of the corresponding boundary value problem exists in the literature and general formulas that describe the displacement and the stress distribution in the vicinity of GSSC have been known for different configurations for a long time, see e.g. [2–5]. Let us, therefore, assume that for the studied cases the functions F_{ijk} and the values p_k (eq. 1) are known as a result of a limiting analytical solution. The general stress intensity factors H_k depend on the boundary conditions (including the external loading), material combination and geometry of the body. Their values have to be estimated numerically. Note that in most cases the number of singular terms in eq. (1) is $n = 1$ or $n = 2$.

3. Linear elastic fracture mechanics of general singular stress concentrators

For a crack in homogeneous materials the eq. (1) has a form (modes I and II are considered)

$$\sigma_{ij} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\, r^{-1/2}\, f_{ij}(\theta) + \frac{K_{\mathrm{II}}}{\sqrt{2\pi}}\, r^{-1/2}\, g_{ij}(\theta) \tag{2}$$

where K_{I}, K_{II} are stress intensity factors and $f_{ij}(\theta)$, g_{ij} are a known function of the polar coordinate θ, see e.g. [6]. Mind that $f_{ij}(\theta)$, $g_{ij}(\theta)$ are (depending on ij) either odd or even functions of θ and the first term corresponds to the normal mode and the second one to the shear mode of loading. The values of the stress singularity exponents are in both cases the same and equal to 1/2.

The energy $G = G_{\rm I} + G_{\rm II}$ (crack driving force) released during a co-linear unit crack extension can then be determined by calculating the work done by the surface forces acting across the length da when the crack is closed from length $(a + \mathrm{d}a)$ to length a,

$$
\begin{aligned}
G_{\rm I} &= \lim_{\mathrm{d}a\to 0} \frac{1}{\mathrm{d}a} \int_0^{\mathrm{d}a} \sigma_{\rm yy}(x, \theta = 0)\, u_{\rm y}(x - \mathrm{d}a)\, \mathrm{d}x \ , \\
G_{\rm II} &= \lim_{\mathrm{d}a\to 0} \frac{1}{\mathrm{d}a} \int_0^{\mathrm{d}a} \sigma_{\rm xy}(x, \theta = 0)\, u_{\rm x}(x - \mathrm{d}a)\, \mathrm{d}y \ .
\end{aligned}
\tag{3}
$$

Upon the substitution for $\sigma_{\rm yy}$, $\sigma_{\rm xy}$ from eq. (2) and the corresponding expressions for displacements $u_{\rm y}$, $u_{\rm x}$, we get the relation between the crack driving force G and the stress intensity factor K in the form $G = K_{\rm I}^2/E^* + K_{\rm II}^2/E^*$, where $E^* = E$ for plane stress and $E^* = E/(1 - \nu^2)$ for plane strain. E and ν are the Young modulus and Poisson's ratio. Note that the crack propagates in a self-similar way, see Fig. 3a and the value of the stress singularity exponent is constant during crack increment da.

Contrary to this, in the case of a general singular stress concentrator, the crack initiation at the tip of GSSC represents a non-self-similar procedure, where the stress singularity exponent changes as a step function during crack initiation, see Fig. 3b. The energy released during unit crack initiation in the vicinity of the generalized singular stress concentrator, as defined by formula (3), leads to $G = 0$ if the value of the stress intensity exponent $p_k < 1/2$ and to $G \to \infty$ if $p_k > 1/2$. Moreover, the values of the singularity exponents p_k are different from 1/2 and each term in (1) represents an inherently combined loading mode (function F_{ijk} contains both odd and even terms). Note that the dimension of generalized stress intensity factors is $[H_k] = \mathrm{MPa\,m}^{p_k}$ and depends on p_k.

All these facts lead to the conclusion that it is not possible to describe the behaviour of a general singular stress concentrator by applying the standard (classic) approaches of crack fracture mechanics. On the other hand in both cases (cracks in homogeneous materials and GSSC) the corresponding stress field has a singular character with respect to the distance r from the tip (of crack or GSSC) and thus the mechanism describing their stability can be the same.

3.1. Conditions of stability

One of the principles underlying standard linear elastic fracture mechanics says that an unstable fracture occurs if the stress intensity factor (SIF) reaches the critical value $K_{\rm IC}$, also called fracture toughness. Thus, a crack will propagate (under pure mode I) whenever the SIF $K_{\rm I}$ (which characterizes the strength of the singularity for a given problem) reaches the material constant $K_{\rm IC}$, see e.g. [6].

Similarly, fatigue crack growth does not occur if the value of the corresponding range of the stress intensity factor ΔK is smaller than the value of the fatigue crack growth threshold $\Delta K_{\rm th}$, which is again a material constant, see e.g. [6].

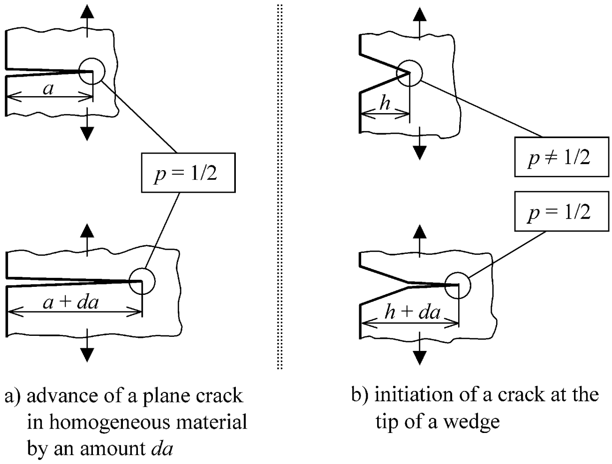

Fig.3: Examples of self-similar (a) and non-self-similar (b) extension of defects

Generally, in both cases, the above conditions of crack stability express the circumstances under which the crack will not propagate and can be written in the form

$$K_{\mathrm{I}}(\sigma_{\mathrm{appl}}) < K_{\mathrm{I\,crit}}(M) \ . \tag{4}$$

$K_{\mathrm{I\,crit}}$ is a critical material parameter depending on the mechanism of the crack propagation (e.g. fracture toughness K_{IC} for brittle fracture, threshold value K_{th} for fatigue crack propagation and so on). Knowing the value of $K_{\mathrm{I\,crit}}(M)$, the critical value of the applied stress, σ_{crit}, can be estimated and the crack will not propagate if $\sigma_{\mathrm{appl}} < \sigma_{\mathrm{crit}}$.

In the same way, conditions of stability for a general singular stress concentrator express the circumstances under which no crack is initiated in the vicinity of the GSSC tip. Again, no crack can be supposed to be initiated if the applied stress $\sigma_{\mathrm{appl}} < \sigma_{\mathrm{crit}}$, where σ_{crit} now depends on the boundary conditions, the type of loading (fatigue, creep, ...), the configuration of GSSC and the material M through which a crack will propagate. In order to find the value σ_{crit}, we assume that the mechanism of the propagation of a crack through material M from the tip of a GSSC is the same as the mechanism of crack propagation in a homogeneous material M and is controlled by the corresponding critical parameter $K_{\mathrm{I\,crit}}(M)$, i.e.

$$\sigma_{\mathrm{crit}} = \sigma_{\mathrm{crit}}[K_{\mathrm{I\,crit}}(M)] \ . \tag{5}$$

Equivalently to (4), for the given geometrical configuration and the material combination corresponding to the GSSC, the critical stress σ_{crit} can be expressed by means of the critical value of GSIF $H_{k\,\mathrm{crit}}(M)$ and the conditions of stability can be expressed in the form

$$H_k(\sigma_{\mathrm{appl}}) < H_{k\,\mathrm{crit}}(M) \tag{6}$$

where $H_{k\,\mathrm{crit}}(M)$ are generalized values of critical material parameters $K_{\mathrm{I\,crit}}$. For a given configuration of GSSC and the corresponding boundary conditions the values of the generalized stress intensity factor H_k can be estimated numerically as a function applied stress σ_{appl}, i.e. $H_k = H_k(\sigma_{\mathrm{appl}})$. The critical value of the applied stress then follows from inequality (6). Then the stability condition for the GSSC has the form $\sigma_{\mathrm{appl}} < \sigma_{\mathrm{crit}}$. To apply the above procedure, two problems have to be solved. The first one is the numerical estimation of the H_{k} value. This is mostly based on an application of direct or integral methods, see e.g. [7–11] for details.

The second problem is to find the relation between $K_{\mathrm{I\,crit}}(M)$ and $H_{k\,\mathrm{crit}}(M)$. Following the assumption that the mechanism of the propagation of a crack from the GSSC in the material M is the same as in the case of the crack propagation in a homogeneous material M we can conclude that the stability of the GSSC and the crack are controlled by the same variable L, which has a clear physical meaning and is well-defined in both cases (e.g. energy density, crack opening, mean stress value and so on). In the case of a homogeneous body M, there is $L = L(\ldots K_{\mathrm{I\,crit}}(M)\ldots)$ and in the case of the GSSC there is analogously $L = L(\ldots H_{k\,\mathrm{crit}}(M)\ldots)$. Let us further assume that in the case of instability $K_{\mathrm{I}} = K_{\mathrm{I\,crit}}(M)$ and $H_k = H_{k\,\mathrm{crit}}(M)$ the variables L reach their critical values $L = L_{\mathrm{C}}$ which are in both cases identical, i.e.,

$$L_{\mathrm{C}}(\ldots K_{\mathrm{I\,crit}}(M)\ldots) = L_{\mathrm{C}}(\ldots H_{k\,\mathrm{crit}}(M)\ldots)\ . \tag{7}$$

Based on the equation (7), the relation between $H_{k\,\mathrm{crit}}(M)$ and $K_{\mathrm{I\,crit}}(M)$ can be found and the critical applied stress σ_{crit} calculated. Contrary to formulation of the crack stability in homogeneous body, the additional length parameter $r = d$ has to be introduced in eqs. (6, 7). The distance d numerically corresponds to r coordinate for which the eqs. (6, 7) are applied. The value d has to be chosen depending on damage mechanism corresponding to studied GSSC.

3.2. Crack propagation direction

To assess the problem of structure failure caused by the existence of a general singular stress concentrator, the direction of a potential crack initiated at the tip of GSSC has to be determined first. Then a criterion for crack initiation and propagation in the selected direction can be applied.

Generally, the stress distribution described by eq. (1) represents an inherently combined mode of normal and shear loading. Functions F_{ijk} contain both odd and even terms and the values of general stress intensity factors H_k are not independent. In most practical cases the stress field consists of two singular terms with different real values of the stress singularity exponent. Each of the singularity terms covers both the normal and the shear types of loading. In such mixed mode loading conditions the generalized strain energy density factor (GSEDF), $\Sigma(r,\theta)$, can be used to describe the direction of the potential crack propagation,

$$\Sigma(r,\theta) = r\,\frac{\mathrm{d}W}{\mathrm{d}V} = r\int_0^{\varepsilon} \sigma_{ij}\,\mathrm{d}\varepsilon_{ij}\ . \tag{8}$$

Here σ_{ij} and ε_{ij} represent stress and strain tensors components. Following the basic assumption of the strain energy density theory [7, 12–17], the potential direction of crack propagation θ_0 will be identical with the direction of the local minimum of the strain of GSEDF $\Sigma(r,\theta)$.

$$\Sigma(r,\theta_0) = \min(\Sigma(r,\theta))\ . \tag{9}$$

The value of the crack propagation angle θ_0 is then given by

$$\left(\frac{\partial\Sigma}{\partial\theta}\right)_{\theta_0} = 0 \;, \qquad \left(\frac{\partial^2\Sigma}{\partial\theta^2}\right)_{\theta_0} > 0 \;. \tag{10}$$

It can be shown [15–17] that the direction of the crack initiation is independent of the absolute value of GSIFs and depends only on their ratio $\Gamma_{21} = H_2/H_1$. The ratio results from the numerical solution of the body with a GSSC, and corresponds to given loading conditions. Mind that contrary to the crack, the strain energy density factor for GSSC depends on the r coordinate, $\Sigma = \Sigma(r)$. Generally, the value of angle θ_0 depends on the distance $r = d$ where the conditions (10) are applied.

In the same way the potential direction of crack initiation can be determined from a maximum of tangential stress (MTS criterion), see e.g. [7, 15].

4. Estimation of the generalized stress intensity factors

The severity of the singularities does not depend only on the exponents of singularity p_k, but also on the value of the generalized stress intensity factors H_k $(k = 1, 2, \dots)$. While the values of the stress intensity exponents p_k follow from asymptotic analysis of the corresponding boundary problem, the values H_k have to be estimated from the numerical solution of the stress distribution in the whole studied body.

Basically, there are two methods for evaluation of the generalized stress intensity factor. The direct method is based on comparison of a numerical and asymptotic analytical solution (see eq. 1) in the vicinity of the general singular stress concentrator tip. The approach is well documented for cracks in homogeneous bodies (see e.g. [9]) and makes it possible to apply standard finite element systems for the estimation.

From the computational point of view, the stresses close to the GSSC tip are unreasonable due to numerical errors and the small region around the tip should be excluded from the estimation process. On the other hand, the stresses at points faraway from the tip are not useful. The reason for this fact is that eq. (1) takes only singular terms into account, while the numerically obtained values of stresses include also higher order regular terms. The absence of these terms undermines the accuracy of the procedure. As a result the dependence $H_1 = H_1(r)$ is obtained and it is supposed that the correct value of H_1 can be obtained by extrapolation of the linear part of the dependence $H_1 = H_1(r)$ to value $r = 0$, see [7, 8] for details.

The integral approach to determination of the GSIF value proceeds from the reciprocal work contour integral method (RWCIM) resulting from the validity of the Betti reciprocal theorem. As a result, the final formula for determination of GSIF H_k reads

$$H_k = \int\limits_{\Gamma_0} (\sigma_{ijk}\, u^*_{ik} - \sigma^*_{ijk}\, u_{ik})\, n_j \,\mathrm{d}s \tag{11}$$

where σ^*_{ijk} and u^*_{ik} are stress and displacement components pertaining to an analytical solution to the complementary problem that must satisfy the identical boundary conditions as the actual problem. Index k denotes the number of the singular term in eq. (1) and Γ_0 is a closed contour surrounding a GSSC tip. The integration should be, in a general case, carried out numerically, see [8] for details.

Examples of practical applications are presented in parts II, III and IV of the paper.

5. Discussion

Composite structures involving interfaces, joints, free edges and cracks generally develop a singular elastic stress field near the intersection of lines of material and geometrical discontinuity. These localized regions of severe stress are possible sites of failure initiation and growth. Generally, the singular stress distribution around the configurations is described by eq. (1). The paper specifies the theoretical considerations concerning linear elastic fracture mechanics of general singular stress concentrators. With this aim the criteria of stability for GSSC are formulated. The suggested approach provides a tool for estimation of the critical stress for crack initiation at the tip of GSSC depending on stress singularity exponent pk and the corresponding boundary conditions. Following the basic assumption of the procedure the mechanism of the propagation of a crack from the GSSC in the material is the same as in the case of the crack propagation in a homogeneous material and the stability of the GSSC and the crack are controlled by the same variable L which has a clear physical meaning and is well-defined in both cases (e.g. energy density, crack opening, mean stress value and so on) and their critical values are identical, see paragraph 3. Owing to the fact that the stress singularity exponent $p_k \neq 1/2$, the problem is confronted with difficulty which does not arise in the case of a standard crack, where $p_k = 1/2$. Namely, the additional length parameter d, introduced directly or indirectly through the plastic zone size dimensions, has to be specified, see 3.1, 3.2. In general, both the potential crack propagation direction value θ_0 and the calculated values of the critical stress σ_{crit} depend on the proper choice of the parameter d. The dependences are weak for the values $p_k \rightarrow 1/2$. In both cases the value of the parameter d depends on the mechanism of crack initiation and has to be chosen depending on the corresponding rupture mechanism and the microstructure of the material. For example for cleavage fracture it can be correlated with the grain size of the corresponding material, see [18]. The sensitivity of the results given by the choice of the parameter d is discussed in parts II and III of the paper.

As has already been stated, the problem of GSSC corresponds to the inherently combined mode of normal and shear loading. This is connected with the number k of singularities in eq. (1). There are only a few examples corresponding to the single value of the stress singularity exponent p ($k = 1$) in eq. (1). A typical example is a crack terminating perpendicularly to a planar interface with normal loading conditions, see Fig. 1c. Most technical problems are connected with singular stress concentrators with two singularities, see [15]. In the following we suppose that the values of the stress singularity exponent are real. A typical example of the singular stress concentrator with a complex exponent represents a crack at an interface, see Fig. 1f. in this case the interpretation of the asymptotic stress field leads to somewhat disturbing conclusions connected with its oscillatory character and the problem is frequently discussed in the literature, see [1].

6. Conclusions

Composite structures involving interfaces, joints, free edges and cracks generally develop a singular elastic stress field near the intersection of lines of material and geometrical discontinuity. These localized regions of severe stress are possible sites of failure initiation and growth. In the contribution the limits of the validity of standard crack linear elastic fracture mechanics are extended to problems connected with failure of structures caused by non-crack-like singular stress concentrators. The general approach to the assessment of the fracture mechanics behaviour of general singular stress concentrators is suggested which makes it possible to answer the question whether or not a crack forms in the vicinity of the stress concentrator and what the parameters controlling crack initiation are. The procedure bridges the existing gap between the fracture assessment of cracks and the general non-crack-like type of a singular stress concentrator. The formulated procedure makes it possible to estimate the critical stress for crack initiation in the region near the general singular stress concentrator and for the determination of the subsequent crack propagation direction. The presented approach follows the basic idea of linear elastic fracture mechanics, i.e. the validity of small scale yielding conditions is assumed, and has a phenomenological character. The applications of the suggested approach will be presented for different types of singular stress concentrators under different loading conditions in parts II, III and IV of the paper.

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