FUZZY PROBABILITY METHODS IN APPLICATIONS TO RELIABILITY ANALYSIS OF EUROCODE RULES FOR STEEL STRUCTURE DESIGN

Zdeněk Kala*

The topic of the paper is the probabilistic analysis of the ultimate limit state of a steel strut loaded by permanent and long-time single variation action. The failure probability misalignment according to the EN 1990 concept of a structure designed according to the EUROCODE 3 is analyzed here. In the stochastic model, material and geometrical characteristics of a hot-rolled steel cross-section are considered according to the experimental research results. The initial curvature shape and size variability of the beam axis is modelled, in detail, by applying the random fields. The functional dependence between the failure probability and the correlation length of a random field is analyzed here. The probabilistic analysis is completed by the fuzzy analysis of the influence of uncertainties on the failure probability. The fuzzification process of coefficients of model uncertainties and the deffuzification process of the fuzzy number of the output failure probability are described. The failure probability fuzzy analysis was evaluated according to the general extension principle, the failure probability having been solved by the Monte Carlo method.

Key words : *fuzzy, random, stochastic, steel, imperfection, buckling*

1. Introduction

Since the European Union has come into existence, a step-by-step moving away the trade obstacles and harmonization of technology specifications have taken place. Within the framework of this programme, it was proceeded to an establishment of a set of harmonized technological rules for the design of building structures – EUROCODES, which should serve, at the beginning, as an alternative to national rules valid in individual EU countries, and substitute them in the event.

At present, the general rules for designing the steel structures are given in the Standard EN 1993-1-1:2005, which was recently accepted, for application in the Czech Republic, in form of literal translation as the ČSN EN 1993-1-1:2006 [24]. The Standard [24] is designated to be applied together with the Eurocode EN 1990 [23], EN 1991 and EN 1992 to EN 1999, as far as steel structures or steel members are concerned. In these standards, the numerical values of partial safety factors and of other reliability parameters are recommended as basic values ensuring an acceptable reliability level on assumption that the corresponding manufacturing level and quality control have been kept. In this context, the elaboration of probabilistic studies is topic at present with the aim to quantify the manufacturing quality effect on reliability from the point of view of transparency and verification of processes step-by-step introduced into practice.

* doc. Ing. Z. Kala, Ph.D., Institute of Structural Mechanics, Faculty of Civil Engineering, BUT

The topic of the present paper is to analyze the load-carrying capacity limit state and to verify the partial safety factors by the probabilistic calculation based on histograms of statistical material and geometrical imperfections in IPE hot-rolled steel members [13]. In the paper, there is given a probabilistic reliability assessment of an IPE 220 steel member under compression on which the stability phenomena manifest themselves clearly in the general load-carrying capacity decrease.

The initial curvature of the steel member axis is one of the major imperfections affecting the load-carrying capacity decrease. Taking into consideration that the published results of the size and shape measurements of this imperfection are not detailed enough and do not reflect the randomness of this phenomenon satisfactorily, in our study, the initial axis curvature measurement realizations were simulated randomly, applying the random fields. Other random material and geometrical imperfections were considered according to the results published in [13]. One of the most important objectives of today's topical technology is providing the reliability and economy at the same time. When solving this task, one cannot make do without statistical and calculation models. The uncertainties related with the structure design can exist in input data, mathematical models, design, manufacturing, maintenance, software, etc. In sophisticated calculation models, the results of probabilistic studies of the load-carrying capacity limit state depend, on model uncertainties in determining the random load action, and load-carrying capacity [5].

Recommended probabilistic models for Model Uncertainties can be taken according to [26]. Apart from classical stochastic methods, also alternative approaches of the model prediction uncertainty representation are recently applied more and more frequently; these are fuzzitivity (vagueness), non-specificity (low determination), and conflict; these are examined within the framework of five theories which include the methods for their quantification (classical sets theory, fuzzy sets theory, probability theory, possibility theory, and Dempster-Shafer theory) [4]. The introduction of these newer representational structures for uncertainty has been accompanied by a lively discussion and debate of their various merits and demerits [10, 11, 12]. At the same time, steadily increasing computational power has made the analysis of uncertainty increasingly practicable, and the increasing use of simulation in support of decision making has created a demand for informative, decision-aiding characterizations of the uncertainty in analysis outcomes relevant to these decisions [4].

In the paper presented, the fuzzy numbers are applied for taking the model uncertainties into consideration. The aim of the fuzzy analysis of the failure probability is to quantify the influence of load-carrying capacity and load action epistemic uncertainties on the failure probability.

2. Random imperfections

2.1. Random field of initial curvature of the axis

In general, the axis of a real beam is a curve, a fully straight beam is practically never concerned. Let us imagine that the initial curvature of the axis would be measured at pre-selected points, e.g, in nodes, see Fig. 1. The node y_i deflection in direction of the axis y is a random quantity, the value of which depends, to a certain extent, on values of quantities y_{i-1} and y_{i+1} in neighbouring points [6]. The size and shape randomness of initial curvature of the axis is reflected by the variability of y_i, and by the corresponding correlation matrix. If the statistical dependence between random characteristics of neighbouring finite elements is described by auto-correlation, the stochastic finite elements and random fields are concerned.

If the deflections of nodes y_i are studied, e.g., by the Monte Carlo method, the correlation among deflections of nodes y_i excludes the unreal initial curvature shapes of beam axis. In theoretical studies, the correlation degree among deflections y_i expresses the auto-correlation function most frequently. The following commonly used exponential form of an isotropic autocorrelation function between y_i and y_j is considered:

$$c_{i,j} = S \, \exp\left[-\left(\frac{|\xi_{i,j}|}{L_{\mathrm{cr}}}\right)^2\right] \tag{1}$$

where L_{cr} is the positive parameter called correlation length. S is the standard deviation of the random field, and $\xi_{i,j}$ is the distance between two nodes x_i a x_j. The correlation coefficient $\varrho_{i,j}$ of the correlation matrix can be determined as:

$$\varrho_{i,j} = \frac{c_{i,j}}{\sqrt{c_{i,i}\, c_{j,j}}} \ . \tag{2}$$

An example of the random field is presented in Fig. 1 in magnified scale. The initial curvature shape of beam axis is substituted by a cubic spline passing through the nodes 0 to 10. For large numbers of measurements, it will be supposed that the mean value m_{y_i} near zero would be determined for each random quantity y_i; i.e., negative and positive deflections occur in identical frequency. Further on, it will be supposed for all the random quantities y_i that their probability density function (PDF) is symmetrical, i.e., their skewness equals zero.

The ray of compressive force of the strut passes through the first and the last node of the random field, see Fig. 2. Buckling of the simple joint member comes out of the direct of axis x^*. The strut load-carrying capacity depends on initial random deviations of nodes in direction of axis y^*. As the statistical characteristics of random imperfections of nodes are other in direction of axis y^* than those in direction of axis y, the transformation equations must be written between coordinates in orthogonal coordinate system y vs. x and y^* vs. x^* [6].

The angle α of adjusting of the local coordinate system with axes x^*, y^* depends on the random position of the first and the last beam node; it can be determined according to the relation:

$$\tan(\alpha) = \frac{\Delta y}{\Delta x} = \frac{y_{10} - y_0}{x_{10} - x_0} \ . \tag{3}$$

For $\Delta y \ll \Delta x$ and $y_i \ll 1$ it holds approximately $x_i^* \approx x_i$, it being expected to be evidently a frequent case in practice.

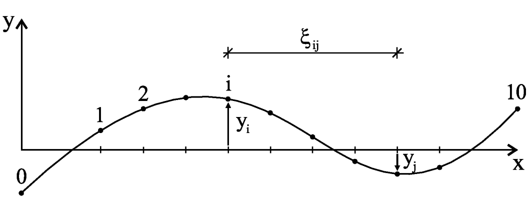

Fig.1: Random field in global coordination system

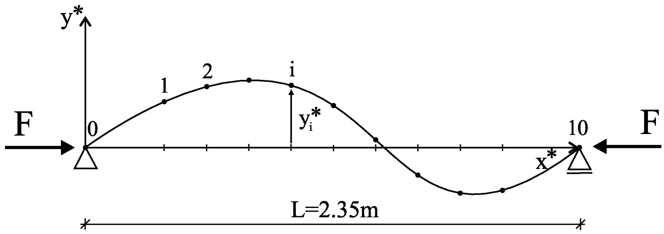

Fig.2: Random field of strut axis in local coordination system

It can be demonstrated (e.g., by the Monte Carlo method) that the random deflection y_i^* of the i^{th} node in direction of the axis y^* has, after evaluation, zero mean value, $m_{y_i^*} = 0$ [6]. Due to marginal conditions, the standard deviation $S_{y_i^*}$ is zero in end nodes, its course along the beam length being determined by the sine function the maximum amplitude of which decreases with increasing correlation length. As the angle α is a random quantity as well, the mutual correlation of quantities y_i^* is lower than that of quantities y_i; however, this decrease is not significant [6].

Taking into consideration the character of beam load action, it is appropriate to introduce the beam random curvature shape in the local coordinate system with x^*, y^*, see Fig. 2. The random quantities y_i^* will be required to be mutually correlated according to the relation (1). Further on, it will be required that standard deviations of quantities y_i^* have sinusoidal course with maximum amplitude $S_{y_5^*}$ in node 5.

$$S_{y_i^*} = S_{y_5^*} \sin\left(\frac{\pi\, x_i^*}{L}\right) . \tag{4}$$

Larger random deformations will be situated nearer the beam middle; also the beam division by finite elements is adapted to this, see Fig. 2. The maximum deflection need not be observed in each case in the beam half; however, its value will be always zero at its ends.

The standard deviation size $S_{y_5^*}$ can be determined according to the Tolerance Standard [25] prescribing the maximum deformation 0.15 % of the beam length L for the IPE 220 beam. In our numerical study, it will be supposed that initial deflections y_i^* have the limited Gaussian distribution, and that 95 % of all the realizations of y_5^* lie within the interval $\langle -0.0015\,L, 0.0015\,L\rangle$; it corresponds with $S_{y_5^*} = 1.8\,\mathrm{mm}$.

For the definition of correlation length L_{cr} of the isotropic autocorrelation function (1), it is possible to use the published experimental research results [3]. In [3], the measured values of initial curvature of the axis of hot-rolled H100 cross-section were approximated by the Fourier series of the type

$$y_i^* = \sum_{n=1}^{3} a_{\mathrm{n}} \sin\left(\frac{n\,\pi\, x_i^*}{L}\right) . \tag{5}$$

Constants a_1 to a_3 are published in [3] as mean values based on 15 measured members, however, it is a rather small statistical sample. The cubic parabola and (5) together with (4) represent hardly comparable model; nevertheless another way how to determine the correlation length L_{cr} in (1) is not practricable but with application of available experimental results. The members slenderness having the ratio of mean value of coefficients

$a_1 : a_2 : a_3 = 15 : 5 : 3$ is nearside to the IPE beam studied here. The parameter L_{cr} was determined by the following heuristic method:

1) Input of correlation length L_{cr}.
2) Simulation of random field for 10 000 runs of Monte Carlo method.
3) Aproximation of y_i^* by Fourier series (5).
4) Calculation of arithmetic means $m_{|a_1|}$, $m_{|a_2|}$, $m_{|a_3|}$ based on 10 000 absolute values of a_1, a_2 and a_3.
5) Error assessment calculation

$$\Delta = \left|\frac{m_{|a_1|}}{m_{|a_2|}} - \frac{15}{5}\right| + \left|\frac{m_{|a_2|}}{m_{|a_3|}} - \frac{5}{3}\right| + \left|\frac{m_{|a_1|}}{m_{|a_3|}} - \frac{15}{3}\right| .$$

Optimum L_{cr} was determined by means of the interval halving method based on the condition of reachung the minimum error Δ. The result of heuristic study is the correlation length $L_{\mathrm{cr}} \approx 1.2\,\mathrm{m}$. As the heuristic method was applied for calculation and as the results [3] were deduced from a small number of samples, the results of stochastic calculations requiring the specification of L_{cr} will be influenced by the epistemic uncertainty of L_{cr}.

2.2. Random quantities

The other imperfections were considered to be random quantities. Statistical characteristics of yield strength were considered according to the experimental research results [13]. The yield strength histogram was evaluated, based on 562 samples taken from one third of the flange of the IPE 160 to IPE 220 profiles. Mechanical characteristics of samples under tension in hot-rolling direction were tested. Statistical geometrical characteristics of IPE 220 cross-section area dimension were considered, as well, according to the experimental research results [13]. The mean value 210 GPa and standard deviation 12.6 GPa were considered for Young's modulus E according to two independent experimental research works [3, 20]. The overview of input random quantities is given in Tab. 1.

Symbol	Value	Density function	Mean value	Standard deviation
h	Cross-section height	Histogram	220.22 mm	0.975 mm
b	Flange width	Histogram	111.49 mm	1.093 mm
t_1	Web thickness	Histogram	6.225 mm	0.247 mm
t_2	Flange thickness	Histogram	9.136 mm	0.421 mm
y_i^*	Initial imperfections	Gauss	0 m	See Eg. (4)
E	Modulus of elasticity	Gauss	210 GPa	12.6 GPa
f_{y}	Yield strength	Histogram	297.3 MPa	16.8 MPa
G	Permanent load	Gauss	G_{k}	$0.1\,G_{\mathrm{k}}$
Q	Variable load	Gumbel-max	$0.6\,Q_{\mathrm{k}}$	$0.21\,Q_{\mathrm{k}}$

Tab.1: Input statistical characteristics

On the last two lines of Tab. 1, there are given statistical characteristics of load action for the purpose of probabilistic analysis. For permanent load, it can be assumed that the characteristic value G_{k} is, at the same time, the mean value of Gaussian distribution with variation coefficient 0.1 . The Gumbel-max distribution with mean value $0.6\,Q_{\mathrm{k}}$ and standard deviation $0.21\,Q_{\mathrm{k}}$ was assumed for single variable load action (50 years)

3. Steel structure reliability

3.1. Reliability conditions by EUROCODES

A simplified problem of compression member loaded by permanent action G combined with single variable action Q will be considered for the elaboration of a parametric study. The standard design reliability condition according to [23, 24] can be written in the form:

$$\gamma_{\mathrm{G}}\, G_{\mathrm{k}} + \gamma_{\mathrm{Q}}\, Q_{\mathrm{k}} \leq R_{\mathrm{A}\chi}\, \frac{f_{\mathrm{yk}}}{\gamma_{\mathrm{M}}} \tag{6}$$

where $R_{\mathrm{A}\chi} = \chi\, A$ is the product of buckling coefficient χ and cross-section area A, γ_{M} is the material partial safety factor, and values G_{k}, Q_{k}, f_{yk} are characteristic values of load action and yield strength. The design reliability is ensured by partial safety factors γ. The design reliability condition (6) can be written more shortly as the inequality $F_{\mathrm{d}} \leq R_{\mathrm{d}}$ of design load action F_{d} and design load-carrying capacity R_{d}.

It is supposed in a numerical reliability study that the design load action is equal to the design load-carrying capacity, $F_{\mathrm{d}} = R_{\mathrm{d}}$, i.e., that the structure has been designed with maximum load-carrying capacity (economic design). Characteristic values G_{k}, Q_{k} are expressed by the ratio δ of random load action Q_{k} to the total load action $G_{\mathrm{k}} + Q_{\mathrm{k}}$:

$$\delta = \frac{Q_{\mathrm{k}}}{G_{\mathrm{k}} + Q_{\mathrm{k}}}\ . \tag{7}$$

Actually, it was proceeded so that in the first stage, the value of parameter δ was selected according to (7). Characteristic values G_{k} and Q_{k} are calculated according to the relation:

$$1.35\, G_{\mathrm{k}} + 1.5\, Q_{\mathrm{k}} = 468.6\,\mathrm{kN}\ . \tag{8}$$

Equation (8) is derived from (6) for partial safety factors $\gamma_{\mathrm{G}} = 1.35$; $\gamma_{\mathrm{Q}} = 1.5$ [23] and $\gamma_{\mathrm{M}} = 1.0$ [24]. The value $R_{\mathrm{d}} = 468.6\,\mathrm{kN}$ on the equation's right side is the design load-carrying capacity of the IPE 220 strut with nondimensional slenderness $\bar{\lambda} = 1.0$ calculated according to [24]:

$$R_{\mathrm{d}} = \frac{\chi_{\mathrm{b}}\, A\, f_{\mathrm{yk}}}{\gamma_{\mathrm{M}}} = \frac{0.597 \cdot 2.34{\times}10^{-3} \cdot 235{\times}10^{6}}{1.0} = 468.6\,\mathrm{kN} \tag{9}$$

where χ_{b} is buckling coefficient for the buckling strength curve b, A is the nominal cross-section area, and f_{yk} is the characteristic value of yield strength. The random characteristics of load action G and Q are thus calculated from characteristic values according to Tab. 1.

The failure probability of a strut designed according to [23] is defined by non-fulfilling the reliability condition (10), in which R is the random load-carrying capacity, and G, Q are random load action effects.

$$G + Q < R\ . \tag{10}$$

3.2. Nonlinear computational FEM model

Member geometries may be modelled by means of the beam element with initial curvature in the form of a parabola of the 3rd degree [7]. The member was meshed into 10 beam elements, see Fig. 2. The steel member was solved by the nonlinear Euler incremental method and combined with the Newton-Raphson method [7]. Geometrical and material nonlinearities were considered. The first criterion for the load-carrying capacity is a loading at which plastification of the flange is initiated. The second criterion for the load-carrying capacity is represented by a loading corresponding to a decrease of the determinant to zero. The second criterion is applied to the calculation of random load-carrying capacity only exceptionally if the imperfection is identical with the second, the third or any further higher eigen mode shape of stability loss of ideal strut. This theoretical phenomenon occurs at high yield strength values with small geometrical member imperfections. Due to real imperfections, the load-carrying capacity is always lower than the Euler critical force. For the strut having the random initial curvature of the beam axis in Fig. 2, the dependence of load action versus deformation is always increasing monotonously, i.e., snap-through and/or snap-back effect does not occur (the methods of arc-length type are not to be applied). In each run of the simulation method, the load-carrying capacity was determined to an accuracy of 0.1 % [7].

3.3. Model uncertainties

The theoretical value of failure probability which serves for comparison and decision making is calculated according to (10). When calculating it, there exist other uncertainties which are not distinguished by the stochastic calculation model.

The first epistemic uncertainty is in determining the correlation length. A further uncertainty is met in boundary conditions which do not correspond with an ideal joint, and therefore they effect on the load-carrying capacity increase. On the contrary, the effect of residual stress and excentricity of load action decreases the load-carrying capacity. Considerable uncertainty exists in determining the load action random variability, the statistical information about which is completely missing at the stage of structure design. Valuable information and thus the conclusions following from the probabilistic solution can be depreciated to a large extent due to the vague (fuzzy) uncertainty of input quantities and calculation models. The error inserted into solution by applying this or that calculation model can be taken into consideration by coefficients of model uncertainties Θ_F, Θ_R according to recommendations [26].

$$\Theta_\mathrm{F}\,(G+Q) < \Theta_\mathrm{R}\,R\ . \tag{11}$$

In an internet document [26], it is recommended for the study carried out here to consider the coefficients Θ_F and Θ_R to be random quantities with lognormal distribution, with mean value 1.0 and standard deviation 0.05.

Another possibility is to consider the coefficients of model uncertainties as fuzzy numbers. Basic terms and definitions relating to the fuzzy-randomness were introduced in [17, 22]. The fuzzy-randomness occurs when the observation under exactly defined boundary conditions cannot be carried out [14]. Let us suppose that, when calculating the random load-carrying capacity, there exists the uncertainty ±3 %, whatever may be its cause.

The aim of the fuzzy analysis is not the searching for the uncertainty causes of calculation models but the theoretical quantification of their influence on failure probability. For this purpose, the coefficients of model uncertainties Θ_F and Θ_R were considered to be linear symmetric fuzzy numbers, see Fig. 3.

The membership function expresses the degree of membership of a value observed on the horizontal axis into a set. The value 1.0 on the vertical axis means that coefficients of model

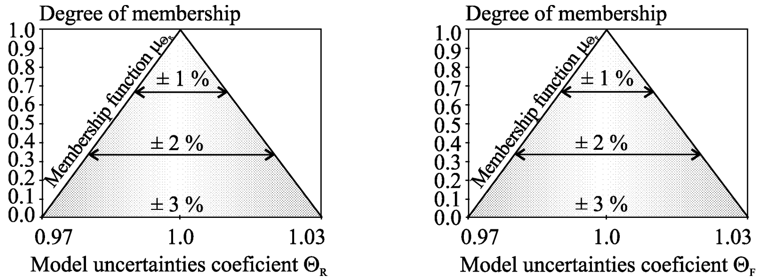

Fig.3: Fuzzy numbers of model uncertainties

uncertainties 1.0 belong completely to the set; on the contrary, the value 0.0 means that the values of coefficients of model uncertainties 0.97 and 1.03 do not belong to the set at all. The triangular form of membership functions of the fuzzy numbers in Fig. 3 guarantees maximum transparency of the fuzzy analysis results of the effect of model uncertainties on failure probability.

4. Analysis of steel structure reliability

4.1. Analysis of failure probability

To determine the failure probability P_f, so many runs of Monte Carlo method were applied that the non-fulfilling the condition (10) took place for 200 runs minimum; this guarantees a balanced error of probability assessment – 7 %. The dependence of failure probability on the correlation length (1) was analyzed first of all. The failure probability was calculated for $L_\mathrm{cr} \in \{0.01, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 2.4, 5, 10, 20, 30, \infty\}$. The initial curvature having the sine function shape corresponds to the correlation length ∞. The courses of functions in Fig. 4 are drawn after the failure probability aproximation by applying Hermite polynoms by the least squares method.

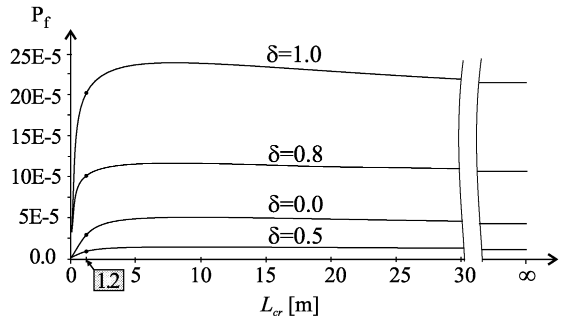

Fig.4: Failure probability vs. correlation length

It is evident from Fig. 4 that for all the values of parameters $\delta \in \{0.0, 0.5, 0.8, 0.99\}$ the value of failure probability for $L_\mathrm{cr} = 1.2\,\mathrm{m}$ is lower than that for $L_\mathrm{cr} = \infty$; however, the differences are not significant. The highest failure probability values were obtained for correlation lengths within the interval $L_\mathrm{cr} \in (6, 9)\,\mathrm{m}$ approximately, see Fig. 4.

The misalignment analysis of failure probability due to change of parameter δ for $L_\mathrm{cr} = 1.2\,\mathrm{m}$ is presented in Fig. 5. The results according to (10) are drawn by the full line, the results with consideration of the functions of random model uncertainties accord-

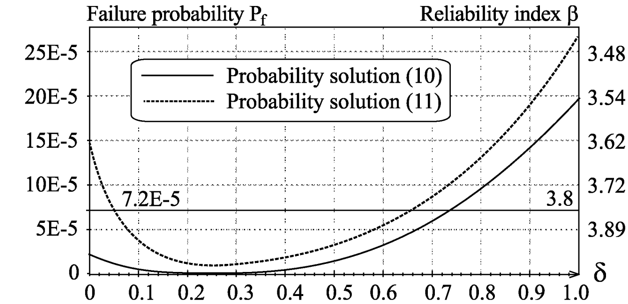

Fig.5: Misalignment of the failure probability for $L_{\mathrm{cr}} = 1.2\,m$

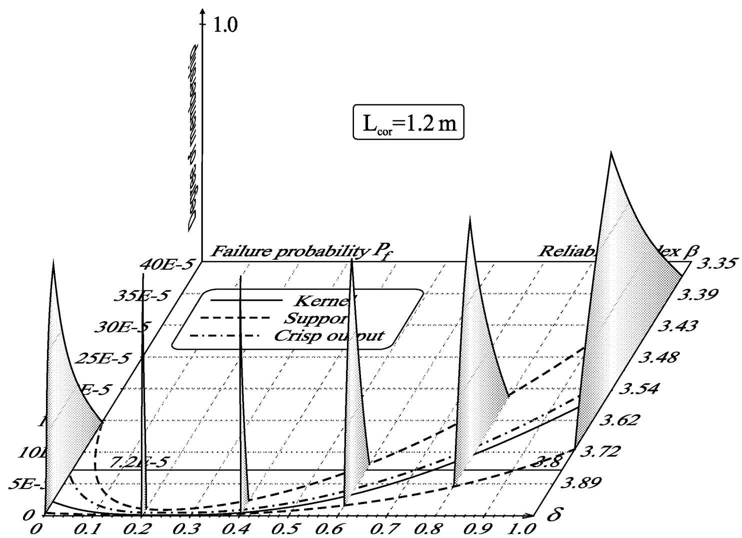

Fig.6: Fuzzy analysis of misalignment of the failure probability

ing to (11), by the dotted one. It is clear from Fig. 5 that the maximum effect on increase in failure probability is that of function of model uncertainties for $\delta = 0$.

4.2. Fuzzy analysis of failure probability

If the model uncertainties are assumed to be fuzzy numbers, the output is formed by fuzzy numbers of failure probability. The fuzzy analysis of failure probability P_{f} was evaluated according to the general extension principle (12) for 10 α-cuts [2, 16].

$$\mu_{\mathrm{P_f}}(\Theta_{\mathrm{F}}, \Theta_{\mathrm{R}}) = \bigvee_{\mathrm{P_f}} (\mu_{\Theta_{\mathrm{F}}} \wedge \mu_{\Theta_{\mathrm{R}}}) \; . \tag{12}$$

The fuzzy probabilistic analysis was evaluated for $\delta \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$, but for lucidity's sake, there are drawn only membership functions for $\delta \in \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$. The correlation length $L_{\mathrm{cr}} = 1.2\,\mathrm{m}$ was assumed.

The fuzzy probabilistic analysis completes the probabilistic study by much new information, see Fig. 6. The failure probability analysis (drawn in horizontal projection) is quantified by values of membership functions of failure probability (on the vertical axis). The sharp peak (kernel) of membership functions is identical with purely stochastic solution drawn by full line in Fig. 5. Dashed lines on horizontal projection represent the support calculated for model uncertainties of load action and load-carrying capacity $\pm 3\,\%$.

The course of membership functions in Fig. 6 is nonlinear, and they are markedly asymmetric. When considering that the membership functions of input fuzzy numbers Θ_{F}, Θ_{R} are linear and symmetric, so the form of membership functions quantifies the nonlinear effect of model uncertainties to be the failure probability. The dash and dot line represents the defuzzified probability value (the so-called crisp output) by COG method (centre of gravity) [2]. It can be observed that the defuzzified value (crisp output) is, due to the above causes, higher than the kernel.

5. Conclusion

The stochastic calculation model taking into consideration the uncertainty of the initial curvature shape of a steel beam axis by the finite element method using random fields was described in the introduction of the present paper. The correlation among initial deflections of nodes was defined by the Gauss auto-correlation function. The correlation length L_{cr} was recommended, based on results of virtual simulations of experiments [3] to be $L_{\mathrm{cr}} \cong 0.5\,L$. For the studied problem of an IPE 220 strut, it has been shown that, for $L_{\mathrm{cr}} < 0.5\,L$, the failure probability decreases rapidly, see Fig. 4. The functions of failure probability in Fig. 4 are convex , their maximum being around for $L_{\mathrm{cr}} \approx 3\,L$.

The probability analysis of a steel strut design according to [24] has shown the misalignment of failure probability in dependence on the parameter δ. When applying the stochastic functions of model uncertainties, the failure probabilities obtained were higher. The maximum effect of model uncertainties was observed for $\delta = 0$; however, it is, in practice, a very little probable case of a structure loaded only by permanent load action. Reliable values of failure probability were obtained for 'reasonable' ratios of permanent and variable load actions.

The fuzzy analysis results appropriately complete the probabilistic solution by valuable information, not offered by the purely stochastic analysis. The output membership functions in Fig. 6 map the nonlinear effect of model uncertainties on failure probability. Dashed lines present the support of the failure probability fuzzy numbers, taking into consideration the uncertainty of random load action and of load-carrying capacity, $3\,\%$. It is evident that the change of failure probability caused by increase of random load action and by decrease of random load-carrying capacity by $3\,\%$ is far higher that the change due to decrease of random load action and increase of random load-carrying capacity by $3\,\%$. The deviations of failure probabilities due to model uncertainties, $3\,\%$, in Fig. 6 are comparable to the deviations due to misalignment of failure probability in Fig. 5.

The stochastic analysis and the fuzzy one of the effect of model uncertainties on failure probability is very valuable from the technology point of view because it is one of effective methods quantifying the uncertainty in design of steel structures [21]. Generally it can be recommended to eliminate the effect of model uncertainties by more accurate modelling, e.g., by nonlinear FEM solution within the frames of the ANSYS programme, applying shell finite elements SHELL181. The residual stress variability which has been neglected in the presented study could be taken into account by an accurate calculation model. It is necessary to add that the fuzzy stochastic analysis with the application of nonlinear solution and SHELL elements would require unrealistic time-demanding processes at present. In this sense the methods Response Surface promise well, which makes it possible to analyse the reliability, applying even very demanding calculation models [1, 18].

Symbols

α .... angle
Δ .... deviation
χ_{b} .... buckling coeficient
γ .... partial safety factor
λ, $\bar{\lambda}$ ...... slenderness, nondimensional slenderness
$\varrho_{i,j}$ ...... correlation coeficient
Θ .... fuzzy number of model uncertainty
$\xi_{i,j}$ ...... distance between nodes x_i a x_j
a .... amplitude of the Fourier series
b .... flange width
$c_{i,j}$ .... co-variation coefficient
E .... modulus of elasticity
f_{y}, f_{yk} ... random yield strength, characteristic yield strength
F, F_{d} .... load action, design load action
G, G_{k} .... random permanent load action, characteristic permanent load action
h .... cross-section height
i, j, n .... index
L, L_{cr} .... member length, correlation length
m .... mean value
P_{f} .... failure probability
Q, Q_{k} .... variable load, characteristic variable load
R, R_{d} .... resistance, design resistance
S .... standard deviation
t_1, t_2 ..... web thickness, flange thickness
x .... distance
y_i .... initial deflections of the i^{th} node in global coordinate system
y_i^* .... initial deflections of the i^{th} node in local coordinate system

Acknowledgement

The paper was elaborated under the research projects MSM 0021630519, GAČR 103/07/1067 and Junior Research Project KJB201720602.

References

[1] Bucher C.G., Bourgung U.: Efficient Use of Response Surface Methods, Inst. Eng. Mech., Innsbruck University, Report No. 9, 1987
[2] Dubois D.: Fuzzy Sets and Systems – Theory and Applications, Academic Press 1980, p. 394, ISBN 0-12-222750-6
[3] Fukumoto Y., Kajita N., Aoki T.: Evaluation of Column Curves Based on Probabilistic Concept, In Proc. of Int. Conference on Stability, Prelim. Rep., publ. by Gakujutsu Bunken Fukyu – Kai, Tokyo, 1976, pp.1–37.
[4] Helton J.C., Johnson J.D., Oberkampf W.L.: An Exploration of Alternative Approaches to the Representation of Uncertainty in Model Predictions, Reliability Engineering & System Safety, Vol. 85, Issues 1–3, 2004, pp. 39–71, ISSN 0951-8320
[5] Holický M., Marková J.: Effect of Alternative Procedures in the EN 1990 on the Reliability of Structures (Vliv alternativních postupů v EN 1990 na spolehlivost konstrukcí), Stavební obzor, Praha: ČVUT, 2002, No. 2, pp.33–36, ISSN 1210-4027 (in Czech)

[6] Kala Z.: The Influence of Initial Curvature of the Axis upon the Member Ultimate Strength, Journal of Structural Mechanics, Vol. 36 – No. 1, 2003, pp. 3–14, ISBN 0783-6104

[7] Kala Z.: Sensitivity Analysis of the Stability Problems of Thin-walled Structures, Journal of Constructional Steel Research 61 (2005), pp. 415-422

[8] Kala Z., Strauss A., Melcher J., Novák D., Fajkus M., Rozlívka L.: Comparison of Material Characteristics of Austrian and Czech Structural Steels, International Journal of Materials & Structural Reliability, Vol. 3 No. 1, 2005, pp. 43–51, ISSN 1685-6368

[9] Kala Z.: A Problem of Fuzzy Uncertainty of the Results of Probabilistic Methods (K otázce fuzzy neurčitosti výsledků pravděpodobnostních metod), In Proc. of the Int. Conf. Probability of Failures in Structures – PPK 2006 (Sborník referátů z konference s mezinárodní účastí Pravděpodobnost porušování konstrukcí – PPK 2006), Brno, 2006, s. 193–204, (in Czech), ISBN 80-214-3251-9

[10] Klir G.J.: Is there more to uncertainty than some probability theorists, might have us believe? Int. J. Gen. Syst., 1989, 15:347–78

[11] Kosko B.: Fuzziness vs. probability, Int. J. Gen. Syst., 1990, 17(2–3):211–40

[12] Laviolette M., Seaman J., Barrett J., Woodall W.: A probabilistic and statistical view of fuzzy methods, Technometrics, 1995; 37:249–92

[13] Melcher J., Kala Z., Holický M., Fajkus M., Rozlívka L.: Design Characteristics of Structural Steels Based on Statistical Analysis of Metallurgical Products, Journal of Constructional Steel Research 60 (2004), pp. 795–808, ISSN 0143-974X

[14] Möller B., Beer M., Reuter U.: Theoretical Basics of Fuzzy Randomness – Application to Time Series With Fuzzy Data, In CD Proc. of 9th Int. Conf. On Structural Safety and Reliability ICOSSAR'05, Rome (Italy), 2005, ISBN 90-5966-040-4

[15] Novák D., Lawanwisut W., Bucher C.: Simulation of Random Fields Based on Orthogonal Transformation of Covariance Matrix and Latin Hypercube Sampling, In Proc. of Int. Conference on Monte Carlo Simulation, Monte Carlo, 2000, pp. 129–136

[16] Novák V.: Fuzzy Sets and Their Application (Fuzzy množiny a jejich aplikace), SNTL 1986 (in Czech)

[17] Puri M.L., Ralescu D.: Fuzzy Random Variables, Journal Math. Anal. Appl. Vol. 114:409–422, 1986

[18] Stemberk P., Kruis J., Bittnar Z.: Fuzzy Distribution of Internal Forces in Seismically Loaded Frame Structure, In National Conference with International Participation Engineering Mechanics 2006, Svratka (CZ), 2006, p. 9, ISBN 80-86246-27-2

[19] Strauss A., Kala Z., Bergmeister K., Hoffmann S., Novák D.: Technologische Eigenschaften von Stählen im europäischen Vergleich, Stahlbau, 75 (2006), Januar 2006, Heft 1, ISSN 0038-9145

[20] Guedes Soares C.: Uncertainty Modelling in Plate Buckling, Structural Safety, 1988, (5), pp. 17–34.

[21] Vičan J., Koteš P.: Bridge Management System of Slovak Railways, In Proc. of the IIIrd Int. Scientific Conf. Quality and Reliability of Building Industry, Levoča, 2003, pp. 539–542, ISBN 80-7099-746-X

[22] Wang G., Zang Y.: The Theory of Fuzzy Stochastic Processes, Fuzzy Sets Syst. Vol. 51:161–178, 1992

[23] EN 1990 Eurocode: Basis of Structural Design, 2002

[24] EN 1993-1-1:2006: Eurocode 3: Design of Steel Structures – Part 1-1: General Rules and Rules for Buildings, CEN, 2006

[25] EN 10034: Structural Steel I and H Sections, Tolerances on Shape and Dimensions, September 1995

[26] JCSS Probabilistic Model Code, Part 3, Resistance Models, Static Properties of Structural Steel (Rolled Sections), JCSS Zurich, 2001, (http://www.jcss.ethz.ch/)

Received in editor's office: February 26, 2007
Approved for publishing: June 12, 2007