CONTRIBUTION TO THE METHODS OF ANALYSIS OF COMPOSITE STEEL-CONCRETE BEAMS, REGARDING RHEOLOGY

Doncho Partov, Vesselin Kantchev*

The paper presents analysis of the stress changes due to creep in statically determinate composite steel-concrete beam. Each beam consists of steel element I – section acting compositely with concrete plate, attached to the upper surface of the beam. The mathematical model involves the equation of equilibrium, compatibility and constitutive relationship, i.e. an elastic law for the steel part and an integral-type creep law of Boltzmann-Volterra for the concrete part. For determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time 't', system of two independent Volterra integral equations of the second kind have been derived, on the basis of the theory of the viscoelastic body of Arutyunian-Trost-Bažant. Numerical method, which makes use of quadrature formulae for solving these equations, is proposed. The computer programs are realized in environment of a high-performance language for technical computing MATLAB. Some relevant examples with the model proposed are investigated and discussed. The creep functions is suggested by the 'CEB-FIP' models code 1970. The elastic modulus of concrete $E_c(t)$ is assumed to be either constant or depend on time 't'. Our results are compared with the corresponding results of Effective modulus method (EMM), Rate of creep method (RCM), Improved Dischinger method (IDM) and Trost method (TM).

Key words: *composite steel-concrete beams, creep, rheology, elastic modulus, viscoelastic body, Volterra integral equation, quadrature formulae*

1. Introduction

The problem of investigating the statically determinate composite plate beam in the time t has for 60 years drawn the attention of engineers who were dealing with the problems of their design. This problem has, however, received a certain currency in the past few years, due to the new facts gathered about the rheological qualities of concrete.

It is known that while in the steel beam, under the effect of the serviceability loads, we see only elastic deformations, in the concrete plate during the time significant plastic deformation takes place as a consequence of creep and shrinkage of concrete.

As a result of these deformations and because of the stiff connection between the two elements of the composite plate beam, in every cross-section subjected to the effect of constantly operating outside bending moment M_o in the time t arises a new additional group of forces and moments $N_{c,r}(t)$, $M_{c,r}(t)$, $N_{s,r}(t)$, $M_{s,r}(t)$ (Fig. 1). The influence of this group of forces and moments over the general stress conditions of the statically determinate com-

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posite plate beam is expressed by the decrease of the stresses in the concrete plate and in the increase of stresses in the steel beam.

The papers dealing with the solution of the problem of finding the unknown normal forces $N_{c,r}(t)$, $N_{s,r}(t)$ and the bending moments $M_{c,r}(t)$, $M_{s,r}(t)$ are numerous and diverse. Their chronological analysis shows from the first aspect the aspirations of various authors to penetrate further into the actual behavior of the structures, which will eventually lead to the creation of more accurate calculation methods; and from the second aspect it shows the aspirations to replace the complicated methods by simple ones for practical usage.

The first works, which give the answer to this problem are based on the Law of Dischinger [21, 22], who had first formulated a time-dependent stress-strain differential relationship for concrete, using the following equation:

$$\frac{d\varepsilon_{ct}}{dt} = \frac{\sigma_{ct}}{E_{c0}} \frac{d\varphi_t}{dt} + \frac{1}{E_{bt}} \frac{d\sigma_{ct}}{dt} ,$$

where φ_t is called creep function.

These books and papers connected with the names of Frohlich [25], Esslinger [23], Kloppel [33], Sonntag [53], Kunert [35], Muller [38], Dimitrov [20], Mrazik [37] and Bujňák [11] represent one independent group for which it is characteristic that for the unknown quantities $N_{c,r}(t)$ and $M_{c,r}(t)$ a system of simultaneous differential equations have been derived and solved. All these methods have been collected and analyzed by Sattler [47] and by the first author of this paper [40].

In parallel with the developed analytical methods, Blaszkowiak [8], Bradford [9], Fritz [24] and Wippel [60] have developed approximate methods, which use Dischinger's idea for applying in the calculation the ideal (ficticious) modulus of elasticity [21, 22]:

$$E_{ci} = \frac{E_{c0}}{1 + \varphi_n} ,$$

where φ_n is the ultimate value of creep.

Another method of the estimate design calculation as described in [49] has been based on the creep fibred method by Busemann [12].

With Wippel's methods [60] the first stage of the development of the analytical methods is based entirely on the works of Dischinger [21, 22], has been completed.

Further development of rheology as a fundamental science and its application to concrete [2, 4, 44, 46, 57] as well as a great number of investigations in the field of creep of concrete have led to new formulations of the time-dependent behavior of concrete [5, 13, 43].

These new formulations that give the relationship between $\varphi_c(t)$ and $\varepsilon_c(t)$ are formulated by integral equations, which present the basis of the theory of linear viscoelastic bodies.

However, in order to avoid the mathematical problems in solving of the integral equations of Volterra for treating the problem connected with the creep of concrete structures, Trost [55] and Zerna [58], have revised the integral relationship into new algebraic stress-strain relationship:

$$\varepsilon_{ct} = \frac{\sigma_{c0}}{E_{c0}} [1 + \varphi_t] + \frac{\sigma_{ct} - \sigma_{c0}}{E_{c0}} [1 + \varrho\, \varphi_t] ,$$

where ϱ is the relaxation coefficient. From the same considerations another revision of integral relationship into new algebraic stress-strain relationship have been made by Kruger [34] and Wolff [61]:

$$E_{c0}\,\varepsilon_{c\varphi,t} = \sigma_{c0}\,\frac{\varphi_{t0}-\varphi_{t1}}{2} + \sigma_{ct}\left[1+\frac{\varphi_{t(t-1)}}{2}\right] + \sum_{i=1}^{t-1}\sigma_{c,i}\,\frac{\varphi_{t,i-1}-\varphi_{t,i+1}}{2}\ .$$

On the basis of that algebraic stress-strain relationship, new methods have been developed connected with the names Wappenhans [58], Wolff [61], Trost [56], Heim [31], Amadio [1], Dezi [15, 16, 17, 18, 19, 54] and Gilbert [27, 28], for solving the problem raised by Fröhlich [25].

In parallel with the methods developed by Furtak [26], Kindman [32], Lapos [36], Pachla [39], Partov [41], on the basis of the theory of linear viscoelastic bodies, Sattler [48], Haenzel [29], and Profanter [42] have recently developed new methods, which are based on the 'modified theory' of Dischinger, called also the theory of Rüsch-Jungwirt [45]. This theory is described by the following equations:

$$\frac{d\varepsilon_{ct}}{dt} = \frac{\sigma_{ct}}{E_{cv}}\,\frac{d\varphi_{f,v}}{dt} + \frac{1}{E_{bt}}\,\frac{d\sigma_{ct}}{dt}\ , \quad \text{where} \quad E_{cv} = \frac{E_c(t_0)}{1.4}\ , \quad \varphi_{f,v} = \frac{\varepsilon_{f,0}[K_f(t)-K_f(t_0)]}{1.4}\ .$$

Different approach to the solving of the formulated problems is applying the FEM by Hering [30] and Cumbo [14].

Since the theory of Rüsch-Jungwirt [45] has been subjected to serious criticism in the works of Alexandrovski-Arutyunyan [3] and [6, 7] the authors of the present paper make an attempt for a new step toward deriving more precise solution of the problem. An effort is made to give an answer to the dispute between Bažant and Rusch-Jungwirt [5].

2. Preliminary assumption

The theory implies the following assumptions to be true:

a) Bernoulli's concerning plane strain of cross-sections.
b) Concrete is not cracked $\sigma_c \leq (0.4 \div 0.5)\,R_c$.
c) Hooke's law applies to steel as well as to concrete under short-time loads.
d) In the range of service ability loads concrete behaves in a way allowing to be treated as a linear viscoelastic body. The stress-strain behavior of concrete can be described with sufficient accuracy by the integral equations (1) by Bolztmann-Volterra [2, 4, 52]

$$\varepsilon_c(t) = \frac{\sigma_c(t_0)}{E_c(t_0)}\,[1+\varphi(t-t_0)] + \int_{t_0}^{t}\frac{d\sigma_c(\tau)}{d\tau}\,\frac{1}{E_c(\tau)}\,[1+\varphi(t-t_0)]\,d\tau\ , \tag{1}$$

where $\varphi(t-\tau) = \varphi_N\,K(\tau)\,f(t-\tau)$ is the so called the creep function and φ_N the ultimate value of creep coefficient, $K(\tau)$ depends on the age increase of concrete. It is called the function of aging, and it characterizes the process of the aging. The increase of τ makes $K(\tau)$ monotonously decrease. The function $f(t-\tau)$ – (where t is the time interval during which the structure is under observation, τ is the running coordinate of time) – characterizes the process of creeping.

e) The modulus of concrete elasticity is invariant in time t [10, 51] i.e. $E_c(\tau) = E_c(t_0) = E_{const}$ and depending on time t

$$E_c(\tau) = E_c(t_0)\,\sqrt{\frac{\tau}{4+0.86\,\tau}}\ . \tag{2}$$

f) According to a proposal by Sonntag [53], the influence of the development of the bending moment $M_{c,r}(t)$ in the concrete member, upon the redistribution of the normal force of concrete $N_{c,r}(t)$ can be neglected.
g) For the service load analysis no slip occurs between the steel and concrete.

3. Deriving of the mechano-mathematical model for constant elasticity module of concrete

Let us denote both the normal forces and the bending moments in the cross-section of the plate and the girder after the loading in the time $t = 0$ with $N_{c,0}$, $M_{c,0}$, $N_{s,0}$, $M_{s,0}$ and with $N_{c,r}(t)$, $M_{c,r}(t)$, $N_{s,r}(t)$, $M_{s,r}(t)$ a new group of normal forces and bending moments, arising due to creep.

For a composite bridge girder with

$$J_c = \frac{A_c (n\, I_c)\, n}{A_s\, I_s} \le 0.2$$

according to the suggestion of Sonntag [44] we can write the equilibrium conditions in time t as follows

$$N(t) = 0\ , \qquad N_{c,r}(t) = N_{s,r}(t)\ , \tag{3}$$

$$\sum M(t) = 0\ , \qquad M_{c,r}(t) + N_{c,r}(t)\, r = M_{s,r}(t)\ , \tag{4}$$

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations (3), (4) are not sufficient to solve it.

It is necessary to produce two additional equations in the sense of compatibility of deformations of both steel girder and concrete slab in time t (Fig. 1).

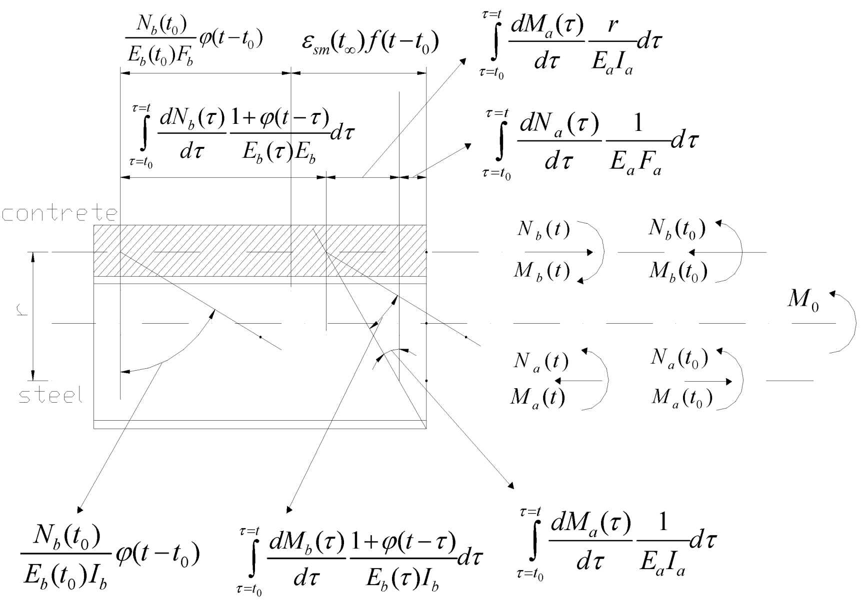

Fig.1: Mechano-mathematical model for deformations in cross-section in composite steel-concrete beam, regarding creep of the concrete

These conditions are as follows.

3.1. Strain compatibility on the contact surfaces between the concrete and steel members of composite girder

$$\frac{N_{\mathrm{c},0}}{E_\mathrm{c}(t_0)\,A_\mathrm{c}}\,[1+\varphi(t-t_0)] - \frac{1}{E_\mathrm{c}(t_0)\,A_\mathrm{c}} \int\limits_{t_0}^{t} \frac{\mathrm{d}N_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\,[1+\varphi(t-\tau)]\,\mathrm{d}\tau + \\ + \frac{N_{\mathrm{s},0}}{E_\mathrm{s}\,A_\mathrm{s}} - \frac{1}{E_\mathrm{s}\,A_\mathrm{s}} \int\limits_{t_0}^{t} \frac{\mathrm{d}N_{\mathrm{s,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau = \frac{M_{\mathrm{s},0}}{E_\mathrm{s}\,I_\mathrm{s}}\,r + r\,\frac{1}{E_\mathrm{s}\,I_\mathrm{s}} \int\limits_{t_0}^{t} \frac{\mathrm{d}M_{\mathrm{s,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\;. \tag{5}$$

Using

$$\frac{N_{\mathrm{c},0}}{E_\mathrm{c}(t_0)\,A_\mathrm{c}} + \frac{N_{\mathrm{s},0}}{E_\mathrm{s}\,A_\mathrm{s}} = \frac{M_{\mathrm{s},0}}{E_\mathrm{s}\,I_\mathrm{s}}\,r$$

and integrating the equation (5) by parts we get

$$\frac{N_{\mathrm{c},0}}{E_\mathrm{c}\,A_\mathrm{c}}\,[\varphi(t-t_0)] - \left[\frac{N_{\mathrm{c,r}}(\tau)}{E_\mathrm{c}\,A_\mathrm{c}}\,[1+\varphi(t-\tau)]\right]\Bigg|_{t_0}^{t} + \\ + \frac{1}{E_\mathrm{c}\,A_\mathrm{c}} \int\limits_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}[1+\varphi(t-\tau)]\,\mathrm{d}\tau - \frac{1}{E_\mathrm{s}\,A_\mathrm{s}}\,N_{\mathrm{s,r}}(t) = r\,\frac{1}{E_\mathrm{s}\,I_\mathrm{s}}\,M_{\mathrm{s,r}}(t)\;, \tag{5a}$$

$$\frac{N_{\mathrm{c},0}}{E_\mathrm{c}\,A_\mathrm{c}}\,[\varphi(t-t_0)] - \frac{N_{\mathrm{c,r}}(t)}{E_\mathrm{c}\,A_\mathrm{c}}\,[1+\varphi(t-t)] + \frac{N_{\mathrm{c,r}}(t_0)}{E_\mathrm{c}\,A_\mathrm{c}}\,[1+\varphi(t-t_0)] + \\ + \frac{1}{E_\mathrm{c}\,A_\mathrm{c}} \int\limits_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}[1+\varphi(t-\tau)]\,\mathrm{d}\tau - \frac{1}{E_\mathrm{s}\,A_\mathrm{s}}\,N_{\mathrm{s,r}}(t) = r\,\frac{1}{E_\mathrm{s}\,I_\mathrm{s}}\,M_{\mathrm{s,r}}(t)\;. \tag{5b}$$

Since $\varphi(0) = 0$ and $N_\mathrm{c}(t_0) = 0$ for assessment of normal forces $N_{\mathrm{c,r}}(t)$ linear integral Volterra equation of the second kind is derived

$$N_{\mathrm{c,r}}(t) = \lambda_\mathrm{N} \int\limits_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}[1+\varphi_\mathrm{N}\,K(\tau)\,f(t-\tau)]\,\mathrm{d}\tau + \lambda_\mathrm{N}\,N_{\mathrm{c},0}\,\varphi_\mathrm{N}\,K(t_0)\,f(t-t_0)\;, \tag{6}$$

where

$$\lambda_\mathrm{N} = \left[1 + \frac{E_\mathrm{c}\,A_\mathrm{c}}{E_\mathrm{s}\,A_\mathrm{s}}\left(1 + \frac{A_\mathrm{s}\,r^2}{I_\mathrm{s}}\right)\right]^{-1}\;. \tag{7}$$

3.2. Compatibility of curvatures when $\tau = t$

$$\frac{M_{\mathrm{c},0}}{E_\mathrm{c}(t_0)\,I_\mathrm{c}}\,[1+\varphi(t-t_0)] - \frac{1}{E_\mathrm{c}\,I_\mathrm{c}} \int\limits_{t_0}^{t} \frac{\mathrm{d}M_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\,[1+\varphi(t-t_0)]\,\mathrm{d}\tau = \\ = \frac{M_{\mathrm{s},0}}{E_\mathrm{s}\,I_\mathrm{s}} - \frac{1}{E_\mathrm{s}\,I_\mathrm{s}} \int\limits_{t_0}^{t} \frac{\mathrm{d}M_{\mathrm{s,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\;. \tag{8}$$

From

$$\frac{M_{\mathrm{c},0}}{E_\mathrm{c}(t_0)\,I_\mathrm{c}} = \frac{M_{\mathrm{s},0}}{E_\mathrm{s}\,I_\mathrm{s}}\;,$$

after integrating the equation (8) by parts and using (4) for assessment of bending moment $M_{c,r}(t)$ linear integral Volterra equation of the second kind is derived

$$\begin{aligned} M_{c,r}(t) &= \lambda_M \int_{t_0}^{t} M_{c,r}(\tau) \frac{d}{d\tau}[1+\varphi_N K(\tau) f(t-\tau)]\, d\tau + \\ &+ \lambda_M M_{c,0}\, \varphi_N K(t_0) f(t-t_0) - \lambda_M \frac{E_c I_c}{E_s I_s} N_{c,r}(t)\, r \ , \end{aligned} \tag{9}$$

in which

$$\lambda_M = \left[1 + \frac{E_c I_c}{E_s I_s}\right]^{-1} . \tag{10}$$

In each of these equations the functions

$$N_{c,0}\, \varphi_N K(t_0) f(t-\tau) \ , \qquad M_{c,0}\, \varphi_N K(t_0) f(t-\tau) \ , \qquad \frac{d}{d\tau}[1+\varphi_N K(\tau) f(t-\tau)]$$

are given.

4. Deriving of the mechano-mathematical model for time depended elasticity module of concrete

Analogically to case with the constant elasticity module using the same notations as in part 3, using the equilibrium equations (3), (4) and strain compatibility on the contact surfaces between the concrete and steel members of composite girder and compatibility of curvatures we obtain the following linear integral Volterra equation of the second kind (12) and (15):

4.1. Strain compatibility on the contact surfaces between the concrete and steel members of composite girder

$$\begin{aligned} &\frac{N_{c,0}}{E_c(t_0) A_c}[1+\varphi(t-t_0)] - \frac{1}{A_c}\int_{t_0}^{t} \frac{1}{E_c(\tau)} \frac{dN_{c,r}(\tau)}{d\tau}[1+\varphi(t-\tau)]\, d\tau + \\ &+ \frac{N_{s,0}}{E_s A_s} - \frac{1}{E_s A_s}\int_{t_0}^{t} \frac{dN_{s,r}(\tau)}{d\tau}\, d\tau = \frac{M_{s,0}}{E_s I_s} r + r\, \frac{1}{E_s I_s}\int_{t_0}^{t} \frac{dM_{s,r}(\tau)}{d\tau}\, d\tau \ . \end{aligned} \tag{11}$$

Using

$$\frac{N_{c,0}}{E_c(t_0) A_c} + \frac{N_{s,0}}{E_s A_s} = \frac{M_{s,0}}{E_s I_s} r$$

and integrating the equation (11) by parts we get

$$\begin{aligned} &\frac{N_{c,0}}{E_{c,0} A_c}[\varphi(t-t_0)] - \left[\frac{N_{c,r}(\tau)}{E_c A_c}[1+\varphi(t-\tau)]\right]\Bigg|_{t_0}^{t} + \\ &+ \frac{1}{A_c}\int_{t_0}^{t} N_{c,r}(\tau) \frac{d}{d\tau}\left[\frac{1}{E_c(\tau)}[1+\varphi(t-\tau)]\right] d\tau - \frac{1}{E_s A_s} N_{s,r}(t) = r\, \frac{1}{E_s I_s} M_{s,r}(t) \ , \end{aligned} \tag{11a}$$

$$\frac{N_{\mathrm{c},0}}{E_{\mathrm{c},0}\,A_{\mathrm{c}}}\,[\varphi(t-t_0)] - \frac{N_{\mathrm{c,r}}(t)}{E_{\mathrm{c}}\,A_{\mathrm{c}}}\,[1+\varphi(t-t)] + \frac{N_{\mathrm{c,r}}(t_0)}{E_{\mathrm{c}}\,A_{\mathrm{c}}}\,[1+\varphi(t-t_0)] + \\ + \frac{1}{A_{\mathrm{c}}}\int_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left[\frac{1}{E_{\mathrm{c}}(\tau)}\,[1+\varphi(t-\tau)]\right]\mathrm{d}\tau - \frac{1}{E_{\mathrm{s}}\,A_{\mathrm{s}}}\,N_{\mathrm{s,r}}(t) = r\,\frac{1}{E_{\mathrm{s}}\,I_{\mathrm{s}}}\,M_{\mathrm{s,r}}(t)\ . \tag{11b}$$

Since $\varphi(0) = 0$ and $N_{\mathrm{c}}(t_0) = 0$ for assessment of normal forces $N_{\mathrm{c,r}}(t)$ linear integral Volterra equation of the second kind is derived

$$N_{\mathrm{c,r}}(t) = \lambda_{\mathrm{N}}(t)\int_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\,E_{\mathrm{c},0}\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left[\frac{1}{E_{\mathrm{c}}(\tau)}\,[1+\varphi_{\mathrm{N}}\,K(\tau)\,f(t-\tau)]\right]\mathrm{d}\tau + \\ + \lambda_{\mathrm{N}}\,N_{\mathrm{c},0}\,\varphi_{\mathrm{N}}\,K(t_0)\,f(t-t_0)\ , \tag{12}$$

where

$$\lambda_{\mathrm{N}} = \left[\frac{E_{\mathrm{c},0}}{E_{\mathrm{c}}(t)} + \frac{E_{\mathrm{c},0}\,A_{\mathrm{c}}}{E_{\mathrm{s}}\,A_{\mathrm{s}}} + \frac{E_{\mathrm{c},0}\,A_{\mathrm{c}}\,r^2}{E_{\mathrm{s}}\,I_{\mathrm{s}}}\right]^{-1}\ . \tag{13}$$

4.2. Compatibility of curvatures when $\tau = t$

$$\frac{M_{\mathrm{c},0}}{E_{\mathrm{c}}(t_0)\,I_{\mathrm{c}}}\,[1+\varphi(t-t_0)] - \frac{1}{I_{\mathrm{c}}}\int_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\,\frac{1}{E_{\mathrm{c}}(\tau)}\,[1+\varphi(t-\tau)]\,\mathrm{d}\tau = \\ = \frac{M_{\mathrm{s},0}}{E_{\mathrm{s}}\,I_{\mathrm{s}}} - \frac{1}{E_{\mathrm{s}}\,I_{\mathrm{s}}}\int_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{s,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\ . \tag{14}$$

From

$$\frac{M_{\mathrm{c},0}}{E_{\mathrm{c}}(t_0)\,I_{\mathrm{c}}} = \frac{M_{\mathrm{s},0}}{E_{\mathrm{s}}\,I_{\mathrm{s}}}\ ,$$

after integrating the equation (14) by parts and using (4) for assessment of bending moment $M_{\mathrm{c,r}}(t)$ linear integral Volterra equation of the second kind is derived

$$M_{\mathrm{c,r}}(t) = \lambda_{\mathrm{M}}\int_{t_0}^{t} M_{\mathrm{c,r}}(\tau)\,E_{\mathrm{c},0}\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left[\frac{1+\varphi_{\mathrm{N}}\,K(\tau)\,f(t-\tau)}{E_{\mathrm{c}}(\tau)}\right]\mathrm{d}\tau + \\ + \lambda_{\mathrm{M}}\,M_{\mathrm{c},0}\,\varphi_{\mathrm{N}}\,K(t_0)\,f(t-t_0) - \lambda_{\mathrm{M}}\,\frac{E_{\mathrm{c},0}\,I_{\mathrm{c}}}{E_{\mathrm{s}}\,I_{\mathrm{s}}}\,N_{\mathrm{c,r}}(t)\,r\ , \tag{15}$$

in which

$$\lambda_{\mathrm{M}} = \left[\frac{E_{\mathrm{c},0}}{E_{\mathrm{c}}(t)} + \frac{E_{\mathrm{c},0}\,I_{\mathrm{c}}}{E_{\mathrm{s}}\,I_{\mathrm{s}}}\right]^{-1}\ . \tag{16}$$

In each of these equations the functions

$$N_{\mathrm{c},0}\,\varphi_{\mathrm{N}}\,K(t_0)\,f(t-\tau)\ ,\qquad M_{\mathrm{c},0}\,\varphi_{\mathrm{N}}\,K(t_0)\,f(t-\tau)\ ,\qquad \frac{\mathrm{d}}{\mathrm{d}\tau}[1+\varphi_{\mathrm{N}}\,K(\tau)\,f(t-\tau)]$$

are given.

5. Numerical method

The integral equations are solved by a numerical method using quadratic formulas. These methods represent a replacement of the integral equations by approximate linear equations with triangle matrix related in view of a discrete value of the unknown function.

The replacement is achieved on the basis of approximation of the integral equation operator by quadrature formulas.

The increase of the parameter τ is related to the growth of the descretizating points, so that the application of certain quadratic formulas of Simpson, Gauss, Markoff, Chebishev is rather troublesome.

That is why integrals are approximated with quadrature formulas of trapeze [50].

6. Numerical results for constant elasticity module of concrete

A demonstration of the numerical method is implemented in MATLAB code. Several practical examples in bridge construction have been solved. One of the examples had its parameters cross-section values and initial section forces are shown in Fig. 2.

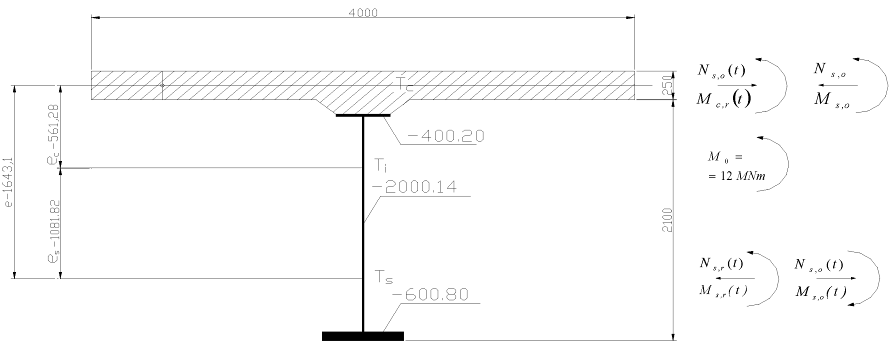

Fig.2: Composite beam with cross-section characteristic: $E_\mathrm{c} = 3.4\times10^4$ *MPa,* $E_\mathrm{s} = 2.1\times10^6$ *MPa,* $A_\mathrm{c} = 10000\,cm^2$, $A_\mathrm{s} = 840\,cm^2$, $n = E_\mathrm{s}/E_\mathrm{c} = 6.176$, $I_\mathrm{c} = 520833\,cm^4$, $I_\mathrm{s} = 4859650\,cm^4$, $r_\mathrm{c} = 56.128\,cm$, $r_\mathrm{s} = 108.182\,cm$, $r = 164.31\,cm$, $A_\mathrm{i} = 2453.05\,cm^2$, $I_\mathrm{i} = 19875408\,cm^4$

On the basis of numerous solved examples the optimal step of three days for solving the integral equations (6) and (9) is found.

Creep coefficient φ_N and the functions $K(\tau)$ and $f(t-\tau)$ are taken according to the recommendations of CEB-FIP(1966–1970). For both functions $K(\tau)$ and $f(t-\tau)$ the approximations of Wolff [61] are chosen:

$$K(\tau) = \begin{cases} \dfrac{10.28}{5+\sqrt{\tau}} & \text{for} \quad \tau \le 857 \\[2ex] 0.3 & \text{for} \quad \tau > 857 \end{cases} \tag{17}$$

and

$$f(t-\tau) = 1 - \exp\left[-0.6\left(\frac{t-\tau}{30} + 0.0025\right)^{0.4} - 0.091\right] . \tag{18}$$

The variations of the inner normal forces $N_{c,r}$, $N_{s,r}$ and moments $M_{c,r}$, $M_{s,r}$ in the example for solving the creep problem are found for $t_0 = 28$, 90, 180, 365 and 730 days till respectively 6025, 6087, 6177, 6362 and 6727 days are shown on figures 3, 4, 5.

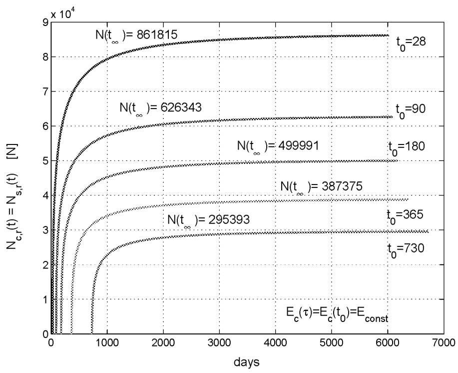

Fig.3: Values of normal forces $N_{c,r}(t) = N_{s,r}(t)$ in time t when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

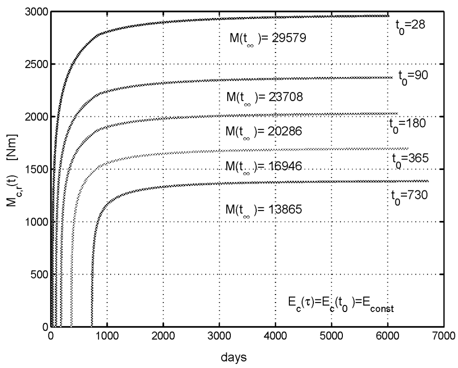

Fig.4: Values of bending moments $M_{c,r}(t)$ in time t when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

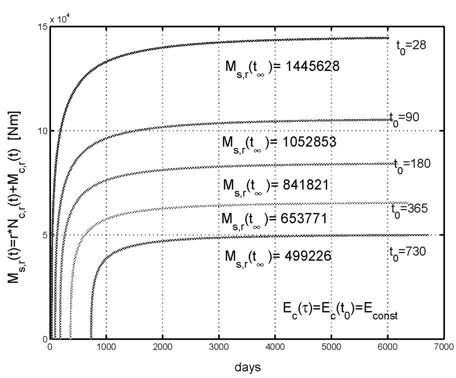

Fig.5: Values of bending moments $M_{s,r}(t) = r\,N_{c,r}(t) + M_{c,r}(t)$ in time t when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

The variations of the normal stresses in time t_∞ in concrete plate σ_c^{up} and in steel girder σ_s^{up}, σ_s^{down} in the example for solving the creep problem are found for $t_0 = 28$, 90, 180, 365 and 730 days till respectively 6025, 6087, 6177, 6362 and 6727 days are shown on figures 6, 7, 8.

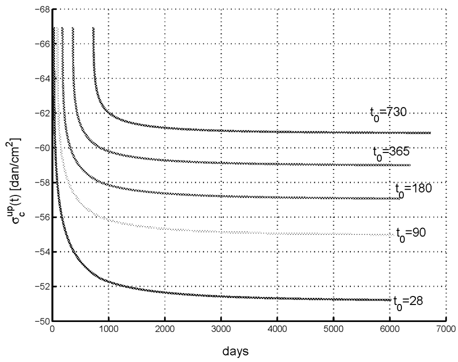

Fig.6: Values of normal stresses in upper fiber of concrete plate $\sigma_c^{up}(t)$ in time t_∞ when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

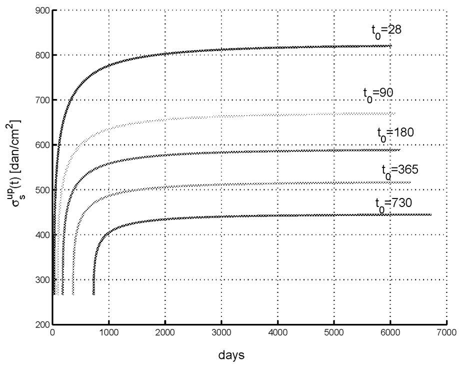

Fig.7: Values of normal stresses in upper fiber of steel girder $\sigma_s^{up}(t)$ in time t_∞ when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

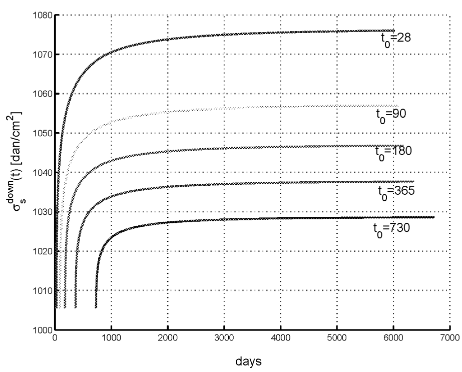

Fig.8: Values of normal stresses in down fiber of steel girder $\sigma_s^{down}(t)$ in time t_∞ when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

7. Numerical results for time depended elasticity module of concrete

The variations of the inner normal forces $N_{\mathrm{c,r}}$, $N_{\mathrm{s,r}}$ and moments $M_{\mathrm{c,r}}$, $M_{\mathrm{s,r}}$ in the example for solving the creep problem are found for $t_0 = 28$, 90, 180, 365 and 730 days till respectively 6025, 6087, 6177, 6362 and 6727 days are shown on figures 9, 10, 11.

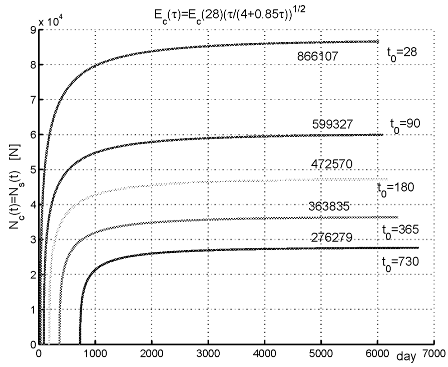

Fig.9: Values of normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{s,r}}(t)$ in time t when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

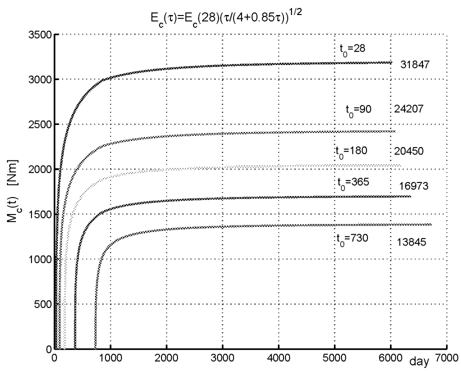

Fig.10: Values of bending moments $M_{\mathrm{c,r}}(t)$ in time t when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

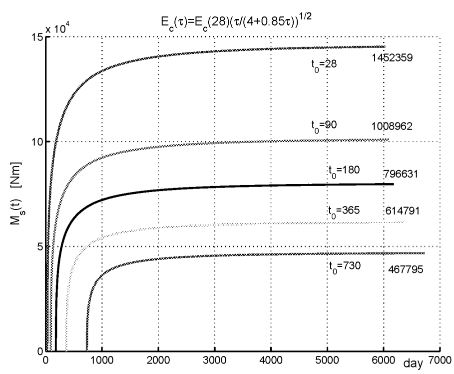

Fig.11: Values of bending moments $M_{\mathrm{s,r}}(t) = r\,N_{\mathrm{c,r}}(t) + M_{\mathrm{c,r}}(t)$ in time t when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days

	t_0	Normal forces and moments	Partov-Kantchev's method	Partov-Kantchev's method	Partov's methods [41]	Trost's methods [56]	Wolf's methods [61]	Haenzel's methods [29]	Sontag's methods [53]
			$E_c(\tau)$	$E_c(\tau)=E_c=const$					
Creep problem	28	$N_{c,r}(t)=N_{s,r}(t)$ [N]	866107	861815	981728	992410	994650	934720	1033930
		$M_{s,r}(t)=N_{c,r}*r+M_{c,r}(t)$ [Nm]	1452359	1445628	1644517	1661340	1665980	1567560	1735180
		$M_{c,r}(t)$ [Nm]	31847	29579	31438	30710	31660	31790	36330
	90	$N_{c,r}(t)=N_{s,r}(t)$ [N]	599327	626343	718010	763970	724950	777870	795320
		$M_{s,r}(t)=N_{c,r}*r+M_{c,r}(t)$ [Nm]	1008962	1052853	1205901	1282500	1217730	1306960	1339890
		$M_{c,r}(t)$ [Nm]	24207	23708	26138	27200	26550	28920	33090
	180	$N_{c,r}(t)=N_{s,r}(t)$ [N]	472570	499991	575429		580330	679970	696330
		$M_{s,r}(t)=N_{c,r}*r+M_{c,r}(t)$ [Nm]	796631	841821	968041		976750	1143960	1175390
		$M_{c,r}(t)$ [Nm]	20450	20286	22853		23210	26760	31240
	365	$N_{c,r}(t)=N_{s,r}(t)$ [N]	363835	387375	447601			581340	605450
		$M_{s,r}(t)=N_{c,r}*r+M_{c,r}(t)$ [Nm]	614791	653771	754943			979410	1024010
		$M_{c,r}(t)$ [Nm]	16973	16946	19489			24270	29180
	730	$N_{c,r}(t)=N_{s,r}(t)$ [N]	276279	295393	342109				
		$M_{s,r}(t)=N_{c,r}*r+M_{c,r}(t)$ [Nm]	467795	499226	578368				
		$M_{c,r}(t)$ [Nm]	13845	13865	16248				

Tab.1: Comparison of results for normal forces $N_{\mathrm{c,r}}$, $N_{\mathrm{s,r}}$ and moments $M_{\mathrm{c,r}}$, $M_{\mathrm{s,r}}$ obtained by various methods; results obtained by numerical methods in time $t_i = 6025$, 6087, 6177, 6362 and 6727 days; results obtained by Wolff's methods [61] in time $t_i = 3584$ days; results obtained by Trost's methods [56], Haenzel's methods [29] and Sonntag's methods [53] in time t_∞

The results for normal forces $N_{\mathrm{c,r}}$, $N_{\mathrm{s,r}}$ and moments $M_{\mathrm{c,r}}$, $M_{\mathrm{s,r}}$ received by the above mentioned method with the results of both methods – of Trost [56] and Haenzel's [29], based theory of the viscoelastic body and the theory of Rüsch-Jungwirt [45](respectively modified theory of Dischinger [21, 22]); as well as with the results of Wolff [61] and Sonntag's methods [53] (according to Dischinger's theory) are compared in Table 1.

The results for normal stresses in upper fiber of concrete plate $\sigma_{\mathrm{c}}^{\mathrm{up}}(t)$ and steel girder $\sigma_{\mathrm{s}}^{\mathrm{up}}(t)$ and in down fiber of concrete plate $\sigma_{\mathrm{c}}^{\mathrm{down}}(t)$ and steel girder $\sigma_{\mathrm{s}}^{\mathrm{down}}(t)$ in time t_∞, when loading is applied in time $t_0 = 28$, 90, 180, 365 and 730 days, received by the above mentioned method are compared in Table 2 with the results of both methods – of Trost [56] and Haenzel's [29], based theory of the viscoelastic body and the theory of Rüsch-Jungwirt [45] (respectively modified theory of Dischinger [21, 22]); as well as with the results of Wolff [60] and Sonntag's methods [53] (according to Dischinger's theory).

8. Summary and Conclusion

Bearing in mind the creep effect, and using the integral equations (7), (9) for constant elasticity module and (12), (15) for time depended elasticity module of concrete, an universal calculating method has been elaborated for statically determinate bridge composite plate girder. The proposed method take into account the vanishing of the creep function for t_0, the beginning of the creep process. This method allows the use of a perfect linear theory of concrete creep i.e. the theory of the viscoelastc body of Maslov-Arutyunyan-Trost-Bažant.

This represents a remarkable step forward in the application of the Volterra integral equations, because in general the creep process is not correct to describe with differential equations.

			$\sigma(t_0)$	Numerical Method (2006) $E_c(\tau)$	Numerical Method (2005) $E_c(\tau)=const$	Numerical Method (1985) $E_c(\tau)=const$	Practical Method (1985) $E_c(\tau)=const$	Trost	Wolf	Haenzel	Profanter, Satler	Sontag
$t_0=28$, $t_\infty=6025$	concrete	σ_c^{up}	- 66.940	-50.988	- 51.22	- 49.71	- 49.30	-49.78 / - 50.09	- 49.52	- 48.42	- 51.50	- 48.01
		σ_c^{down}	-42.791	-42.397	- 41.238	- 40.37	- 40.54	- 40.1 / - 40.93	- 40.3	- 42.62	- 40.50	- 41.03
	steel	σ_s^{up}	-266.067	-793.026	- 820.26	- 894.00	-902.22	- 900.5 / - 864.4	- 902.25	- 865.60	-835.00	- 928.54
		σ_s^{down}	1005.51	1072.90	1076.05	1084.53	1085.60	1085.22 / 1080.9	1083.56	1081.70	1077.00	1089.18
$t_0=90$, $t_\infty=6087$	concrete	σ_c^{up}	- 66.940	-55.214	-54.98	-53.62	-53.27	- 52.80 / - 52.35		- 50.80		- 51.18
		σ_c^{down}	-42.791	-42.757	-42.21	-41.735	-41.91	- 41.70 / - 41.81		- 43.4		- 42.64
	steel	σ_c^{up}	-266.067	-645.552	-669.53	-725.58	-731.77	- 755.17 / - 764.30		- 765.30		- 776.65
		σ_c^{down}	1005.51	1054.125	1057.02	1063.41	1064.26	1067.10 / 1068.41		1069.10		1070.24
$t_0=180$, $t_\infty=6177$	concrete	σ_c^{up}	- 66.940	-57.336	-57.071	-55.84	-55.51	/ - 53.85	- 55.68	- 52.4		- 52.61
		σ_c^{down}	-42.791	-43.034	-42.06	-42.38	-42.55	/ - 42.27	- 42.44	- 43.9		- 43.19
	steel	σ_s^{up}	-266.067	-568.457	-588.56	-634.40	-639.48	/ - 701.7	- 637.69	- 702.6		- 713.31
		σ_s^{down}	1005.51	1044.344	1046.79	1051.53	1052.65	/1060.5	1052.38	1061.2		1062.31
$t_0=365$, $t_\infty=6362$	concrete	σ_c^{up}	- 66.940	-59.239	-58.99	-57.93	-57.62	/ - 55.44		-54.09		- 54.01
		σ_c^{down}	-42.791	-43.2499	-42.98	-42.85	-43.04	/ - 42.66		-44.05		- 43.59
	steel	σ_c^{up}	-266.067	-500.3799	-516.40	-552.52	-556.61	/ - 639.2		-639.9		- 655.60
		σ_c^{down}	1005.51	1035.693	1037.66	1041.60	1012.18	/1052		1051.15		1055.09
$t_0=730$, $t_\infty=6727$	concrete	σ_c^{up}	- 66.940	-60.8587	-60.65	-59.75	-59.48	/ - 57.35		- 56.2		- 55.09
		σ_c^{down}	-42.791	-43.36	-43.16	-43.13	-43.29	/ - 42.93		- 44.17		-43.82
	steel	σ_c^{up}	-266.067	-444.71	-457.18	-484.80	-488.01	/ - 567.42		- 567.8		- 615.83
		σ_c^{down}	1005.51	1028.596	1030.13	1033.00	1033.49	/ 1043.47		1043.9		1050.03

Tab.2: Comparison of results for normal stresses in upper fiber of concrete plate $\sigma_c^{up}(t)$ and steel girder $\sigma_s^{up}(t)$ and in down fiber of concrete plate $\sigma_c^{down}(t)$ and steel girder $\sigma_s^{down}(t)$, obtained by various methods; results obtained by numerical methods in time $t_i = 6025$, 6087, 6177, 6362 and 6727 days; results obtained by Wolff's methods [61] in time $t_i = 3584$ days; Results obtained by Trost's methods [56], Haenzel's methods [29] and Sonntag's methods [53] in time t_∞; all dimensions for stresses are in [daN/cm^2]

The system of integral equations of Volterra is modified by approximating quadrature formulae. In this manner it reduces to triangle algebraic system.

The proposed approach, used in the in the presence of rigid connections and constant cross section allows both a correct interpretation of the viscous elastic problem and solution characterized by a high precision.

Comparing the results calculated by the numerical method [41] and the new improved method elaborated above, and the methods of Trost and Wolff, we can say, that the numerical method gives very reasonable results from a practical point of view. So we can draw the conclusion, that the Trost's method is quite applicable and up to date. That is why, as to a new relaxation coefficient, calculated from the integral equation is not proposed, this method for a long time will be the only one for solving the formulated problem.

Dealing with the methods [29, 42, 48] based on the 'modified' Dischinger's theory, we can say, on the basis of the difference in the results, calculated according to the methods of both groups (confirm Bažant's comment [6]) – that as to the Rusch-Jungwirth's theory no improvement has taken place, especially regarding the approximation of the creep function. The latter should not be recommended for practical use.

From Table 1 it's clearly seen, that according to the proposed method the forces $N_{\mathrm{c,r}}$, $M_{\mathrm{s,r}}$ are lower than the same ones in the other methods. The reason is consequence of the assumptions of the viscoelastic body theory. According to this theory, which takes into account the delayed elastic strain, developing in constrained conditions, it leads to the appearance of recovery of the stresses. They themselves decrease the relaxation of stresses in concrete of composite beams. That's why this fact leads to lower $N_{\mathrm{c,r}}$ and respectively $M_{\mathrm{s,r}}$. So as result we have less stresses in the steal beam, which lead to the more economic designing of composite beam.

Consequently, it should be said that the numerical method, based on the theory of the viscoelastc body, gives more reasonable value of the composite girder behavior, under sustained service and leads to smaller dimensions of the steel beam, especially for time-depended module of elasticity of concrete $E_{\mathrm{c}}(\tau)$.

Notations

E_{s} .... modulus of elasticity of steel,
E_{c} .... instantaneous modulus of elasticity of concrete in time t,
$\varepsilon_{\mathrm{c}}(t)$, $\sigma_{\mathrm{c}}(t)$ ...... strain and stress in concrete, respectively in time t,
t, τ .... time elapsed from casting of concrete, in days,
t_0 .... instant of the first load application,
$K(\tau)$ .... function of τ,
$f(t-\tau)$ .... function of $t-\tau$,
M_{o} .... bending moment of composite section,
$N_{\mathrm{c},0}$, $M_{\mathrm{c},0}$ ...... initial section forces of instant and loading of concrete section or steel section, respectively,
$N_{\mathrm{c,r}}$, $M_{\mathrm{c,r}}$ .... redistribution of section forces, of concrete, of steel section, respectively,
A_{c}, A_{s}, I_{c}, I_{s} ... areas, inertia moments of parts of composite cross-section, respectively,
A_{i} .... transformed area of composite section related to modulus of elasticity of structural steel,
I_{i} .... transformed second moment of area of composite section related to modulus of elasticity of structural steel,
λ .... parameter of Volterra's integral equations,
c, s .... (subscripts) for concrete and steel respectively.

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