INVERSE METHOD FOR CUTTING FORCES PARAMETERS EVALUATION

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Simulation of the milling process is a widespread method to improve productivity in the machining process and several phenomena can be studied and controlled by this mean. All these methods need input parameters that characterize the interaction between the tool and the workpiece in order to evaluate the cutting forces. Many models exist, but the input parameters are often difficult to find out from intrinsic properties of the materials (Young's modulus, yield strength, hardness, . . .). The aim of this article is to set out a simple and efficient method to compute cutting coefficient for two different cutting forces models from a single efforts measurement. The method is validated using both simulated and measured cutting forces. The adequacy is good and allows simulating the whole cutting process.

Key words : *milling, cutting forces, specific pressure, chatter vibration*

1. Background

Chatter vibration during machining is a well know issue which limits productivity. The regenerative effect has been widely studied and several simulation methods have been developed, ranging from analytic approach (leading to the classical 'stability lobes' diagram) to dynamic simulation.

The common point between all these methods is the fact that they all need information about the cutting force amplitude. The models based on the study of the microscopic phenomen [9] lead to a set of equations which are difficult to manipulate for simple computation.

The common approach is thus to model cutting force in machining by mean of relationship to technological 'macroscopic' parameters such as depth of cut or feed (see for example [6], [4], [8]). These empirical laws depend upon several parameters which are often difficult to correlate to physical properties of the material such as Young's modulus, yield strength or hardness. Identification of those parameters from experimental analysis is thus a compulsory step to simulate machining process.

2. Cutting force measurement

Most common sensors to measure cutting forces during machining are based on piezoelectric ceramics linked together to measure up to four components (forces along three directions and torque). Two main types have been developed in milling : rotating and stationary dynamometers.

Rotating dynamometers (Fig. 1) are composed of two parts: the sensor itself which acts as a toolholder interface between the tool and the spindle and a static part linked to the machine

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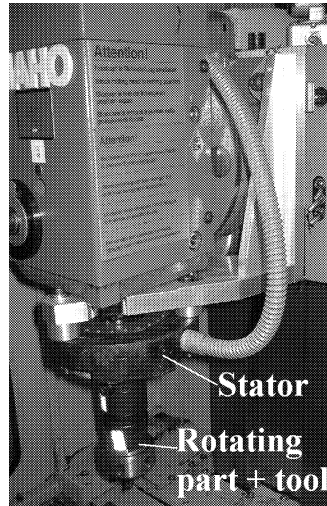
Fig.1: Rotating dynamometer

dealing with the acquisition of data. The reference frame for cutting forces measurement is attached to the cutter and thus rotates with respect to the spindle speed. This type of sensor is able to measure cutting forces in one to three directions (parallel to the spindle axis and two orthogonal directions in the normal plane) and torque along spindle axis. This type is not universal because the interface between the machine and the sensor (ISO cone, HSK interface, . . .) and the fastening of the stator part of the sensor must be dedicated to a particular machine tool. The mass and the overhang added at the tip of the spindle disturb the dynamic behaviour of the machine and may add spurious frequency content.
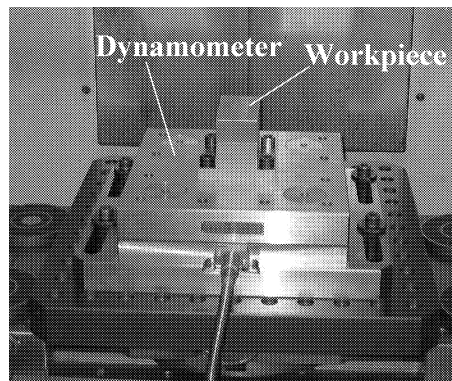

Fig.2: Dynamometric plate

Stationary dynamometers (Fig. 2) are made of piezoelectric rings supporting a plate where the workpiece is clamped. Cutting forces are measured in a reference frame linked to the clamping plate.

For both devices the bandwidth is limited to few kilohertz so the tooth passing period must be small enough to ensure correct measurement. These devices are thus not suitable for high speed cutting measurement.

3. Cutting forces computation

3.1. Introduction

Mathematical relationships allowing cutting force computation are often valid for simple geometrical configuration (tool modelled as a wedge, cutting speed of constant orientation) which cannot be directly applied a real tool. Therefore the model of the tool has to be divided in slices along its axis and the relationships are applied locally with a good approximation. On a local point of view, efforts are stu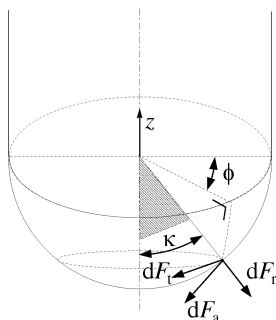died along three directions (see Figure 3): tangential to the cutter rotation (suffix t), radial (along normal direction of the cutter, suffix r) and axial (third direction for an orthogonal frame). Those elementary efforts are then projected in the appropriate reference frame and numerically integrated along each cutting edge to get the global efforts.

Fig.3: Geometry of the cutter

Modelling of cutting force from macroscopic data is a commonly used technique. This class of method is often called 'mechanistic approach'. Many mechanistic forces models are listed in the literature. Simplest of them consider forces to be proportional to chip thickness. Some authors model the efforts as a non-linear function of chip thickness; other authors proposed models that take friction and damping of the cutting process into account [4].

For this paper, we would consider two different models which are widely used in machining simulation. The first one is the linear model assuming that forces are directly proportional to the section of undeformed chip. This model allows linear approximation of machining process in order to get the stability lobes in turning [7] or in milling [2]. In this model, three parameters must be determined to compute cutting forces. These coefficients are often called 'specific pressure'.

$$
\begin{aligned}
\mathrm{d}F_\mathrm{t} &= K_\mathrm{t}\, h(\phi, z)\, \mathrm{d}b\ ,\\
\mathrm{d}F_\mathrm{r} &= K_\mathrm{r}\, h(\phi, z)\, \mathrm{d}b\ ,\\
\mathrm{d}F_\mathrm{a} &= K_\mathrm{a}\, h(\phi, z)\, \mathrm{d}b\ .
\end{aligned}
\tag{1}
$$

The second model has been proposed by Armarego [5] and considers that the efforts are produced by two main mechanisms: shearing of the chip (effort proportional to undeformed chip section $h\,\mathrm{d}b$ and parameter $K_{.\mathrm{c}}$) and friction of the chip along cutting edge (proportional to elementary length of cutting edge ds and coefficient $K_{.\mathrm{e}}$). Six cutting coefficients must be identified for this model. This model is one of the most common in dynamic simulation of the cutting process.

$$
\begin{aligned}
\mathrm{d}F_\mathrm{t} &= K_\mathrm{te}\, \mathrm{d}s(z) + K_\mathrm{tc}\, h(\phi, z)\, \mathrm{d}b\ ,\\
\mathrm{d}F_\mathrm{r} &= K_\mathrm{re}\, \mathrm{d}s(z) + K_\mathrm{rc}\, h(\phi, z)\, \mathrm{d}b\ ,\\
\mathrm{d}F_\mathrm{a} &= K_\mathrm{ae}\, \mathrm{d}s(z) + K_\mathrm{ac}\, h(\phi, z)\, \mathrm{d}b\ .
\end{aligned}
\tag{2}
$$

4. Specific pressure identification

Parameters from the cutting forces model are often difficult to deduct from intrinsic properties of the materials. It is thus necessary to develop algorithms to retrieve those

parameters from experimental studies. Araujo and Silveira [3] developed a method based on reversal of relationships from equation 2. At each time step, matrix relationship can be determined linking efforts and specific pressure. For Armarego's model, the system itself cannot be solved because there are more unknowns (6) than equations (3). The authors propose to combine two consecutive time steps to get a solvable system. The final coefficients are the mean value of all the results during computation.

This method has two main disadvantages:
  – for some time steps, the linear system is ill-conditioned, this lead to loss of precision in the computation of the cutting parameters;
  – this method was developed to model cylindrical end mills only.

We have developed a new method to improve the precision and to enlarge the field of application in order to take into account the geometry of the general end mill. This method performs a least square fitting between measured and computed signal to get the optimal cutting coefficients.

The model is based on discretisation of time into time steps dt and of the tool into discs of thickness da. At each time step, the local geometry is considered for each disk and each tooth. If the tooth is in cut, local chip thickness can be computed using classical relationship:

$$h_{\text{chip}} = s_t \, \sin\phi \, \sin\kappa \; , \tag{3}$$

where s_t is the feed per tooth, ϕ is the angle defining the angular position of the current point on cutting edge and κ is the orientation of the local normal vector (see figure 3). The local cutting edge length dS and the projected depth of cut db can be computed for any cutter geometry (more details are available in reference [6]). The system of equations 1 can be summarized in the following matrix relationship for the linear effort model:

$$\left\{ \begin{matrix} dF_t \\ dF_r \\ dF_a \end{matrix} \right\} = \overbrace{\begin{bmatrix} h\,db & 0 & 0 \\ 0 & h\,db & 0 \\ 0 & 0 & h\,db \end{bmatrix}}^{\mathbf{A}} \overbrace{\left\{ \begin{matrix} K_t \\ K_r \\ K_a \end{matrix} \right\}}^{\boldsymbol{K}} \; . \tag{4}$$

In this first approach only three unknowns have to be determined (K_t, K_r and K_a).

For the second model (equation 2), the matrix $\mathbf{A}$ is a 3×6 matrix and the vector $\boldsymbol{K}$ contains six unknowns:

$$\mathbf{A} = \begin{bmatrix} h\,db & 0 & 0 & dS & 0 & 0 \\ 0 & h\,db & 0 & 0 & dS & 0 \\ 0 & 0 & h\,db & 0 & 0 & dS \end{bmatrix} , \qquad \boldsymbol{K} = \left\{ \begin{matrix} K_{tc} \\ K_{rc} \\ K_{ac} \\ K_{te} \\ K_{re} \\ K_{ae} \end{matrix} \right\} . \tag{5}$$

The elementary efforts are then projected in the reference frame of the measurement device. The classical transformation matrix performs the projection.

$$\left\{ \begin{matrix} dF_x \\ dF_y \\ dF_z \end{matrix} \right\} = \overbrace{\begin{bmatrix} -\cos\phi & -\sin\phi\,\sin\kappa & -\sin\phi\,\cos\kappa \\ \sin\phi & -\cos\phi\,\sin\kappa & -\cos\phi\,\cos\kappa \\ 0 & -\cos\kappa & -\sin\kappa \end{bmatrix}}^{\mathbf{B}} \left\{ \begin{matrix} dF_t \\ dF_r \\ dF_a \end{matrix} \right\} . \tag{6}$$

While using static a dynamometer, the angle ϕ takes three parameters into account:
- rotation of the tool ($\Omega\,\mathrm{d}t$ with Ω the spindle speed);
- shift of each cutting edge around the tool ($2\pi/N$ for a tool with N edges uniformly distributed);
- shift of the cutting edge due to helix angle ($2\,z\,\tan i/D$ for a cylindrical mill, see [6] for other geometries).

While using a rotating dynamometer, the reference frameis fixed with respect to the tool so the rotation of hasn't to be taken into account.

These relationships are then added for each tooth and each disc to perform numerical integration along the cutting edges:

$$
\left\{ \begin{array}{c} F_\mathrm{x} \\ F_\mathrm{y} \\ F_\mathrm{z} \end{array} \right\} = \sum_{i=1}^{n_\mathrm{d}} \sum_{j=1}^{n_\mathrm{t}} \left\{ \begin{array}{c} \mathrm{d}F_\mathrm{x}(i,j) \\ \mathrm{d}F_\mathrm{y}(i,j) \\ \mathrm{d}F_\mathrm{z}(i,j) \end{array} \right\} = \overbrace{\left(\sum_{i=1}^{n_\mathrm{d}} \sum_{j=1}^{n_\mathrm{t}} \mathbf{B}\,\mathbf{A} \right)}^{\mathbf{C}} \boldsymbol{K} \ . \tag{7}
$$

Matrix $\mathbf{C}$ (dimension $3{\times}3$ or $3{\times}6$) links cutting coefficients to cutting forces.

At each time step a matrix $\mathbf{C}^k$ can be build (k is the index of the current time step). All these matrix are then assembled to get the global system:

$$
\overbrace{\left\{ \begin{array}{c} F_\mathrm{x}^1 \\ F_\mathrm{y}^1 \\ F_\mathrm{z}^1 \\ \hline F_\mathrm{x}^2 \\ F_\mathrm{y}^2 \\ F_\mathrm{z}^2 \\ \vdots \end{array} \right\}}^{\boldsymbol{F}} = \overbrace{\left[\begin{array}{c} \mathbf{C}^1 \\ \hline \mathbf{C}^2 \\ \vdots \end{array} \right]}^{\mathbf{D}} \boldsymbol{K} \ . \tag{8}
$$

The specific pressure can be computed by filling the vector $\boldsymbol{F}$ with the measured forces and by applying least square optimization method to solve the overdetermined system:

$$
\boldsymbol{K} = \left(\mathbf{D}^\mathrm{T}\,\mathbf{D} \right)^{-1} \left(\mathbf{D}^\mathrm{T}\,\boldsymbol{F} \right)
$$

$\boldsymbol{K}$ is the matrix containing specific pressure, $\mathbf{D}$ the assembly of all $\mathbf{C}^k$ matrix and $\boldsymbol{F}$ the measured cutting forces.

4.1. Practical consideration

In order to ensure accurate results, the initial position of the first cutting edge must be determined precisely. This is often difficult practically, especially while using a static dynamometer. The solution is to determine this shift using an optimization method that minimizes the RMS value of variation between simulated and computed signal as a function of the unknown shift. RMS value of error is defined by the following relationship:

$$
\mathrm{RMS}_\mathrm{error} = \sqrt{\frac{\sum_{i=0}^{n_\mathrm{points}} \left[\left(F^i_\mathrm{computed} - F^i_\mathrm{measured} \right) \Delta\theta \right]^2}{\theta_\mathrm{end} - \theta_\mathrm{begin}}} \ . \tag{10}
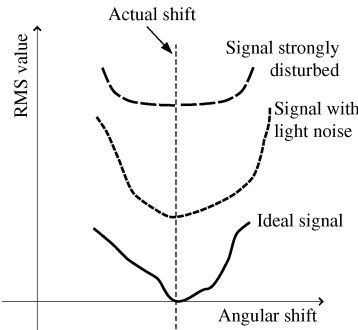$$

Fig.4: RMS versus initial shift

The best shift is computed as the shift giving the minimum value to that criterion. The minimum is sharper when the measured signal is less noisy (Fig. 4).

5. Validation on a simulated example

5.1. Testcase description

The first test is performed on simulated signals in order to check the accuracy of the method. The testcase is extracted from reference [1]. It computes cutting forces for half immersion downmilling (spindle speed 263 RPM, 5.08 mm axial depth of cut, feed 0.05 mm/tooth) of a titanium alloy with a cylindrical end mill (diameter 18.1 mm, four teeth, 30° helix angle).

Two different sets of parameters are considered: the original signal and the same signal disturbed by a random white noise (the evolutions of both signals are shown in Figure 5). The white noise is obtain by multiplying random signal (mean value of zero, standard deviation of 1) by 10 % of the maximum amplitude of the simulated force. In both cases, we get coefficients close to the data of the simulation and a simulated signal with small RMS error. Cutting coefficient and results from extraction are summarized in Table 1.

Coefficient	Data	Extraction from simulation	
		without noise	with 10 % noise
K_{tc} [N/mm^2]	1478	1479	1508
K_{rc} [N/mm^2]	247	246	241
K_{ac} [N/mm^2]	577	577	579
K_{te} [N/mm]	24	24	23
K_{re} [N/mm]	43	43	43
K_{ae} [N/mm]	0	-8×10^{-6}	7×10^{-2}

Tab.1: Cutting coefficient

While using perfect simulated signal as input, the difference between computed and original coefficient is only due to rounding errors; the algorithm also give good results for the disturbed signal.

The RMS value of the gap between input signal and recomputed signal follows the theoretical trend with a minimum at the actual value of the shift (see Figure 6).
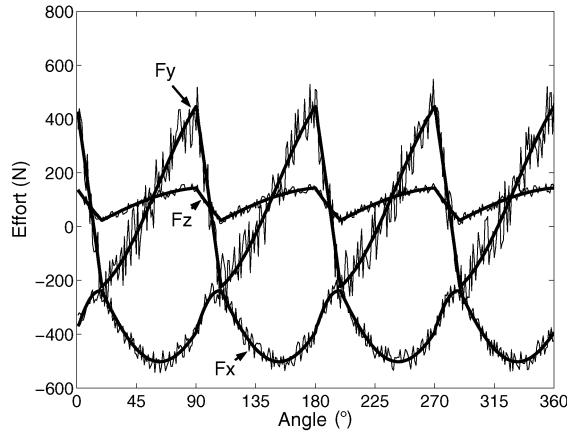
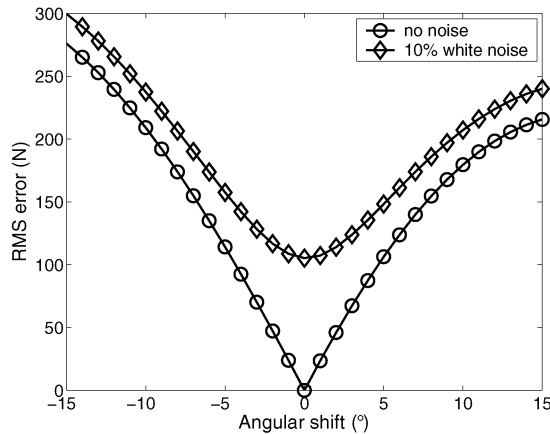Fig.5: Cutting forces with 10% white noise and recomputed signal

Fig.6: Evolution of RMS value of error; minimum
value is at the theoretical value of 0°

6. Experimental results

6.1. Data acquisition

Several tests using a high speed steel cylindrical end mill (diameter 8 mm, 2 flutes, helix angle 30°). Slots and shoulders were machined into St52-3 steel block. Data were acquired using rotating dynamometer Kistler 9123B. The measuring chain is composed of the rotating part of the sensor (acting as a toolholder), the stator (collecting the signals), charge amplifier (conditioning the signal) and the data acquisition card (saving the amplified signal on a PC). The sample rate is 20 kHz.

The acquired signal is noisy, we assumed that this noise is due to the dynamic behaviour of the tool/sensor/spindle assembly that is no rigid within the measured bandwidth. We decided to test our method on two series of data: the raw signal and the signal filtered using Butterworth 6th order filter with cutting frequency of 300 Hz.

6.2. Analysis of a specific measure

In this section, we develop the results of the analysis a particular measure. This is a slot cutting with an axial depth of cut of 1 mm. The spindle speed is 875 RPM, the feed is 0.046 mm/tooth (these values correspond to testcase 15, see annex Table 4).

We compared the results given by our method to the results given by the method proposed by Araujo and Silveira. Both methods are tested on samples of about 20.000 measured points (the samples cover at least five complete rotation of the cutter); the tool is divided into ten slices along its axis. The cutting coefficient and corresponding RMS error are given in Table 2.

Method	Inverse method		Araujo and al
Coefficient	Armarego	Linear	Armarego
K_{tc} [N/mm^2]	3850	5974	4160
K_{rc} [N/mm^2]	2529	3669	2174
K_{ac} [N/mm^2]	363	1484	1246
K_{te} [N/mm]	66	0	163
K_{re} [N/mm]	36	0	30
K_{ae} [N/mm]	46	0	11
RMS error (N)	37	44	167

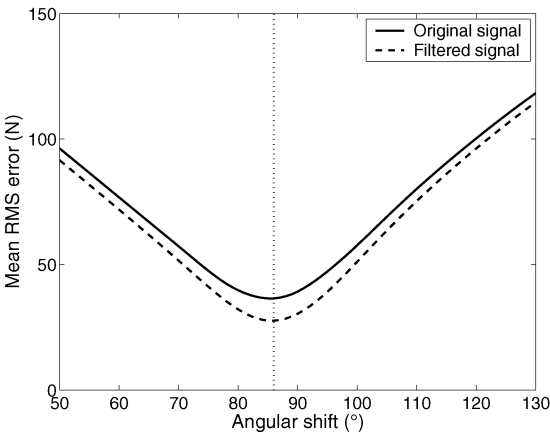*Tab.2: Cutting coefficient retrieved using measured signal (sample 15)*

Fig.7: Evolution of the error RMS value for ideal shift evaluation (sample 15)

The RMS value of the error between measured and simulated signal is more than four times smaller while using our method. We can see from Figure 7 that the optimisation for ideal shift is good.

Figures 8 to 11 show the comparison between measured and simulated signal for the four different cases (filtered or original signa, linear or Armarego's model).

6.3. Overall results

Our method of cutting coefficient evaluation was tested on both models (six and three coefficients) for both series of measurements (original and filtered signal). The RMS error value of the error for each cutting tests are drawn in Figure 12.
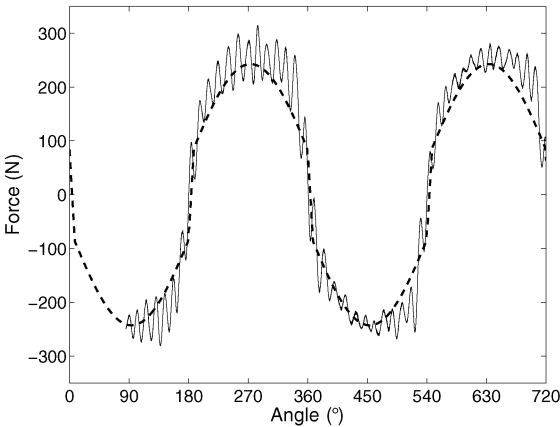
Fig.8: Experimental measurement and fitting using Armarego's model (F1, sample 15)
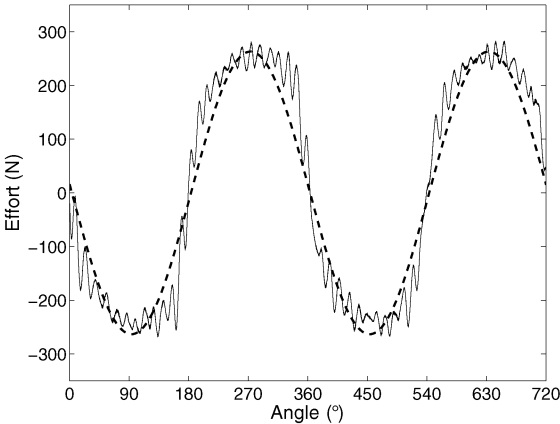

Fig.9: Experimental measurement and fitting using linear model (F1, sample 15)
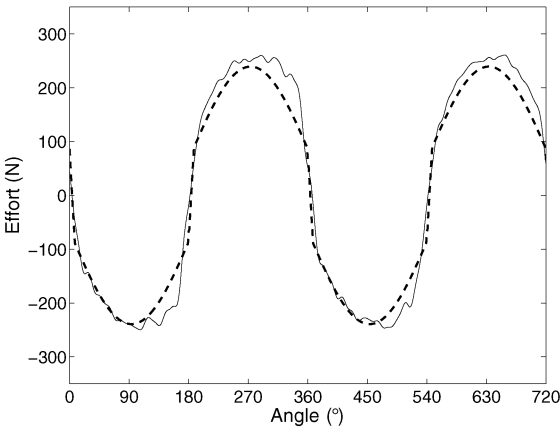

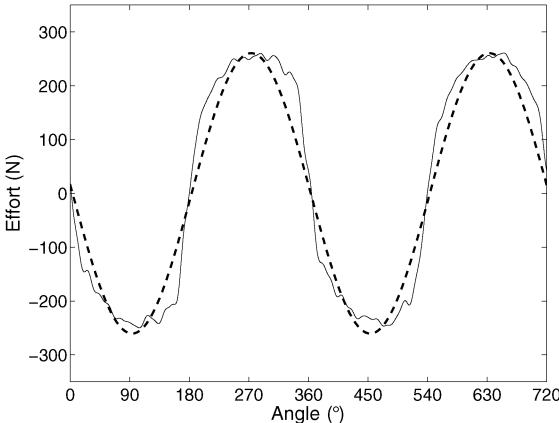*Fig.10: Filtered signal and fitting using Armarego's model (F1, sample 15)*

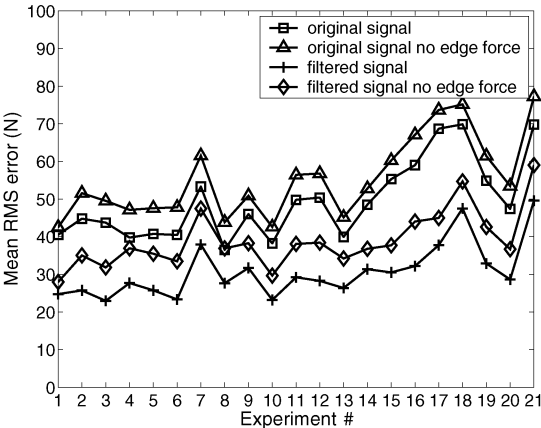*Fig.11: Filtered signal and fitting using linear model (F1, sample 15)*

Fig.12: Evolution of RMS value of error

We can see that for each case, fitting using the six coefficients method is more precise than fitting with three coefficients. This is an obvious result because the optimization process has more degree of freedom; the error is also smaller while considering filtered signal.

6.4. Mean value

For all the experiments, signal was simulated using the overall mean values of the coefficients found for each case (see Table 3).

	Armarego		Linear
direction	K_e [MPa]	K_c [N/mm]	K [MPa]
t	3605	79	6427
r	2748	30	3748
a	214	46	1769

Tab.3: Mean value of the cutting coefficients for both models

The error between filtered signal and simulated signal is of the same order of magnitude than the error with individual fit (see Figure 13). The method is thus able to find reliable cutting coefficient for a given couple tool/workpiece material with reasonable error.
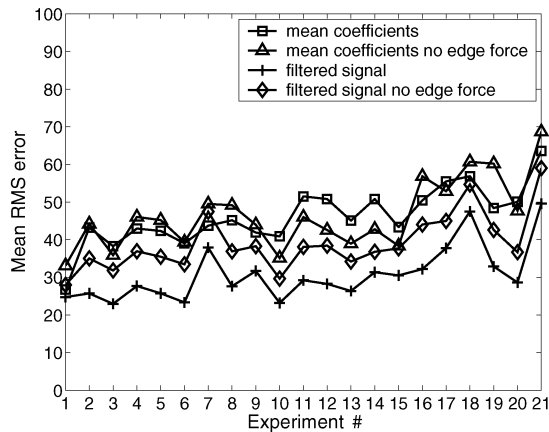

Fig.13: Evolution of RMS value of error while using mean coefficients

6.5. Simulation of the machining process

Retrieving parameters for cutting force simulation is often a preliminary step for the dynamic simulation of the whole process. The cutting coefficients extracted with this method can be used as an input for the simulation of the cutting process. We have simulated an exhaustive example with a software developed at the 'Faculte Polytechnique de Mons' which performs dynamic simulation of the milling process (see [10] to get more details about the simulator).

Testcase 3 (machining of a shoulder, 2 mm radial depth of cut upmilling) was simulated taking dynamic of the system into account. The modelling parameters are divided into several sets :
 – geometrical parameters (tool geometry);
 – technological parameters (feed, spindle speed, depth of cut, ...);
 – material properties (cutting coefficients);
 – dynamic properties (modal mass, eigenfrequency, damping);
 – simulation parameters (time step, number of discs to discretize the tool, ...).

The geometrical and technological parameters describe the simulation case. The material cutting coefficients are computed using the method described heretofore. The dynamic system is modelled as a single degree of freedom system. Natural frequency is identified using FFT of the disturbed signal; damping ratio is computed using logarithmic decrement method; modal mass is the mass of the sensor.

Figure 14 compares the measured signal and the simulated signal. We can obviously see on this interrupted cutting example that the sensor measured the cutting forces and the inertia forces. The adequacy between the simulation and the measure is good. Our method to deduce cutting parameters can be a good starting point to get parameters for dynamic simulation of the milling process. Those simulations can be useful to predict stability of the process or quality of the final part.
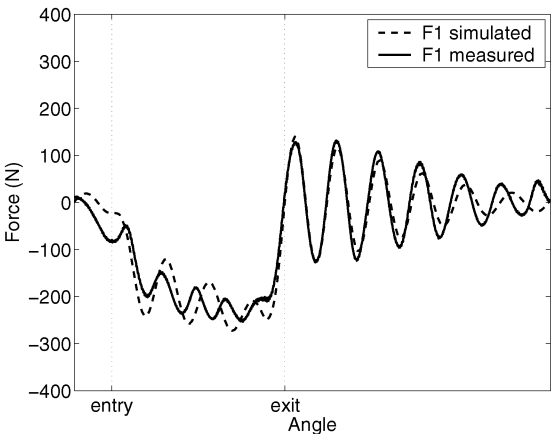
Fig.14: Simulated and measured effort, direction 1

Test	ADOC	RDOC	Feed per tooth	Spindle speed
1	0.75 mm	2 mm	0.04 mm/tooth	875 RPM
2	0.75 mm	8 mm	0.04 mm/tooth	875 RPM
3	0.75 mm	8 mm	0.036 mm/tooth	962 RPM
4	0.75 mm	8 mm	0.044 mm/tooth	875 RPM
5	0.75 mm	8 mm	0.04 mm/tooth	962 RPM
6	0.75 mm	8 mm	0.04 mm/tooth	875 RPM
7	0.75 mm	8 mm	0.046 mm/tooth	875 RPM
8	1 mm	2 mm	0.04 mm/tooth	875 RPM
9	1 mm	8 mm	0.04 mm/tooth	875 RPM
10	1 mm	8 mm	0.036 mm/tooth	962 RPM
11	1 mm	8 mm	0.044 mm/tooth	875 RPM
12	1 mm	8 mm	0.04 mm/tooth	962 RPM
13	1 mm	8 mm	0.04 mm/tooth	875 RPM
14	1 mm	5 mm	0.046 mm/tooth	875 RPM
15	1 mm	8 mm	0.046 mm/tooth	875 RPM
16	1.25 mm	8 mm	0.04 mm/tooth	875 RPM
17	1.25 mm	8 mm	0.036 mm/tooth	962 RPM
18	1.25 mm	8 mm	0.044 mm/tooth	875 RPM
19	1.25 mm	8 mm	0.04 mm/tooth	962 RPM
20	1.25 mm	8 mm	0.04 mm/tooth	875 RPM
21	1.25 mm	8 mm	0.046 mm/tooth	875 RPM

Tab.4: Data for the cutting tests (ADOC: axial depth
of cut, RDOC: radial depth of cut)

7. Conclusions

In this paper we developed an improved method to identify the cutting parameters from cutting tests. This method is able to retrieve cutting coefficients from the measurement of cutting forces in milling. It has been validated on simulated signals and on signals disturbed with white noise.

Several tests using the same pair tool/workpiece material were performed in order to validate the method with real measurements. The result of each identification gives a RMS error between measured an recomputed signal about one third smaller than the result given by a method available in the literature.

The mean coefficients retrieved from all the identification were used to compute the cutting forces. The mean RMS errors between these simulations and the measurements are of the same order of magnitude than the mean RMS errors for each individual identification.

The cutting coefficients given by this method can be a good starting point for dynamic simulation of the whole machining process (dimensional tolerances, tool wear or chatter vibrations).

Acknowledgement

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