AN ATTEMPT TO MODELLING THE CRUDE OILS FLOW IN A PIPE WITH THE MPTT MODELS

Madjid Meriem Benziane*, Abdelkrim Liazid**, Benyebka Bousaid†, Olivier Bonneau‡

The crude oil is a non-Newtonian fluid presenting, during its flow, phenomena which cannot be represented by the traditional models which considers the homogeneous fluid hypothesis. Hence, it is necessary to take into account the microstructure characteristics of crude oil in the behaviour law, what is more realistic. The application of a non-linear differential model in the case of crude oil flow won't be adopted. This paper deals with the modelling approach resulting from rheology of the polymer solutions. The Model of Phan-Thien and Tanner (MPTT) is considered in this paper. However, this model presents certain restrictive assumptions which seem non representative of the observed phenomena. The impact of the modification of one of these assumptions was studied and no modification on the computed stress results was observed. The obtained results of stresses are presented and discussed.

Key words : *waxy crude oils, rheology, modelling, constraints*

1. Introduction

The relation between the rheological properties and the structure of waxy crude oils cooled in quiescent conditions and under flow is a fundamental knowledge. The crude oil flow generates normal and tangential stresses. In order to improve the wall behaviour of the technical installations in the transport and refinement domain of the crude oil, rheological researchers need to know these constraints. As measurements in vivo are not realizable, it is thus interesting to resort to the modelling of the crude oil flow, allowing, in the most realistic possible context, to evaluate physical properties, like shear stresses being exerted on the level of the wall. The crude oil flow is very particular and cannot be represented by the traditional models. The simplest approach is to consider the crude oil as a homogeneous fluid : these are the models of viscosity whose characteristic is only to take into account the viscosity variation according to the rate of shearing. The theory of the diluted suspensions constitutes the first stage towards the consideration of the micro-structural characteristic of crude oil which is considered as a solution of rigid particles in different composites. The non-linear differential model make possible to consider the unstable character of the flow. Lastly, a very significant branch of rheology is interested in the solutions of polymers: there are many analogies in the flow behaviour between these polymer solutions and crude oils. There are two theories, rising from this branch, allowing a micro-structural analysis: The Dumbbell theory and the network analysis. The model of Phan-Thien and Tanner [1], resulting from the network

---

* M. Meriem Benziane, Département de Génie Mécanique, Université Hassiba Ben-Bouali, BP. 151, Chlef, Algérie

** A. Liazid, Laboratoire LTE, ENSET-Oran, BP 1523, El Mnaouer 31000-Oran, Algérie

† B. Bousaid, Laboratoire de Mécanique des Contacts et Solides, INSA de Lyon, France

‡ O. Bonneau, Laboratoire de Mécanique et des Solides, Université de Poitiers, France

analysis, is studied here. Some assumptions, related to the creation and the destruction of the micro-structural connections as well as the space distribution seems approximate and non-representative of the observed phenomena. A new contribution called modified MPTT model is proposed to avoid these assumptions.

2. Interest of the study

The crystallisation of wax in crude oils has been investigated for more than 30 years, first because of the severe difficulties that it causes in pipelining and storage and second because evaluation of the flow properties of oils containing wax is difficult. Initially, poor agreement was found between different rheological measurements and between laboratory and pilot plan data. During the last years, however, substantial progress has been made thanks to the efforts for the determination of the appropriate measuring techniques and to the systematic study of the physical parameters affecting the rheology. The role of the shear history on the flow properties, in particular, was clearly put into evidence. At high temperatures, the crude oils containing linear paraffins behave like Newtonian liquids, the paraffins being in the molten state. Below the Wax Appearance Temperature (WAT), which corresponds to incipient crystallisation of the paraffins during cooling in static conditions, oils turn into gels? The amount of solid wax responsible for gelation was recognised to be very low, around 2–4 % [2]. Cooling under flow leads to different states, which are non-Newtonian fluids, with a very rapid increase in viscosity below the WAT.

3. Crude oil description

Crude oil is a complex fluid in perpetual renewal, ensuring of multiple functions essential to the industrial maintenance. Different composite is an aqueous solution (60–70 %). Water occupy about (30–40 %) of the total volume of crude oil.

Deformability is, with the capacity of stacking, a significant characteristic in the dynamics of the crude oil flow. The methodology for measuring the rheological properties is now well established. To understand the complex behaviour of waxy crude oils, a more fundamental knowledge of the relationship between microstructure and rheology is, however, necessary. In addition, the influence of the molecular composition on the mechanisms of crystallisation should help to explain the differences between crude oils from various sources. This investigation was based on the following guide lines:

- In static conditions of cooling, a precise determination of the storage and loss moduli, measured under small deformations can be undertaken. The storage moduli of the gelled oils are in general very high, (106 Pa) in the range of temperatures between 08C and the room temperature. One can establish experimentally the relation between the amount of crystals and the shear moduli for oils with different molecular compositions.
- The morphology of the crystals is an important aspect which is in direct relation with the rheology of crude oils and which is determined by the crystallisation process. The rheological behaviour of the studied crude oil is shown in the figure 1.

The figure 2 shows the general shape of the viscosity evolution of crude oil [2]. This non linearity characterizes the non-Newtonian phenomena.

In addition to the characteristics of viscosity, other non-Newtonian phenomena characterize the behaviour of crude oil.

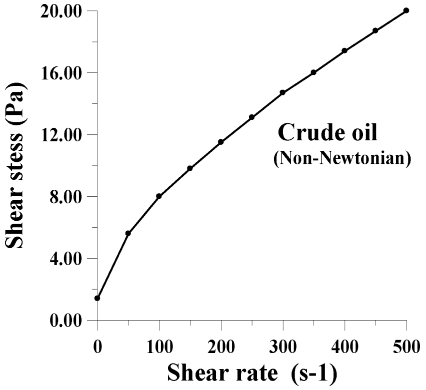

Fig.1: Rheological behaviour of crude oil [2]

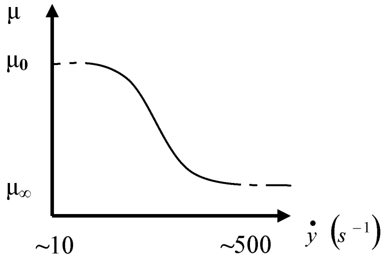

Fig.2: Viscosity of crude oil according to the rate of shearing

First of all, crude oil has the essential characteristic of the non-Newtonian fluids to generate normal stresses differences (or stress overshoot) during a flow in shearing. With the notations of figure 3, the first and second stresses differences are defined by [3] :

$$N_1 = \tau_{\mathrm{xx}} - \tau_{\mathrm{yy}} \ , \qquad N_2 = \tau_{\mathrm{yy}} - \tau_{\mathrm{zz}} \ .$$

In addition, when the viscous stresses are week, the different composite agglomerate face to face by forming rollers figure 2, and the presence of these rollers deeply modifiers the rheological behaviour of crude oil [2]. The formation of the rollers would be due to electric phenomena. The rollers assemblies are broken easily under the action of low shear stresses.

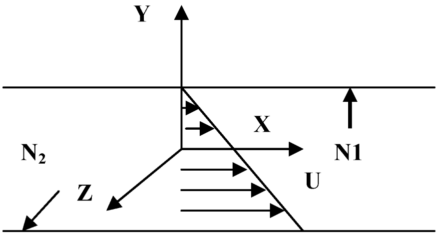

Fig.3: Normal stresses differences

4. Phan-Thien and Tanner model

The modern lubricants incorporate a broad variety of additives; the purpose is to improve the properties of basic oils of mineral origin. The addition of these polymers leads to a modification of the rheological behavior of the lubricant: a linear relation between the rates of deformations and the stresses is not enough any more to describe the lubricant. Its behavior is non-Newtonian what is mainly illustrated by the shear thinning effect. The additives imply viscoelastic effect. First of all let us point out the various choices of viscoelastic laws to describe the behavior of the lubricant. The behavior of the lubricant can be approached in two ways :

- By considering the lubricant as continuous, viscoelastic medium: Theory of the continuous mediums.
- Analysis the behavior of the long macromolecular chains, responsible for the viscoelastic effects: Microstructural analysis.

The theory of the continuous mediums is at the origin of the determinism principles. The approach of the continuous mediums does not distinguish the microstructure from material. The constitutive laws, formulated starting from the only principles quoted previously, are applicable to any type of fluid. For the microstructural analysis, the step followed by these theories to obtain the constitutive law has for major aims :

- To clarify the different microstuctural mechanisms which constitute the formulation of the behaviour law.
- To provide a physical justification of the terms appearing in the behaviour law.
- To define the limits of the behavior law. The modeling of the microstructure leads to various concepts according to whether the fluid is considered as a diluted polymer solution, concentrated polymer solution or even as dispersion :
  - In the theory of diluted solutions, the long polymer chains are modeled by springs which characterize the deployment of the molecule. Hydrodynamic resistance due to the movement of the solvent (basic oil) and the Brownian resistance (thermal agitation), caused by the random movement of the molecule, are applied at each end of the spring. The polymer chains are supposed not to have any interaction between them, and are influenced only by the flow of solvent Dumbbell theory [4].
  - The theories of concentrated solution seek on the contrary to take into account the entanglements of the polymer chains. One regards the lubricant as a polymer network bathing in a solvent. The junctions of the girl are not clearly identified and can represent nodes or loops between the various polymer chains. During the flow, certain junctions disappear but evolution leads to the formulation of the constitutive equation : Network analysis.

The applicability of these various theories depends on the concentration and the length of the polymer chains.

The model of Phan-Thien and Tanner belongs to the network analysis. This model was developed for the study of lubricant subjected to sudden requests.

Under this type of stress, the lubricant loses its Newtonian character : its viscosity falls for raised rates of shearing, and stress overshoots are observed.

These properties thus show the possibility for interesting resemblances to the crude oil flow.

4.1. Basic elements of the network analysis [7]

The network analysis of the macromolecular solids, the chemical articulation are described like points or junctions on the level of which, potions of molecules are always obliged to move together. By adapting this theory to the liquids, it is supposed that the junctions are not permanent but are continuously being created and destroyed. The principal approximation in the network analysis is the fact that the interactions between polymers are localized on the level of the junctions. A junction is a place of strong attraction between two isolated point which each one belongs to a different polymeric chain, and which are obliged to coincide. A macromolecular chain joining two successive junctions is called ‘segment’. A macromolecular chain connected to only one junction is called ‘loose end’. A macromolecular chain not attached to the network is called ‘parasitic chain’. The totality of the segments forms the molecular network. Let us recall that these junctions depend on time. The figure 4 shows a part of the macromolecular network.

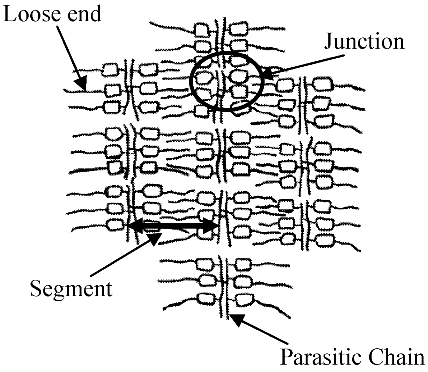

Fig.4: Representation of the network

The modelling of this network is based on the following consideration: The polymer chains which are not segments, the parasitic chain and loose ends are omitted. So, the figure 5 shows the model representation of the network.

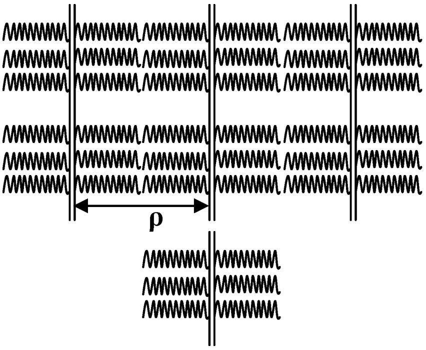

Fig.5: Modelling of the network

There are six assumptions common to the network theories for both solids and liquids:

1) A concentrated polymer solution can be represented by a molecular network with polymer-polymer interactions occurring only as local interactions at isolated points along the chains. Only this network is responsible for the rheological behaviour of the material.
2) The polymer solution is assumed to be incompressible.
3) The network junctions move affinely.
4) The phase space distribution function for each segment is equilibrated at all times.
5) The polymer solution is subjected to either a homogeneous deformation or a homogeneous flow.
6) Each segment of the network can be modelled as a Gaussian chain.

These six assumptions cannot be modified without changing the theory basically. The network analysis for the solids rests essentially upon these six assumptions. For the liquids one adopts the following additional assumptions:

- Segments are lost and created during the flow. The network will consist of segments with a distribution of age.
- The stress is assumed to be the sum of contribution from all the existing segments at the present time, these having been created at various past times.

- It is supposed that the distribution function for the segments at the moment of creation is identical to the equilibrium distribution function for a freely jointed chain with no constraints on the end points.
- Segments can be characterized by only one positive integer I that indicates the 'complexity' of the segment.

For fluid not much is know about the mechanism for formation and loss of the temporary entanglements. This is why the theory focuses on the segments. It is expected that many different kinds of physical entanglements can be formed and that these will exhibit a wide range of degree of permanence. The idea of segment complexity is used to describe the relative permanence of different segments. One distinguishes the segments resulting from temporary physical entanglements with those formed between permanent chemical cross links. For the solids, the segments have all same complexity since they are all permanent. For the fluid the complexity of segment is tied to the topology of the 'knot' formed by Brownian motion and which constitutes the junction.

4.2. Basic formulation [5]

Isothermal flow is considered. A chains part of molecule located between two junctions. At each time, one can find chains of various lengths in an element of volume. The length of the chain is represented by the number density of subunits. N_i is the number density of chains containing i subunits (i-chains). A chain is represented by its number of subunits and by the vector ϱ between its endpoints. $\psi_i(\varrho)$ is the distribution function of end-to-end vectors. $\psi_i(\varrho)\,\mathrm{d}^3\varrho$ is the number of chains per unit volume having i subunits and with a chain vector in a neighbourhood $\mathrm{d}^3\varrho$ around ϱ. The flow of the solution introduces a deformation of the network and then a destruction of junctions thus of chains, just like a formation of new junction and chains. We are going to describe this process. For that, in a unit of volume, one has to take into account:

- The number of i-chains which leave and which enter the volume during the interval $\mathrm{d}t$.
- The rate of formation k_i of these i-chains.
- The rate of disappearance l_i of these i-chains.
- The total derivative of ψ is the equal to the rate of creation minus the rate of destruction.
- The balance equation for the distribution function of the chains containing i segments is:

$$\frac{\mathrm{d}\psi_i}{\mathrm{d}t} = \frac{\partial}{\partial\varrho}\left(\psi_i\,\dot{\varrho}\right) + k_i - l_i\ . \tag{1}$$

The model of Phan-Thien and Tanner is different from the other models of the network theory by taking for assumption a non-affine deformation of the polymer chains within solvent. The viscosity $\dot{\varrho}$ can be related to the end-to-end vector ϱ through a transformation. If the motion of the junctions coincides with the motion of the material particles of the equivalent macroscopic continuum, then the transformation is linear: $\dot{\varrho} = L\,\varrho$. L is the gradient speed tensor of the macroscopic flow ($\nabla\nu$). The development is unchanged if the following non-affine deformation is employed:

$$\dot{\varrho} = (L - \xi\,D)\,\varrho = \zeta\,\varrho\ . \tag{2}$$

D is the deformation rate tensor (the symmetrical part of L). ξ is a measure of the deformation-induced slip between the network and the continuum. ζ is the speed gradient tensor of the local flow.

It is necessary to specify the rates k_i and l_i. The rate of disappearance is taken to be proportional to the number of the present chains:

$$l_i = h_i\, \psi_i \ . \tag{3}$$

h_i is a function of the chain size, configuration and local flow parameters. The formation of the segments follows a Brownian process. The rate of creation is proportional to the equilibrium distribution function $k_i = g_i\, \psi_{i,0}$.

It is assumed here that the flow will reduce the rate of creation of these junctions. This reduction is represented by a term that is first order in ψ_i:

$$k_i = g_i\, \psi_{i,0} - \alpha_i\, \psi_i \ . \tag{4}$$

g_i and α_i depend on the same variables as h_i.

At equilibrium, in the lack of any deformation, the rates of creation and destruction must be equal:

$$h_{i,0} = g_{i,0} - \alpha_{i,0} \ . \tag{5}$$

One then agrees to express h_i

$$h_i = (g_i - \alpha_i) + p_i \ , \tag{6}$$

$p_{i,0} = 0$.

With the specific constitutive assumptions made above, the equilibrium equation for the distribution function (1) becomes:

$$\frac{\mathrm{d}\psi_i}{\mathrm{d}t} = \frac{\partial}{\partial \varrho}\,(\psi_i^*\, \zeta\, \varrho) + g_i\,(\psi_{i,0} - \psi_i) - p_i\, \psi_i \ . \tag{7}$$

The number density N_i of i-chains is obtained by integration of the distribution function over the configuration space:

$$N_i = \int \psi_i\, \mathrm{d}^3\varrho \ . \tag{8}$$

According to Phan-Thien and Tanner, it is assumed that the parameters g_i, p_i and ξ depend at most on the distribution. In this case the integral of the equilibrium equation (7) gives the kinetic equation:

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = g_i\,(N_{i,0} - N_i) - p_i\, N_i \ . \tag{9}$$

It is convenient to define the fraction relating to the equilibrium of i-chains:

$$x_i = \frac{N_i}{N_{i,0}} \ . \tag{10}$$

One obtains then:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = g_i\,(1 - x_i) - p_i\, x_i \ . \tag{11}$$

The equation containing stresses is obtained by multiplying the equilibrium equation (7) by the dyadic $\varrho_i\, \varrho_i$ and integrating over the configuration space:

$$\frac{\mathrm{D}}{\mathrm{D}t}\langle \varrho_i\, \varrho_i \rangle = \frac{g_i}{x_i}\,(\langle \varrho_{i,0}\, \varrho_{i,0}\rangle - \langle \varrho_i\, \varrho_i \rangle) \tag{12}$$

with

$$\frac{\mathrm{D}}{\mathrm{D}t}(\,) = \frac{\mathrm{d}}{\mathrm{d}t}(\,) - \zeta\,(\,) - (\,)\,\zeta^t = \text{non affine differential operator.}$$

The fluid being incompressible, the constraints are given with an arbitrary isotropic pressure p_i.

$$\pi_i = \tau_i + p_i \ . \tag{13}$$

One wants to calculate τ_i. The theory of the molecular kinetics provides the following expression:

$$\tau_i = N_i\,\frac{3\,k\,T}{i\,a^3}\,\left(\langle\varrho_i\,\varrho_i\rangle - \langle\varrho_{i,0}\,\varrho_{i,0}\rangle\right) \ . \tag{14}$$

To determine $\langle\varrho_{i,0}\,\varrho_{i,0}\rangle$, it is necessary to define the distribution function of the segments at the initial moment taken as equilibrium. It is considered that the state of the initial constraints is isotropic. It is supposed that the function of distribution $\psi_{i,0}$ has the following form:

$$\psi_{i,0} = \left(\frac{3}{2\pi\,i\,a^3}\right)^{\frac{3}{2}} \exp\left(-\frac{3}{2\pi\,i\,a^2}\,\varrho^2\right) \ . \tag{15}$$

Hence:

$$\langle\varrho_{i,0}\,\varrho_{i,0}\rangle = \left(\frac{i\,a^2}{3}\right) I \ . \tag{16}$$

With I the unit tensor and $i\,a^2$ the end-to-end distance of a segment. k is the Boltzman constant and T the temperature.

By taking into account the equation (14) and (16), one can rewrite the equation (12):

$$\frac{x_i}{g_i}\,\frac{\mathrm{D}}{\mathrm{D}t}\,\frac{\tau_i}{N_i\,k\,T} + \frac{\tau_i}{N_i\,k\,T} = \frac{x_i}{g_i}\,2\,(1-\xi)\,D \ . \tag{17}$$

The total extra stress is given as the sum of the partial stresses:

$$\tau = \sum \tau_i \ . \tag{18}$$

The relaxation time is given by:

$$\lambda_i = \frac{1}{g_i + p_i} \ . \tag{19}$$

This relaxation time involves both the time constant of network destruction and the retardation of net work build-up associated with the deformation.

4.3. Phan-Thien and Tanner

Two assumptions are made. The first assumption is a pseudo-steady state of the structure:

$$\frac{\mathrm{d}N}{\mathrm{d}t} \approx 0 \ . \tag{20}$$

So, according to the equation (9):

$$N_i \approx \frac{g_i}{g_i + p_i}\,N_{i,0} \ . \tag{21}$$

This assumes that structural transients are slow relative to a time scale of time equal to the relaxation time. One can rewrite the equation of stresses (7) :

$$\frac{\mathrm{D}\tau_i}{\mathrm{D}t} + (g_i + p_i)\,\tau_i = \frac{g_i}{g_i + p_i}\,N_{i,0}\,k\,T\,2\,(1-\xi)\,D\ . \tag{22}$$

The relaxation time can be written according to the stresses :

$$(g_i + p_i) = \frac{\sigma_i(\mathrm{tr}\,\tau_i)}{\lambda_{i,0}}\ . \tag{23}$$

The second assumption, which is not fundamental in terms of basic behaviour, is that the functions g_i and $g_i + p_i$ are in a constant ratio :

$$\frac{g_i}{g_i + p_i} = r_i = cte\ . \tag{24}$$

With equations (23) and (24), the equation (22) can be written :

$$\lambda_{i,0}\,\frac{\mathrm{D}\tau_i}{\mathrm{D}t} + \sigma(\mathrm{tr}\,\tau_i)\,\tau_i = 2\,\lambda_{i,0}\,G_{i,0}\,D \tag{25}$$

with : $G_{i,0} = r_i\,(1-\xi)\,N_{i,0}\,k\,T$.

One obtains the constitutive equation of the model of Phan-Thien and Tanner (PTT). Phan-Thien uses :

$$\sigma_i = \exp\left(\in\frac{\mathrm{tr}\,\tau_i}{G_{i,0}}\right)\ .$$

It is necessary to make a light modification to the model of Phan-Thien and Tanner in order to take into account the shear thinning effect. The shear thinning effect described the fall of viscosity at high rate of shearing. One obtains the constitutive equation of the model of modified Phan-Thien and Tanner (MPTT) [1] :

$$\lambda_{i,0}\,\frac{\mathrm{D}\tau_i}{\mathrm{D}t} + \sigma(\mathrm{tr}\,\tau_i)\,\tau_i = 2\,\eta_\mathrm{m}\,D \tag{26}$$

with η_m the viscosity given by the law of Carreau [3] :

$$\eta_\mathrm{m} = \eta_\mathrm{p}\,\frac{1+\xi\,(2-\xi)\,\lambda^2\,\dot{\gamma}^2}{(1+\Gamma^2\,\dot{\gamma}^2)^{(1-n)/2}} \tag{27}$$

where η_p is the zero-rate shearing viscosity of the crude oil, λ is the relaxation time of the fluid, Γ is a time dimension parameter, fixed numerically to $11.08\,\lambda$, n is the power law index ($n < 1$).

4.4. Modifications to be made

The model of MPTT is today, the only model able to take into account such a broad number of specificities of crude oil and crude oil circulation. This model contains nevertheless a given number of limitations and points which do not seem to correspond as well as possible to reality the case of crude oil flow :

- Rate of creation and rate of destruction independent of time.
- A short time of request implying negative shear stresses.

- Restrictive assumption with regard to the statistical distribution : the Gaussian distribution used here is in contradiction with the fact that element are in movement.
- The first point results from the first assumption taken to write the model of PTT : $\mathrm{d}N/\mathrm{d}t \approx 0$.

However, in an aneurysm, there are phenomena of agglomerate which cannot be represented by this assumption. We will try to rewrite a new model on the basis of Phan-Thien and Tanner model, by dismissing this assumption $\mathrm{d}N/\mathrm{d}t \approx 0$ and by taking a number of chains N_i variable according to time. It means that one considers the rates of creation and destruction varying in time. By applying this new model to the case of the crude oil in the pipe, we hope to obtain results closer to reality.

4.5. Modification of the first assumption

Lets us set out again of the equation (17) :

$$\frac{x_i}{g_i}\frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i\,k\,T} = \frac{x_i}{g_i}\,2\,(1-\xi)\,D \ . \tag{28}$$

Let us bring some change to the form of this equation. We have :

$$\frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i} + \frac{g_i}{x_i}\frac{\tau_i}{N_i} = k\,T\,2\,(1-\xi)\,D \ .$$

However, one has (7) : $x_i = N_i/N_{i,0}$ from where :

$$\frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i} + \frac{g_i\,N_{i,0}}{N_i^2}\,\tau_i = k\,T\,2\,(1-\xi)\,D \ . \tag{29}$$

We wish to have a non-nil variation of the number of chains : $\mathrm{d}N_i/\mathrm{d}t \neq 0$. We thus remove the first assumption of the model of Phan-Thien and Tanner, the assumption (20).

The evolution of the number of chain in time is given by the equation (9) :

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = g_i\,(N_{i,0} - N_i) - p_i\,N_i \ .$$

This gives us :

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} + N_i\,(g_i + p_i) = g_i\,N_{i,0} \ . \tag{30}$$

We will modify the form of this equation (30).

According to the second assumption of the model of Phan-Thien and Tanner, (24), one has :

$$\frac{g_i}{g_i + p_i} = r_i = cte \ .$$

In addition, know that :

$$\lambda_i = \frac{1}{g_i + p_i}$$

from where :

$$\lambda_i\,\frac{\mathrm{d}N_i}{\mathrm{d}t} + N_i = r_i\,N_{i,0} \ . \tag{31}$$

According to (19), (23) and (25):

$$\lambda_i = \frac{\lambda_{i,0}}{\sigma_i(\operatorname{tr}\tau_i)} = \frac{\lambda_{i,0}}{\exp\left(\in \dfrac{\operatorname{tr}\tau_i}{G_{i,0}}\right)} \ . \tag{32}$$

The new model is thus reduced to the following system:

$$\begin{gathered} \frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i} + \frac{g_i\,N_{i,0}}{N_i^2}\,\tau_i = k\,T\,2\,(1-\xi)\,D\ , \\ \lambda_i\,\frac{\mathrm{d}N_i}{\mathrm{d}t} + N_i = r_i\,N_{i,0} \end{gathered} \tag{33}$$

with:

$$\lambda_i = \frac{\lambda_{i,0}}{\exp\left(\in \dfrac{\operatorname{tr}\tau_i}{G_{i,0}}\right)} \ .$$

We will call this model PTT2.

We now wish to use the shear thinning effect in the new model resulting from PTT2. Let us recall that this shear thinning effect represents a decrease of viscosity to rate of high shearing. Let us set out again of the system (33):

$$\begin{gathered} \frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i} + \frac{g_i\,N_{i,0}}{N_i^2}\,\tau_i = k\,T\,2\,(1-\xi)\,D\ , \\ \lambda_i\,\frac{\mathrm{d}N_i}{\mathrm{d}t} + N_i = r_i\,N_{i,0} \end{gathered}$$

with:

$$\lambda_i = \frac{\lambda_{i,0}}{\exp\left(\in \dfrac{\operatorname{tr}\tau_i}{G_{i,0}}\right)} \ .$$

We wish to use the viscosity η_m given by the law of Carreau in the equation (27).

$$\eta_\mathrm{m} = \eta_\mathrm{p}\,\frac{1+\xi\,(2-\xi)\,\lambda^2\,\dot\gamma^2}{(1+\Gamma^2\,\dot\gamma^2)^{(1-n)/2}}$$

We thus will reveal in the system (27) the product $\lambda_{i,0}\,G_{i,0}$. We know that:

$$G_{i,0} = r_i\,(1-\xi)\,N_{i,0}\,k\,T\ .$$

We obtain:

$$\frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i} + \frac{g_i\,N_{i,0}}{N_i^2}\,\tau_i = 2\,D\,\lambda_i\,G_{i,0}\,\frac{1}{r_i\,N_{i,0}\,\lambda_{i,0}}\ .$$

As for the development between model PTT and model MPTT, we obtain:

$$\frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i} + \frac{g_i\,N_{i,0}}{N_i^2}\,\tau_i = 2\,\eta_\mathrm{m}\,D\,\frac{1}{r_i\,N_{i,0}\,\lambda_{i,0}}\ .$$

We thus obtain the following system

$$\begin{aligned} \frac{\mathrm{D}}{\mathrm{D}t}\frac{\tau_i}{N_i} + \frac{g_i\,N_{i,0}}{N_i^2}\,\tau_i &= 2\,\eta_{\mathrm{m}}\,D\,\frac{1}{r_i\,N_{i,0}\,\lambda_{i,0}}\ , \\ \lambda_i\,\frac{\mathrm{d}N_i}{\mathrm{d}t} + N_i &= r_i\,N_{i,0} \end{aligned} \tag{34}$$

with:

$$\lambda_i = \frac{\lambda_{i,0}}{\exp\left(\in \dfrac{\mathrm{tr}\,\tau_i}{G_{i,0}}\right)}\ .$$

This new model will be called MPTT2

5. Application

We will apply the models MPTT and MPTT2 to the crude oil in the pipe.

We consider the axisymmetric problem, with a parabolic velocity profile as schematized by the figure 6.

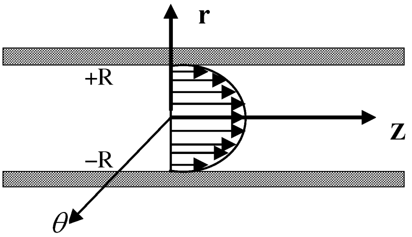

Fig.6: Flow velocity profile

While considering a pulsed flow, the instantaneous velocity is expressed as [5, 6]:

$$u_{\mathrm{z}} = U_{\max}\left(1 - \frac{r^2}{R^2}\right)\cos(w\,t)\ , \qquad u_{\mathrm{r}} = u_\theta = 0\ .$$

From the velocity profile, we can calculate the gradient rate tensor of the global flow L, the gradient rate tensor of the local flow ζ, the macroscopic rate of deformation tensor D.

$$L_{ij} = \frac{\partial u_i}{\partial u_j}\ , \qquad D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial u_j} + \frac{\partial u_j}{\partial u_i}\right)\ , \qquad \zeta = L - \xi\,D\ .$$

These tensors are introduced in the constitutive equation of the studied models to obtain a system of differential equation. This type of system can be solved numerically. In order to be able to compare the results obtained with former work, dimensionless variables are defined by:

$$\tau^* = \frac{\tau}{A}\ , \qquad t^* = \frac{t}{\lambda_{i,0}}\ , \qquad r^* = \frac{r}{R} \qquad \text{with} \qquad A = \frac{\mu\,U_{\mathrm{moy}}}{2\,R}\ .$$

where μ is viscosity of crude oil, U_{moy} mean velocity, R radius of the pipe. In the case of the model MPTT2, we have to change the number of chains:

$$N_i^* = \frac{N_i}{N_{i,0}}$$

with $N_{i,0}$ chains number at balance.

Here are the final dimensionless systems (* means dimensionless).

Model MPTT*

$$\begin{aligned}
\frac{\partial\tau^*_{\mathrm{rr}}}{\partial t^*} &= -\sigma^*\,\tau^*_{\mathrm{rr}} + \frac{2\,U_{\max}}{R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\,\xi\,\lambda_{i,0}\,\tau^*_{\mathrm{rz}}\ ,\\
\frac{\partial\tau^*_{\mathrm{rz}}}{\partial t^*} &= -\sigma^*\,\tau^*_{\mathrm{rz}} + \frac{2\,U_{\max}}{R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\,\lambda_{i,0}\left[\xi\,\tau^*_{\mathrm{zz}} + (\xi-2)\,\tau^*_{\mathrm{rr}}\right] -\\
&\qquad\qquad - 2\,\eta^*_{\mathrm{m}}\,\frac{2\,U_{\max}}{A\,R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\ ,\\
\frac{\partial\tau^*_{\mathrm{zz}}}{\partial t^*} &= -\sigma^*\,\tau^*_{\mathrm{zz}} + \frac{2\,U_{\max}}{R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\,\lambda_{i,0}\,(\xi-2)\,\tau^*_{\mathrm{rz}}
\end{aligned}$$

with

$$\sigma^* = \exp\left(\in A\,\frac{\mathrm{tr}(\tau^*_i)}{G_{i,0}}\right)\ , \qquad \eta^*_m = \eta_{\mathrm{p}}\,\frac{1+\xi\,(2-\xi)\,\lambda_i^2\,\dot\gamma^2(t^*\,\lambda_{i,0})}{[1+\Gamma^2\,\dot\gamma^2(t^*\,\lambda_{i,0})]^{(1-n)/2}}\ .$$

Model MPTT2*

$$\begin{aligned}
\frac{\partial\tau^*_{\mathrm{rr}}}{\partial t^*} &= -\frac{\lambda_{i,0}}{\lambda^*_i}\,\tau^*_{\mathrm{rr}} + \frac{2\,U_{\max}}{R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\,\xi\,\lambda_{i,0}\,\tau^*_{\mathrm{rz}}\ ,\\
\frac{\partial\tau^*_{\mathrm{zz}}}{\partial t^*} &= -\frac{\lambda_{i,0}}{\lambda^*_i}\,\tau^*_{\mathrm{zz}} + \frac{2\,U_{\max}}{R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\,(\xi-2)\,\tau^*_{\mathrm{rz}}\,\lambda_{i,0}\ ,\\
\frac{\partial\tau^*_{\mathrm{rz}}}{\partial t^*} &= -\frac{\lambda_{i,0}}{\lambda^*_i}\,\tau^*_{\mathrm{rz}} + \frac{2\,U_{\max}}{R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\,\lambda_{i,0}\left[\xi\,\tau^*_{\mathrm{zz}} + (\xi-2)\,\tau^*_{\mathrm{rr}}\right] -\\
&\qquad\qquad - 2\,\eta^*_{\mathrm{m}}\,\frac{2\,U_{\max}}{A\,R}\,r^*\,\cos(w\,\lambda_{i,0}\,t^*)\,\frac{N_i}{A\,r_i}\ ,\\
\frac{\partial N^*_{\mathrm{zz}}}{\partial t^*} &= -\frac{\lambda_{i,0}}{\lambda^*_i}\,N^*_i + r_i\,\frac{\lambda_{i,0}}{\lambda^*_i}
\end{aligned}$$

with

$$\lambda^*_i = \frac{\lambda_{i,0}}{\exp\left(\in\frac{\mathrm{tr}(\tau^*_i)}{G_{i,0}}\right)}\ , \qquad \eta^*_m = \eta_{\mathrm{p}}\,\frac{1+\xi\,(2-\xi)\,\lambda_i^{*2}\,\dot\gamma^2(t^*\,\lambda_{i,0})}{[1+\Gamma^2\,\dot\gamma^2(t^*\,\lambda_{i,0})]^{(1-n)/2}}\ .$$

6. Results

The figures 7 and 8 represent the dimensionless shear stress and the first normal stress difference evolution according to time and to the width, for the model MPTT*.

The first normal stress difference varies parabolically according to the width of the conduit R and oscillates at the angular speed according to time. Its maximum value is 0.03 Pa.

We thus note that the result obtained with models MPTT* and MPTT2* are identical. It means that the additional equation representing the variation of N_i according to time does not have, in this case, a significant influence on the result for these models. The interest that we had to study the model of Phan-Thien and Tanner was to test a model which takes into account many rheological specificities of crude oil: interactions between oil and water, the non Newtonian effect. We noted that the first assumption leading to the writing of the PTT model was not satisfactory and we decided to use a new model not taking anymore into account this assumption. This new model, with an additional equation, applied to the case

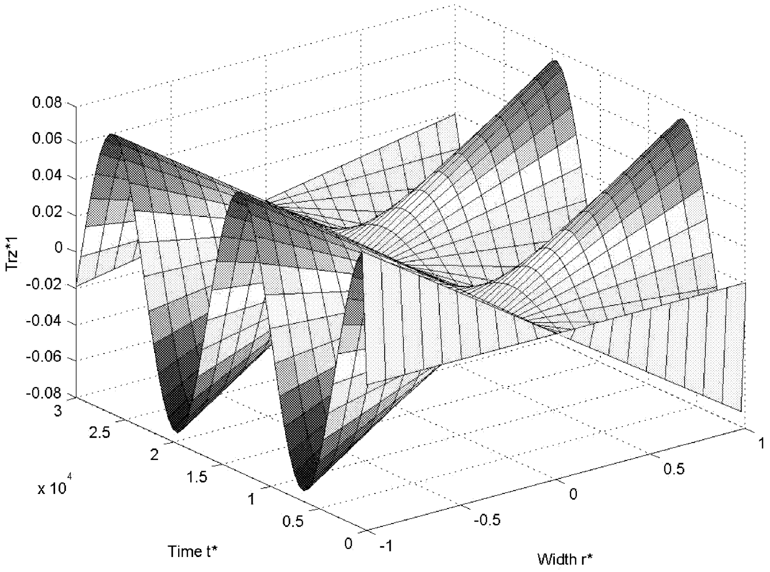

Fig.7: Shear stress τ^*_{rz} according to time and width

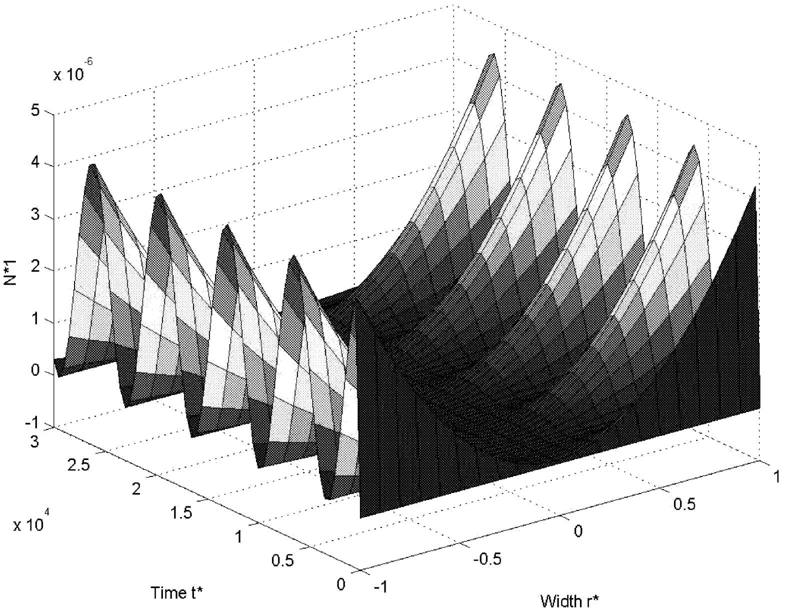

Fig.8: Shear stress N^* according to time and width

of the crude oil, nevertheless provided the same results as these of MPTT in this specific case. Then we can conclude that in the case of the crude oil, the model of PTT is not influenced by this first assumption. We can also highlight the importance of the taking into account of the shear thinning effect. On the interval of rate of shearing used, we obtain an oscillating shear stress between 0 Pa and 4×10^{-6} Pa.

7. Conclusion

Crude oil is a fluid presenting a non-Newtonian behavior. Its modelling is thus not easy. We decided to use the model of Phan-Thien and Tanner, looking more realistic, because based on a microstructural description of the fluid. However, some assumptions of the model of Phan-Thien and Tanner seem no representative of the phenomena observed. We were thus interested in the modification of one of these assumptions, and studied the impact of this modification on the computation of the stresses. However it proved that the application of this model to the case of crude oil flow was not sensitive to this modification. We, all the same, observed the influence of the shear thinning effect on the result of the computation of the stresses. It should be noticed that the model of Phan-Thien and Tanner offer the greatest flexibility on the level of the choice of the configuration and the boundary conditions. Next step is to apply more complex boundary conditions in order to have a better modelling of the crude oil flow and to work on the statistical distribution. The result of the computation of the stresses should be closes to reality.

Notations

Symbol	**Designation**	**Symbol**	**Designation**
ε	Strain due to shearing	$\psi_i(\varrho)$	Distribution function of the vector end
$\dot{\varepsilon}$	Shearing rate	ν	Cinematic viscosity
η	Dynamic viscosity	ϱ	Volumic mass
τ_{ij}	Shearing stress	h	Effective shearing height
τ_p	Shearing stress to the wall	L	Tensort of gradient rate of the local flow
ω	Rotation speed	l_i	Formation rate of i-chains
D	Tensor of macroscopic rate of strain	k	Boltzmen constant
D_h	Hydraulic diameter	K	Coefficient of consistency
F	Friction force	k_i	Formation rate of i-chains
n	Behavior index	R	Raduis
N_i	Stress surmodulaire	Re	Reynolds number
$N_{i,0}$	Equilibrium chains number	Re_c	Critical Reynolds number
P	Pressure	S	Pipe section
Q	Flow	t	Time of flow
r, r'	Raduis variable	T	Temperature
$\Delta P/L$	Pressure gradient	x, y, z	Space variables
u	Fluid speed	$G_{i,0}$	Shearing Module crude oil
U_max	Fluid maximum speed	U_moy	Fluid average speed
α	Angle	θ	Flow angle
η_0	Flow viscosity	η_∞	Limit viscosity
η_app	Apparent viscosity	ξ	Parameter defining the friction rate
ζ	Speed tensor of local flow	τ	Shearing stress

References

[1] Phan Thien N., Tanner R.I.: A new constitutive equation derived from network theory, Journal of Non-Newtonian Fluid Mechanics, Elsevier Scientific Publishing Company, 1977

[2] Kane M., Djabourov M.: Rheology and structure of waxy crude oils in quiescent and under shearing conditions, Journal of Fuel, Elsevier Scientific Publishing Company, 2004, p. 1591–1605

[3] Amblard A., Bousaid B.: Modelling of the blood flow in an aorta : an attempt. The MPTT and Modified MPTT models, published in the Journal of Engineering Tribology, IME

[4] Bousaid B., Py C.: Modelling of the blood flow in the aorta : an attempt. Part I1: The phenomenological and Dumbbell models, published in the Journal of Engineering Tribology, IME

[5] Py C. : Modélisation de l'écoulement sanguin dans l'aorte, Mémoire de DEA, Institut National des Sciences Appliquées de Lyon, 2002

[6] Bousaid B., Py C.: Modelling of the blood flow in the aorta : an attempt, Part I: General considerations, published in the Journal of Engineering Tribology, IME

[7] Mewuis J., Morton M.D.: Constitution equation based on the transient network concept, Journal of Non-Newtonian Fluid Mechanics, Elsevier Scientific Publishing Company, 1983

[8] Ehret P. : Contribution à l'étude du comportement de mécanismes lubrifiés sous chargement transitoire, Mémoire de Doctorat, Institut national des Sciences Appliquées de Lyon, 1993

Received in editor's office: March 16, 2006
Approved for publishing: October 6, 2006