ON THE PHENOMENON OF FRACTURE IN LOADING-FREE BODIES

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The author's mesomechanical concept is applied to modeling the phenomenon of fracture in bodies that are neither mechanically nor thermally loaded. According to this model, this phenomenon is caused by rheological redistribution of internal stresses that result from production. Our analysis takes into account two scales:
(i) The first scale distinguishes between the substructure that remains elastic throughout the process, and the other substructure that creeps.
(ii) The other scale distinguishes between two microstructures in the creeping substructure: the creep-yielding microstructure and the creep-resistant microstructure. In the course of the process under discussion, the creep-resistant microstructure is more and more stressed, and then more and more damaged. Its damage can finally lead to an overall fracture.

Key words: *creep, fracture, mesomechanics*

1. Introduction

Many years ago, Professor A. Farlík from the Brno University of Technology drew my attention to his observations of fracture in loading-free bodies, to a phenomenon that was not easy to explain. The metallic bodies in question, and the conditions in which they were stored, excluded the possibilities of thermal or mechanical loading; the influence of irradiation, dehydration or chemical energy was out of question as well. Professor Farlík passed away long ago, but his observations still represent an interesting scientific topic.

In isothermal processes, it is generally assumed that fracture is caused by a supply of mechanical energy to the body. In the case under study, the body – looked upon as a thermodynamic system – is without any input of energy; on the contrary, there is an output of dissipated thermal energy into the surrounding medium. This is the essential difference from the current cases of fracture.

It seems that processes of this kind can well be explained and modeled on the basis of an analysis on the mesoscale. In our publications [2], [3], [5] a general mesomechanical model has been elaborated and applied to different materials and bodies. The main idea consisted in describing a macroscopically homogeneous material as a two-constituent (or many-constituent) medium, in which the differing constituents have their specific mechanical properties, volume fractions, and geometric configurations of their microstructures. On the macroscale, such an approach leads to constitutive equations with tensorial internal variables.

In the case of metallic materials, a relatively simple model medium has been worked out as consisting of two material kinds: the first one characterized by easy glide or creep, the other

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one corresponding to resistant barriers that hinder the inelastic deformation process. It is an essential feature of our mesomechanical approach that the physical nature and microscale description of these two material constituents need not be specified. It is sufficient to assume that they exist, and the necessary model parameters are determined from simple macroscopic experiments performed with specimens of the material in question.

In the current study, this approach is applied to the creep-yielding substructure in loading-free bodies with internal stresses that resulted from their production.

2. General mathematical model for the creep-yielding material constituent

The basic formulae of the model are similar as they appear in our preceding papers [2], [5], [3] :

$$v^{\mathrm{e}}\,\sigma^{\mathrm{e}}_{ij} + v^{\mathrm{n}}\,\sigma^{\mathrm{n}}_{ij} = \bar{\sigma}_{ij}\ , \tag{1}$$

$$v^{\mathrm{e}}\,\varepsilon^{\mathrm{e}}_{ij} + v^{\mathrm{n}}\,\varepsilon^{\mathrm{n}}_{ij} = \bar{\varepsilon}_{ij}\ , \tag{2}$$

$$\dot{e}^{\mathrm{e}}_{ij} = \mu\,\dot{s}^{\mathrm{e}}_{ij}\ , \qquad \varepsilon^{\mathrm{e}} = \varrho\,\sigma^{\mathrm{e}} = \varrho\,\bar{\sigma} = \bar{\varepsilon}\ , \tag{3}$$

$$\dot{e}'^{\mathrm{e}}_{ij} = \dot{e}^{\mathrm{e}}_{ij} - \dot{\bar{e}}_{ij}\ , \qquad \varepsilon'^{\mathrm{e}} = \varepsilon^{\mathrm{e}} - \bar{\varepsilon}\ , \tag{4}$$

$$\dot{e}'^{\mathrm{e}}_{ij} = \mu\,\dot{s}'^{\mathrm{e}}_{ij}\ , \qquad \varepsilon'^{\mathrm{e}} = 0\ , \tag{5}$$

$$\dot{e}^{\mathrm{n}}_{ij} = \mu\,\dot{s}^{\mathrm{n}}_{ij} + \frac{s^{\mathrm{n}}_{ij}}{2\,H^{n}}\ , \qquad \varepsilon^{\mathrm{n}} = \varrho\,\sigma^{\mathrm{n}} = \varrho\,\bar{\sigma} = \bar{\varepsilon}\ , \tag{6}$$

$$\dot{e}'^{\mathrm{n}}_{ij} = \dot{e}^{\mathrm{n}}_{ij} - \dot{\bar{e}}_{ij}\ , \qquad \varepsilon'^{\mathrm{n}} = \varepsilon^{\mathrm{n}} - \bar{\varepsilon}\ , \tag{7}$$

$$\dot{e}'^{\mathrm{n}}_{ij} = \mu\,\dot{s}'^{\mathrm{n}}_{ij} + \frac{s'^{\mathrm{n}}_{ij}}{2\,H^{n}}\ , \qquad \varepsilon'^{\mathrm{n}} = 0\ , \tag{8}$$

$$s^{\mathrm{e}}_{ij} - s^{\mathrm{n}}_{ij} + \frac{s'^{\mathrm{e}}_{ij}}{\eta^{\mathrm{e}}} - \frac{s'^{\mathrm{n}}_{ij}}{\eta^{\mathrm{n}}} = 0\ . \tag{9}$$

If the changes of η^{e}, η^{n} are taken into account, the differential form of the last equation reads :

$$\dot{s}^{\mathrm{e}}_{ij} - \dot{s}^{\mathrm{n}}_{ij} + \frac{1}{\eta^{\mathrm{e}}}\,\dot{s}'^{\mathrm{e}}_{ij} - \frac{s'^{\mathrm{e}}_{ij}}{(\eta^{\mathrm{e}})^2}\,\dot{\eta}^{\mathrm{e}} - \frac{1}{\eta^{\mathrm{n}}}\,\dot{s}'^{\mathrm{n}}_{ij} + \frac{s'^{\mathrm{n}}_{ij}}{(\eta^{\mathrm{n}})^2}\,\dot{\eta}^{\mathrm{n}} = 0\ . \tag{10}$$

The density of elastic energy comprised in the whole material is :

$$W = \frac{1}{2}\left\{\mu\left[v^{\mathrm{e}}\left(s^{\mathrm{e}}_{ij}\,s^{\mathrm{e}}_{ij} + \frac{1}{\eta^{\mathrm{e}}}\,s'^{\mathrm{e}}_{ij}\,s'^{\mathrm{e}}_{ij}\right) + v^{\mathrm{n}}\left(s^{\mathrm{n}}_{ij}\,s^{\mathrm{n}}_{ij} + \frac{1}{\eta^{\mathrm{n}}}\,s'^{\mathrm{n}}_{ij}\,s'^{\mathrm{n}}_{ij}\right)\right] + 3\,\varrho\,\bar{\sigma}^2\right\} \tag{11}$$

and its part comprised in the e-constituent :

$$W^{\mathrm{e}} = \frac{1}{2}\,v^{\mathrm{e}}\left[\mu\left(s^{\mathrm{e}}_{ij}\,s^{\mathrm{e}}_{ij} + \frac{1}{\eta^{\mathrm{e}}}\,s'^{\mathrm{e}}_{ij}\,s'^{\mathrm{e}}_{ij}\right) + 3\,\varrho\,\bar{\sigma}^2\right]\ . \tag{12}$$

The meaning of the symbols is explained in the 'Nomenclature' section.

3. Specific mathematical model

The structures of bodies with the observed loading-free fractures can be very different, but for modeling the gist of the phenomenon a very simple scheme has been chosen and shown in Fig. 1. It consists of a creep-resistant elastic ring (R) (modeling the creep-resistant substructure), and a creep-yielding drawbar (B) (modeling the creep-yielding substructure). It is assumed that due to some process in the way of production the elastic ring has been compressed in a way that corresponds to an effect of two opposite forces (P), leading to displacements δ. This deformed configuration is assumed to be held fixed by a drawbar B. After removing the forces P, there remains tensile stress in the drawbar. The ring is assumed to remain in elastic state throughout the investigated process. The tensile stress in the drawbar is assumed to be high enough to start its creep. Hence, a time-dependent process of decreasing δ and of decreasing tensile stress in the drawbar will start after removal of the forces P. There is no input of energy into the system from the surrounding medium, and the character of the process is relaxation – the stresses in the ring as well as in the drawbar are decreasing. Generally, this is not a situation in which fracture is expected. But yet, under some circumstances fracture can appear even in this case.

The relation between the tensile stress in the bar and the displacement δ is (according to [6]) linear and given by the following formula:

$$\bar{\sigma}_{11} = C\,\delta \ , \tag{13a}$$

and in the process of decreasing δ the decrease of $\bar{\sigma}_{11}$ results as follows:

$$\dot{\bar{\sigma}}_{11} = C\,\dot{\delta} \ , \tag{13b}$$

where:

$$C = \frac{E\,S_\mathrm{R}\,h^2}{3\,\pi\,S_\mathrm{B}\,r^3} \ . \tag{14}$$

The constitutive equation for the creep-yielding bar results from the above-presented set of equations as follows:

$$\dot{\bar{\varepsilon}}_{11} = \dot{\bar{e}}_{11} + \dot{\bar{\varepsilon}} = \mu\,\dot{\bar{s}}_{11} + v^\mathrm{n}\,\frac{s^\mathrm{n}_{11}}{2\,H^n} + \varrho\,\dot{\bar{\sigma}} \ , \tag{15}$$

$$\dot{s}^\mathrm{e}_{11} = \dot{\bar{s}}_{11} + v^\mathrm{n}\,\frac{p\,s^\mathrm{n}_{11} - \eta^\mathrm{e}\,s'^\mathrm{n}_{11}}{2\,H^n\,\mu\,q} - \frac{v^\mathrm{n}\,\eta^\mathrm{e}}{\eta^\mathrm{n}\,q}\,s'^\mathrm{n}_{11}\,\dot{\eta}^\mathrm{n} + \frac{v^\mathrm{n}\,\eta^\mathrm{n}}{\eta^\mathrm{e}\,q}\,s'^\mathrm{e}_{11}\,\dot{\eta}^\mathrm{e} \ , \tag{16}$$

$$\dot{s}^\mathrm{n}_{11} = \dot{\bar{s}}_{11} - v^\mathrm{e}\,\frac{p\,s^\mathrm{n}_{11} - \eta^\mathrm{e}\,s'^\mathrm{n}_{11}}{2\,H^n\,\mu\,q} + \frac{v^\mathrm{e}\,\eta^\mathrm{e}}{\eta^\mathrm{n}\,q}\,s'^\mathrm{n}_{11}\,\dot{\eta}^\mathrm{n} - \frac{v^\mathrm{e}\,\eta^\mathrm{n}}{\eta^\mathrm{e}\,q}\,s'^\mathrm{e}_{11}\,\dot{\eta}^\mathrm{e} \ , \tag{17}$$

$$\dot{s}'^\mathrm{e}_{11} = -v^\mathrm{n}\,\eta^\mathrm{e}\,\frac{\eta^\mathrm{n}\,s^\mathrm{n}_{11} + s'^\mathrm{n}_{11}}{2\,H^n\,\mu\,q} - \frac{v^\mathrm{n}\,\eta^\mathrm{e}}{\eta^\mathrm{n}\,q}\,s'^\mathrm{n}_{11}\,\dot{\eta}^\mathrm{n} + \frac{v^\mathrm{n}\,\eta^\mathrm{n}}{\eta^\mathrm{e}\,q}\,s'^\mathrm{e}_{11}\,\dot{\eta}^\mathrm{e} \ , \tag{18}$$

$$\dot{s}'^\mathrm{n}_{11} = \eta^\mathrm{n}\,\frac{v^\mathrm{e}\,\eta^\mathrm{e}\,s^\mathrm{n}_{11} - (v^\mathrm{n} + \eta^\mathrm{e})\,s'^\mathrm{n}_{11}}{2\,H^n\,\mu\,q} + \frac{v^\mathrm{e}\,\eta^\mathrm{e}}{\eta^\mathrm{n}\,q}\,s'^\mathrm{n}_{11}\,\dot{\eta}^\mathrm{n} - \frac{v^\mathrm{e}\,\eta^\mathrm{n}}{\eta^\mathrm{e}\,q}\,s'^\mathrm{e}_{11}\,\dot{\eta}^\mathrm{e} \ , \tag{19}$$

where for simplification of the form the following formal expressions have been defined:

$$p = v^\mathrm{n}\,\eta^\mathrm{n} + v^\mathrm{e}\,\eta^\mathrm{e} \ , \qquad q = v^\mathrm{n}\,\eta^\mathrm{n} + v^\mathrm{e}\,\eta^\mathrm{e} + \eta^\mathrm{n}\,\eta^\mathrm{e} \ . \tag{20}$$

The evolution equations (16) to (19) of the internal variables are not independent due to the validity of equations (1) and (9), but the above-presented forms are clearer than combinations of all these equations together.

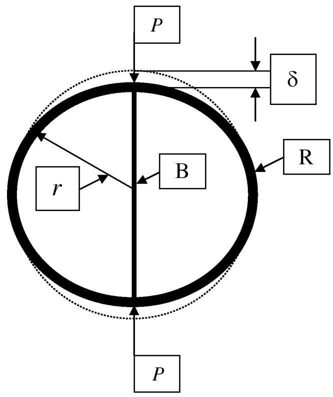

Fig.1: Schematic configuration of interactive parts of the body

It is evident from our scheme shown in Fig. 1 that the value of $\bar{\sigma}_{11}$ monotonically decreases with time, which results from the permanent decrease of δ according to equation (13b). The rates of our symbols are then related as follows:

$$\dot{\bar{\sigma}} = \frac{1}{3}\,\dot{\bar{\sigma}}_{11} = \frac{1}{3}\,C\,\dot{\delta}\;, \qquad \dot{\bar{s}}_{11} = \frac{2}{3}\,\dot{\bar{\sigma}}_{11} = \frac{2}{3}\,C\,\dot{\delta}\;. \tag{21}$$

4. The primary stage of the creep of the drawbar

In our model, the primary stage of creep is described as a process in which the structural parameters $\eta^{\rm e}$, $\eta^{\rm n}$ do not change ($\dot{\eta}^{\rm e} = \dot{\eta}^{\rm n} = 0$), the structure remains essentially undamaged and the changes of the state of the material are described by changes of the internal stress variables $s_{11}^{\rm e}$, $s_{11}^{\rm n}$, $s_{11}^{\prime\rm e}$, $s_{11}^{\prime\rm n}$ only. For a classical process of creep, where $\bar{\sigma}_{11}$ is constant and high enough to cause the creep process, the value of $s_{11}^{\rm e}$ increases and the value of $s_{11}^{\rm n}$ decreases. For a concrete material (cf. Section 5), the evolution of the values of $s_{11}^{\rm e}$ and $s_{11}^{\rm n}$ at the beginning of the *primary* stage of creep is shown in Fig. 2. Thus, the stress in the resistant e-microstructure increases, and the density of its elastic energy $W^{\rm e}$ (cf. equation (12)) increases as well. Similarly as in our previous paper [3], the end of the primary stage is characterized by a specific value of $W^{\rm e}$ that is named $(W^{\rm e})_{\rm EP}$.

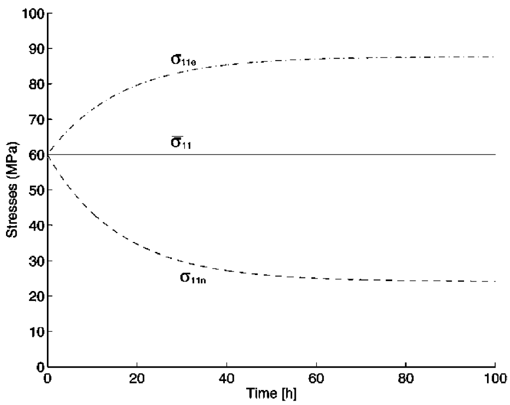

Fig.2: Incipient development of deviatoric stresses in the creep-yielding (n) and creep-resistant (e) microstructures of the material of the drawbar

For the process under study, the value of $(\bar{\sigma}_{11})_0$ at the beginning of the process is assumed to be high enough to start the creep, but it monotonically decreases, i.e. $\dot{\bar{\sigma}}_{11} < 0$ permanently. It is natural to expect that the end of the primary stage (characterized by $(W^{\mathrm{e}})_{\mathrm{EP}}$) steps in after a longer time than with $\dot{\bar{\sigma}}_{11} = 0$.

5. The tertiary stage of the creep of the drawbar

The start of the tertiary stage of creep is characterized by the begin of cumulative damage, which is described in our model by changes of η^{e}, η^{e} and H^n. This is a model approximation that describes the macroscopic behavior of the process. Microscopic damage formation can sometimes be observed even prior to the start of the tertiary stage.

Let us assume that the criterion limiting the strength of the barriers can be described in the simplest way – by von Mises' criterion:

$$s^{\mathrm{e}}_{ij}\, s^{\mathrm{e}}_{ij} \le \frac{2}{3}\, (c^{\mathrm{e}})^2 \ , \tag{22}$$

where c^{e} is some material constant that need not be determined if the end of the primary stage is specified by the value $(W^{\mathrm{e}})_{\mathrm{EP}}$. For an active process, we receive by differentiation of equation (22):

$$s^{\mathrm{e}}_{ij}\, \dot{s}^{\mathrm{e}}_{ij} = 0 \ , \tag{23}$$

which is the relation that is further used for modeling the development of the tertiary stage of creep.

In our case of equal elastic constants, the rates $\dot{\eta}^{\mathrm{e}}$, $\dot{\eta}^{\mathrm{n}}$ are bound by the following simple equation (cf. [4]):

$$\dot{\eta}^{\mathrm{n}} = -\frac{v^{\mathrm{n}}\, (\eta^{\mathrm{n}})^2}{v^{\mathrm{e}}\, (\eta^{\mathrm{e}})^2}\, \dot{\eta}^{\mathrm{e}} \ . \tag{24}$$

With the use of equations (16), (23) and (24) it turns out:

$$\dot{\bar{s}}_{11} + \frac{v^{\mathrm{n}}}{q} \left[\frac{p\, s^{\mathrm{n}}_{ij} - \eta^{\mathrm{e}}\, s'^{\mathrm{n}}_{ij}}{2\, H^n\, \mu} + \dot{\eta}^{\mathrm{e}} \left(\frac{\eta^{\mathrm{n}}}{\eta^{\mathrm{e}}}\, \frac{v^{\mathrm{n}}}{v^{\mathrm{e}}}\, s'^{\mathrm{n}}_{ij} + \frac{\eta^{\mathrm{n}}}{\eta^{\mathrm{e}}}\, s'^{\mathrm{e}}_{ij} \right) \right] = 0 \ , \tag{25}$$

which leads to:

$$\dot{\eta}^{\mathrm{e}} = -\frac{v^{\mathrm{e}}\, \eta^{\mathrm{e}}}{\eta^{\mathrm{n}} (v^{\mathrm{n}}\, s'^{\mathrm{n}}_{11} + v^{\mathrm{e}}\, s'^{\mathrm{e}}_{11})} \left(\frac{q}{v^{\mathrm{n}}}\, \dot{\bar{s}}_{11} + \frac{p\, s^{\mathrm{n}}_{11} - \eta^{\mathrm{e}}\, s'^{\mathrm{n}}_{11}}{2\, H^n\, \mu} \right) \ . \tag{26}$$

Hence, the differential constitutive equation for the tertiary stage of creep is given by equation (15), with the evolution equations for the internal tensorial variables given by equations (16), (17), (18), (19), (24) and (26).

The change of H^n models the cumulative damage in the n-microstructure itself – i.e. a progressive weakening of its inner bonds. Similarly as in our previous paper [3] the change of H^n is assumed to be related to the change of η^{e} by the following form:

$$\frac{\dot{H}^n}{H^n} = -K\, \frac{\dot{\eta}^{\mathrm{e}}}{\eta^{\mathrm{e}}} \ . \tag{27}$$

The above-presented relations describe the course of the *tertiary* stage of creep. In its course the cumulative damage increases and finally is ended by fracture.

The degree of damage is usually measured by a parameter that increases from zero value (undamaged material) to unity (complete damage). It has been shown and substantiated in [2] that such a parameter can be related to our structural parameters as follows:

$$D = \frac{\eta^{\mathrm{e}} - (\eta^{\mathrm{e}})_0}{1 + \eta^{\mathrm{e}} - (\eta^{\mathrm{e}})_0} , \tag{28}$$

where $(\eta^{\mathrm{e}})_0$ is the value in the virgin undamaged state, η^{e} the value in the current state.

According to experimental findings, the value of D at real fracture does not reach full unity but some specific value $(D)_{\mathrm{FR}}$ that is lower than unity, but very near to it.

6. Demonstration for a concrete material

The possibility of fracture depends on the concrete structure, material and temperature. For a demonstration, let us use for the drawbar the material parameters determined in our paper [3] for one heat (labeled I) of steel 0,5Cr0,5Mo0,3V (Czech Standard ČSN 41 5128) that was experimentally investigated in [1] at temperature 600 °C and 60 MPa tensile loading.

It is clear enough that this material cannot be characterized as creep-yielding for current temperatures, and temperature 600 °C is not a commonly met temperature, but the aim of the current study is not a quantitative description of a concrete technical process, but a qualitative demonstration of the essence of the process under study. The demonstration is based on a randomly chosen material, whose model parameters have been determined in our paper [3]. These parameters are given in Table 1.

v^{n}	0.4363	$(H^n)_0$	814360
v^{e}	0.5637	$(W^{\mathrm{e}})_{\mathrm{EP}}$	0.0818
$(\eta^{\mathrm{e}})_0$	10.1233	K	1.4
$(\eta^{\mathrm{n}})_0$	0.0373	$(D)_{\mathrm{FR}}$	0.9924

Tab.1: Model parameters of the investigated material

For a *constant* tensile macroscopic stress $\bar{\sigma}_{11} = 60\,\mathrm{MPa}$, the respective creep curve up to fracture (confronted with experimental data received in [1]) is shown in Fig. 3. The end of the *primary* stage of creep is visualized by symbol ▷, the end of the *tertiary* stage – i.e. fracture – by symbol ∗. Similarly as in our preceding papers – in accordance with experimental findings published in [7] – the *primary* stage of creep is assumed to be followed immediately by the *tertiary* stage, i.e. by a stage accompanied by internal continuum damage of the material. Between the *primary* and the *tertiary* stages rather a minimum strain-rate than a *secondary* state value is generally observed.

In the current study, the main attention is paid to the cases of *decreasing* values of $\bar{\sigma}_{11}$ according to Section 3. The decrease of $\bar{\sigma}_{11}$ is dependent on the value of C (cf. equations (13b) and (14)), which in turn depends on the properties of the structure and of the materials. Three demonstrative examples are shown for three values of C: 0, 0.11 and 0.13 in Fig. 4. The zero value of C means that $\bar{\sigma}_{11}$ is constant. With increasing C, the creep strain at fracture as well as the time at fracture do increase. This process must not be mistaken for cases in which different initial stresses remain constant, where the creep-curves can differ.

It is evident that if the value of C is high enough, no fracture will occur at all, as the drop of $\bar{\sigma}_{11}$ will be so quick that the increasing value of D will never reach up to the critical

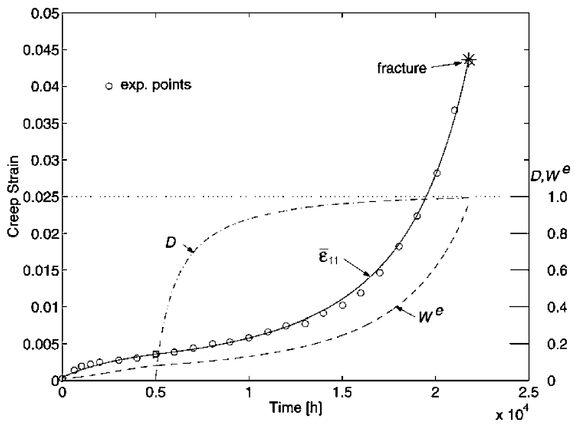

Fig.3: The whole experimental and theoretical course of the creep curve up to fracture under a constant macroscopic loading stress

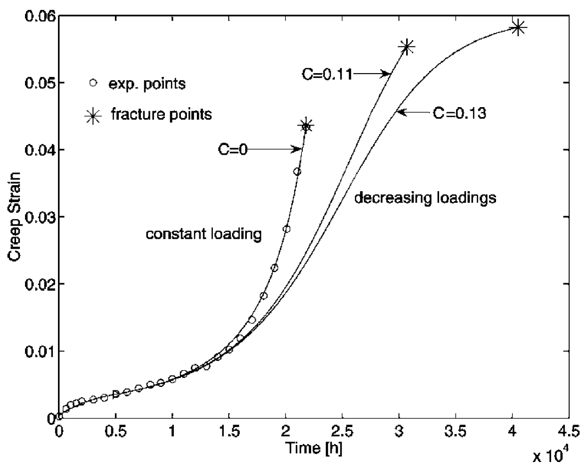

Fig.4: The effect of the decrease of macroscopic loading stress on the creep curve of the drawbar; for $C = 0$, 0.11 and 0.13

value $(D)_{\mathrm{FR}}$. With a good knowledge of the structure of the body, of the material properties of its parts and of their stress state after production, it is then possible to predict the danger of the loading-free fracture.

7. Discussion

With the use of our mesomechanical model, the background of the process of fracturing of loading-free bodies has been described. This process is caused by rheological redistribution of internal stresses resulting from production. Generally, creep leads to relaxation of stresses, and our model describes relaxation as well – on the first scale. However, on the second scale – on the microscale of the creep-yielding substructure, there are two microstructures, where stress decreases in one of them and increases in the other one. Damage of this last named microstructure can result in fracture.

If the body under consideration is looked upon as a thermodynamic system, there is no input of energy from the surrounding medium into the body, which is the essential difference from classical fracture process. On the contrary, there is an output of energy resulting from dissipation of elastic energy in the yielding material. In spite of it, the resulting redistribution of internal stresses can lead to fracture. With a deep knowledge of the properties of the body in question, the basic scheme of this approach can be used for a prediction of the danger of the respective loading-free fracture.

Nomenclature

$\bar{\sigma}_{ij}$ $\{\bar{\varepsilon}_{ij}\}$ – macroscopic stress {strain} in the drawbar
$\sigma^{\rm e}_{ij}/\sigma^{\rm n}_{ij}$ $\{\varepsilon^{\rm e}_{ij}/\varepsilon^{\rm n}_{ij}\}$ – mesoscopic stress {strain} in the elastic/inelastic microstructures in the drawbar – averaged values in the respective material constituent labeled e and n
$\bar{s}_{ij}$ $\{s^{\rm e}_{ij},\, s^{\rm n}_{ij}\}$ – macroscopic {mesoscopic} deviatoric stress
$\bar{e}_{ij}$ $\{e^{\rm e}_{ij},\, e^{\rm n}_{ij}\}$ – macroscopic {mesoscopic} deviatoric strain
$\delta_{ij}\,\bar{\sigma}$ $\{\delta_{ij}\,\sigma^{\rm e},\, \delta_{ij}\,\sigma^{\rm n}\}$ – macroscopic {mesoscopic} isotropic part of the stress tensors
$\delta_{ij}\,\bar{\varepsilon}$ $\{\delta_{ij}\,\varepsilon^{\rm e},\, \delta_{ij}\,\varepsilon^{\rm n}\}$ – macroscopic {mesoscopic} isotropic part of the strain tensor
δ_{ij} – Kronecker delta
$\varepsilon'^{\rm e}_{ij} = \varepsilon^{\rm e}_{ij} - \bar{\varepsilon}_{ij}$ $\{\varepsilon'^{\rm n}_{ij} = \varepsilon^{\rm n}_{ij} - \bar{\varepsilon}_{ij}\}$ – definition of variables $\varepsilon'^{\rm e}_{ij}$ $\{\varepsilon'^{\rm n}_{ij}\}$
$e'^{\rm e}_{ij}$ $\{e'^{\rm n}_{ij}\}$ – deviatoric parts of $\varepsilon'^{\rm e}_{ij}$ $\{\varepsilon'^{\rm n}_{ij}\}$
$\delta_{ij}\,\varepsilon'^{\rm e}$ $\{\delta_{ij}\,\varepsilon'^{\rm n}\}$ – isotropic parts of $\varepsilon'^{\rm e}_{ij}$ $\{\varepsilon'^{\rm n}_{ij}\}$
$\sigma'^{\rm e}_{ij}$ $\{\sigma'^{\rm n}_{ij}\}$ – stresses related to $\varepsilon'^{\rm e}_{ij}$ $\{\varepsilon'^{\rm n}_{ij}\}$ similarly as are $\sigma^{\rm e}_{ij}$ $\{\sigma^{\rm n}_{ij}\}$ related to $\varepsilon^{\rm e}_{ij}$ $\{\varepsilon^{\rm n}_{ij}\}$
$s'^{\rm e}_{ij}$ $\{s'^{\rm n}_{ij}\}$ – deviatoric parts of $\sigma'^{\rm e}_{ij}$ $\{\sigma'^{\rm n}_{ij}\}$
$\delta_{ij}\,\sigma'^{\rm e}$ $\{\delta_{ij}\,\sigma'^{\rm n}\}$ – isotropic parts of $\sigma'^{\rm e}_{ij}$ $\{\sigma'^{\rm n}_{ij}\}$
i, j – indices that can take on the values $1, 2, 3$; their repetition means summation
W – density of elastic energy comprised in the drawbar
$W^{\rm e}$ – density of elastic energy comprised in its e-constituent
$(W^{\rm e})_{\rm EP}$ – the value of $W^{\rm e}$ at the end of the primary stage of creep in the drawbar
$v^{\rm e}$, $v^{\rm n}$, $(v^{\rm e} + v^{\rm n} = 1)$ – volume fractions of the respective material constituents
$\eta^{\rm e}$, $\eta^{\rm n}$ – non-negative 'structural parameters' that characterize the structure of the material of the drawbar; $\eta^{\rm e} = \eta^{\rm n} = 0$ correspond to Voigt's homogeneous strain model, $\eta^{\rm e} = \eta^{\rm n} = \infty$ correspond to Reuss' homogeneous stress model; generally, an increase {decrease} of one of these parameters describes a decrease {increase} of connectivity of the respective substructure
E – Young's modulus of the material of the drawbar
ν – Poisson's ratio
$\mu = (1 + \nu)/E$ – deviatoric elastic compliance
$\varrho = (1 - 2\,\nu)/E$ – isotropic elastic compliance
t – time
$c^{\rm e}$ – limit value of $s^{\rm e}_{11}$ at the end of the primary stage of creep of the drawbar
p, q – formal expressions defined by equations (20); used for simplification of the equations
H^n – coefficient of viscosity of the n-constituent of the drawbar
K – coefficient relating the change of H^n to the change $\eta^{\rm e}$
R – creep-resistant ring
B – creep-yielding drawbar
$S_{\rm R}$ – rectangular cross-section of the ring
C – formal expression given by equation (14)
δ – displacements of the upper and lower points of the creep-resistant ring (cf. Fig. 1)
r – radius of the ring

h – thickness of the ring
S_{B} – cross-section of the drawbar
D – degree of damage of the drawbar
$(D)_{\mathrm{FR}}$ – value of D at the point of fracture of the drawbar

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