JOINTED TIME AND FREQUENCY TRANSFORMS IN TESTING MATERIAL DEFECTS

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The paper deals with the non-destructive testing of structural elements by means of the acoustic response using the jointed time and frequency transforms. These methods make it possible to localise the beginning and the end of frequency components contained in the measured signal, and in this way, they enable us to analyse perfectly the spectrum of the non-stationary noise. In this way, this mathematical procedure enables us to distinguish a good specimen from a defective one.

Key words : *time frequency transform, wavelet transform, acoustic response analysis, ambiguity function*

1. Foreword

Many times we have already been able to convince ourselves that utilising the experience and knowledge having their roots even in the far past have brought surprisingly good results. One of such experience is the knowledge that noise resulting from the shock applied to a structure with cracks (disturbances) is significantly different from the same subject without cracks. This phenomenon has been known for a long time. As early as in the Middle Ages this phenomenon was used to detect cracks in ceramic pots after their firing. The phenomenon mentioned makes it possible to detect cracks in metallic materials as well. Its very old application in the railway transport is generally known. However, in the development and application of the methods used to detect defects in structural elements and materials, this phenomenon has often been neglected. The absence of the advanced measuring techniques and appropriate mathematical instruments necessary for the evaluation of measured signals were the main reasons.

There are only the methods of the time-frequency analysis in connection with the classic spectral analysis that make a thorough analysis of the signals measured with good possibilities of classifying and identifying possible defects. The noise excited by force impulse to the tested construction or element is a very interesting phenomenon since it contains a number of mutually independent and well recognisable symptoms according to which it is possible to differentiate materials with cracks from those without cracks [8, 9].

These symptoms are particularly included multi-spectra of the measured noise signals. The composition of the spectrum of noise of each material is given by several characteristics with their own side elements with respect to time and frequency. The spectrum of the measured noise is changed for the duration of the process. The symptoms of cracks are as follows :

– changes of the amplitudes of particular characteristic, frequency components
– frequency shifts of characteristic frequency components,

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– appearance of new frequency components,
– presence of modified spectrum in comparison with a good product.

2. The theory of time frequency analysis

Information relating to any technical or physical occurrence is represented in the signal by time changes of immediate values or physical phenomena described. The direct evaluation of the time-amplitude representation often appears not to be easy. That is why there is a practice of the signal transform from the time domain to some different ones. In some cases, some important pieces of information from the frequency area may be obtained. There are a lot of various transforms applied for transition between time, frequency and jointed time frequency domain. The best known method is the Fourier transform and some of its modifications.

The Fourier transform and some of its modifications are the techniques which are especially suitable for processing stationary signals. These methods can analyse transient and non-stationary signals as well as in the cases when we are interested only in the frequency components contained in the whole time behaviour of signal. However, these do not provide us with information on the occurrence of important frequency components in the time flow.

One of the possible way how to analyse the time occurrence relating to the frequency components of transient and non-stationary signals is the use of the so-called time-frequency analysis method.

The goal of this paper is to demonstrate some less-known methods for creating functions representing the energy of the signal simultaneously in time and frequency domain.

The Short Time Fourier and Wavelet Transforms represent the examples of the linear time-frequency distributions. The main idea of the Short Time Fourier Transform (STFT) is to split a non-stationary signal into segments where the signal is considered to be stationary. The Fourier transform on each of these segments is computed. The STFT is defined by equation [3]

$$STFT(\tau, f) = \int_{-\infty}^{\infty} [x(t)\, g^*(t-\tau)]\ \mathrm{e}^{-\mathrm{j}\, 2\pi\, f\, t}\, \mathrm{d}t\ , \tag{1}$$

where '*' denotes the complex conjugate, $g(t)$ is a short time window, $x(t)$ is a signal, τ is a time location parameter, f is a frequency and t is a time. In the two dimensional time-frequency joint representation, the vertical slice of the complex valued STFT coefficients $STFT(\tau, f)$ correspond to the Fourier spectra of the windowed signal with the window shifted by time τ. The time frequency resolution is limited to the Heisenberg principle, which is the main disadvantage of the linear time-frequency transform. The signal component cannot be presented as a point in the time frequency space. Only its position inside the $\Delta t\, \Delta f$ rectangle region may be determined. This is due to the imposition of local time window $g(t)$. If the width of the window is increased, frequency resolution improves but time resolution becomes poor and vice-versa.

The Wavelet Transform (WT) is a new mathematical tool developed mainly since the middle of the 1980s. It is efficient for the local analysis of non-stationary and fast developing transient signals. Similarly to the STFT, the Wigner distribution, ambiguity function and the Wavelet transform map the signal to the time-scale (frequency) joint presentation. The temporal aspect of the signal is preserved. The WT provides a multi-resolution analysis with a dilated window. The high frequency analysis is made using a narrow window and the lower frequency analysis is made using a wide window. The Wavelet analysis is similar to the Fourier analysis because it breaks a signal down into its constituent parts for the analysis. Whereas the Fourier transform decomposes the signal into a set of sine waves of different frequencies, the Wavelet transform decomposes the signal into its 'wavelets', scaled and shifted versions of the 'parent Wavelet'. The Wavelet transform allows us an outstanding localisation in both the time domain via translation of the parental wavelet and in the scale (frequency) domain via dilatation. The translation and dilatation operations applied to the parent wavelet are performed to calculate the wavelet coefficients representing the correlation between the wavelet and the localised section of the signal. The wavelet coefficients are calculated for each wavelet segment, giving a time-scale function relating to the wavelets correlation to the signal [6, 7].

$$WT(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t)\, \psi^*\!\left(\frac{t-\tau}{s}\right) \mathrm{d}t \ , \tag{2}$$

where '*' denotes the complex conjugate, $x(t)$ is a signal, t is a time, τ is a translation factor, s is a scale factor (frequency) and $\psi(x)$ is a wavelet. This transform is called Continuous Wavelet Transform because the analysing wavelet can be used at any scale, and its position can also be shifted continuously over the entire time domain of the signal being analysed.

As the parent wavelet (which is obviously the most suitable especially for transient processes) the Morlet Wavelet and the Mexican Hat Wavelet are often used. The parent Morlet Wavelet is preferred for the analysis of the vibration measurement. The Morlet's basic wavelet function is a multiplication of the Fourier basis with a Gaussian window according to equation (3)

$$\psi = \mathrm{e}^{-\mathrm{j}\,\omega_0\,t}\, \mathrm{e}^{-\frac{t^2}{2}} \ . \tag{3}$$

Its real part is a Cosine-Gaussian function and the imaginary part is a Sine-Gaussian function. The Wavelet transform is of a particular interest for the analysis of non-stationary and transient signals. The Wavelet transform provides an alternative to the classic Short Time Fourier Transform or the Gabor transform, and it is more efficient than the Short Time Fourier Transform.

The Wigner-Ville transform [2, 4] is another alternative method for a Short-Time Fourier Transform and the Wavelet Transform for processing both the stationary and non-stationary signals. The Wigner distribution was proposed by Wigner in 1932 for the region of quantum physics, and about 15 years later, this was modified for the region of the signal analysis by Ville. The Wigner-Ville transform is defined for the time frequency region by relation

$$WVT_{\mathrm{x}}(t, f) = \int_{-\infty}^{\infty} x\!\left(t + \frac{\tau}{2}\right) x^*\!\left(t - \frac{\tau}{2}\right) \mathrm{e}^{-\mathrm{j}\,2\pi\,f\,\tau}\, \mathrm{d}\tau \ , \tag{4}$$

where '*' is a complex conjunction, t is a time, τ is a shift along the time axis, $x(t)$ is a time representation of the signal and $WVT_{\mathrm{x}}(t, f)$ is a jointed time and frequency representation of the signal. In contrast to the linear time frequency transforms in which the resolution is limited by the window function, the Wigner-Ville spectrum offers us an excellent resolution both in the frequency and time domain. The calculation is not limited by the Heisenberg principle of uncertainty which is its important characteristic since it is a more general transformation that does not utilise the weighing function.

Quadratic (non-linear) methods represent the second fundamental class of time frequency distributions. The quadratic methods are based upon estimating an instantaneous power (or energy) spectrum using a bilinear operation on the signal $x(t)$ itself. The class of all quadratic time-frequency distributions to time-shift and frequency-shift is called the Cohen's class. Similarly, the class of all quadratic time-frequency distributions covariant to time-shift and scale is called the Affine class. The intersection of these two classes contains time-frequency distributions like the Wigner-Ville distribution that are covariant to all operators [2].

Cohen [1] describes the non-linear time frequency transformation (especially shift-invariant class) that can be derived from the Wigner distribution.

Cohen [1] generalised the definition of the time frequency distributions in such a way which includes a wide variety of different distributions. These different distributions can be represented in several ways. The Cohen's class definition like the Fourier Transform, with respect to τ, of the generalised local correlation function, is the most common. With a two-dimensional kernel, the bilinear time-frequency distribution of the Cohen's class is defined according to equation [1, 10]

$$C_{\mathrm{x}}(t,f) = \iiint \mathrm{e}^{-\mathrm{j}\,2\pi\,\theta\,t' - \mathrm{j}\,2\pi\,f\,\tau + \mathrm{j}\,2\pi\,\theta\,t}\,\psi(\theta,\tau)\,x\Big(t+\frac{\tau}{2}\Big)\,x^{*}\Big(t-\frac{\tau}{2}\Big)\,\mathrm{d}\theta\,\mathrm{d}t\,\mathrm{d}\tau\ , \tag{5}$$

where $x(t)$ is a signal, t is a time, τ is a time location parameter, ω is an angular frequency, θ is a shift frequency parameter, a function $\psi(\theta,\tau)$ is called the kernel of the time frequency distribution. Distribution $C_{\mathrm{x}}(t,\omega,\psi)$ from the Cohen's class can be interpreted as the two-dimensional Fourier Transform of a weighted version of the ambiguity function of signal

$$C_{\mathrm{x}}(t,f) = \iint A_{\mathrm{x}}(\theta,\tau)\,\psi(\theta,\tau)\,\mathrm{e}^{-\mathrm{j}\,2\pi\,f\,\tau}\,\mathrm{e}^{-\mathrm{j}\,2\pi\,\theta\,t}\,\mathrm{d}\tau\,\mathrm{d}\theta\ , \tag{6}$$

where $A_{\mathrm{x}}(\theta,\tau)$ is the ambiguity function of signal $x(t)$ given by equation

$$A(\theta,\tau) = \int x\Big(t+\frac{\tau}{2}\Big)\,x^{*}\Big(t-\frac{\tau}{2}\Big)\,\mathrm{e}^{-\mathrm{j}\,\theta\,t}\,\mathrm{d}t\ . \tag{7}$$

Note that all integrals run from $-\infty$ to ∞. The weighted function $\psi(\theta,\tau)$ is called the kernel. It determines the specific properties of the distribution. The product $A_{\mathrm{x}}(\theta,\tau)\,\psi(\theta,\tau)$ is known as the characteristic function. Since function $A_{\mathrm{x}}(\theta,\tau)$ represents a bilinear operation for processing the signal, the contributions from the so-called cross components are exhibited during its calculation, which consequently deteriorates the differentiation of the given transformation. This effect may be limited by a suitable choice of the so-called kernel function. Then the kernel function unambiguously determines the properties of a given transformation.

Therefore, the kernel is often selected to weight the ambiguity function such that the auto-elements that are centred at the origin of the (θ,τ) ambiguity plane are passed, while the cross-elements that are located away from origin are suppressed. This means that the suppression of cross-elements might be understood as the frequency response of a two-dimensional low-pass filter.

When a low-pass kernel is employed, there is a trade-off between the cross-elements suppression and the auto-element concentration. Generally, as the band-pass region of the kernel is made smaller, the amount of the cross-element suppression increases, however, at the expense of the auto-element concentration. Table 1 presents definitions of the kernels for various time frequency distributions [5].

Distribution	**Kernel function** $\psi(\theta,\tau)$
Rihaczek	$\psi(\theta,\tau) = \mathrm{e}^{\frac{\mathrm{j}\,\theta\,\tau}{2}}$
Margenau-Hill	$\psi(\theta,\tau) = \cos\left(\frac{\theta\,\tau}{2}\right)$
Page	$\psi(\theta,\tau) = \mathrm{e}^{\frac{\mathrm{j}\,\theta\,\lvert\tau\rvert}{2}}$

Tab.1: Definitions of the kernels for various time frequency distributions

Equation 6 can also be rewritten into the following form [5, 10]

$$C_{\mathrm{x}}(t,f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Pi(\tau - t, \theta - f)\; WVT(\tau,\theta)\,\mathrm{d}\tau\,\mathrm{d}\theta\ , \tag{8}$$

where

$$\Pi(t,f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi(\theta,\tau)\,\mathrm{e}^{-\mathrm{j}\,2\pi\,(f\,\tau-\theta\,t)}\,\mathrm{d}t\,\mathrm{d}\omega \tag{9}$$

is the two-dimensional Fourier transform of the kernel ψ and WVT presents the Wigner-Ville transform. Cohen's class has a simple interpretation as a smoothed Wigner-Ville distribution.

3. Laboratory measurement

The text to follow pursues the determination of the quality of structural elements (in this case the 'hurdis' brick) by the method of acoustic response analysis. The basis for the methodology designed by the author is the analysis of the response of the noise signal to a mechanical impulse, particularly by time-frequency procedures. The measured specimen was placed on a special device fixed to supports (Fig. 1 and Fig. 2). The mechanical shock was excited by a special pendulum with a defined choice of the shock intensity. The measuring device (made by the Bruel & Kjaer Company) consisted the PULSE 3360C signal analyser, a microphone and the measuring software. The sampling frequency was 12 kHz. The electric signal detected and digitally recorded resulted from the measurement, and it was adequate to the amplitude of the acoustic pressure in the place of the microphone location. After completing the analysis, the check measurement and calculations, the following methods and parameters were used to the analysis of the response to the mechanical shock:

1. diagram time history of the acoustic pressure,
2. frequency analysis with help of behaviour of the power spectral density (the algorithm of the fast Fourier transform was used),

3. linear jointed time and frequency methods of the spectral analysis (the algorithms of the Short-Time Fourier transform and the Wavelet transform were used),
4. non-linear jointed time and frequency methods of the spectral analysis (the algorithms of the Margenau-Hill transform, the Rihaczek transform and the Page transform were used).

The analysed figures are composed of three diagrams. The upper diagram shows the time history of the acoustic pressure. In the left diagram, the amplitude spectrum of the acoustic response calculated by the direct employing of the Fourier transform on measured signal is shown. The middle diagram shows the 3D view of jointed time and the frequency spectrum

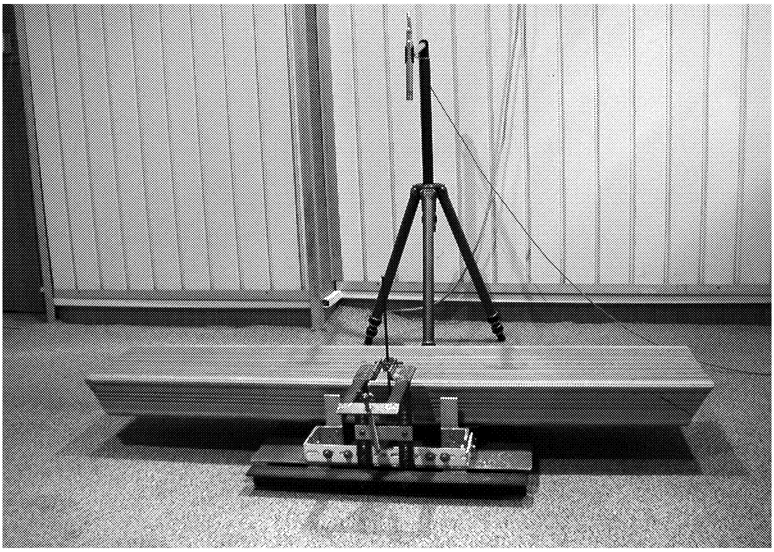

Fig.1: General view of the working place

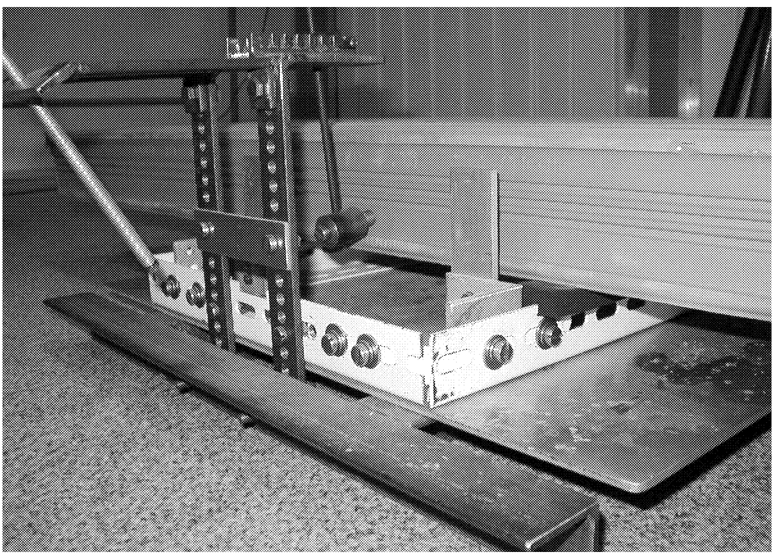

Fig.2: Detail view of the working place

of the amplitude spectrum of the acoustic pressure. The spectra in particular diagrams in Figs. 4 to 7 were gradually calculated by means of the Short-Time Fourier transform or more precisely by the Rihaczek transforms.

The values of the acoustic pressure in the decibel scale are depicted in the middle diagrams in different colours. It should be said that the maximum value is in black colour. Figures 4 to 7 present three groups of diagrams of the signals measured by the microphone using the impact-echo method used to a defective product and for the defect-free one. Comparing the time records of a good and a defective specimen, it may be considered that the signal coming from the good product has a lower damping, i.e. the specimen sounds for a longer time.

Then the frequency characteristic shows that in a defective specimen (Fig. 3) the distinctive frequency components have 'moved' in the direction of the lower frequency values and that the spectrum is wider compared with defect-free specimens. The comparison of the course of the time-frequency transform clearly shows the different characters of the spectra of the defective specimens and the defect- free specimens.

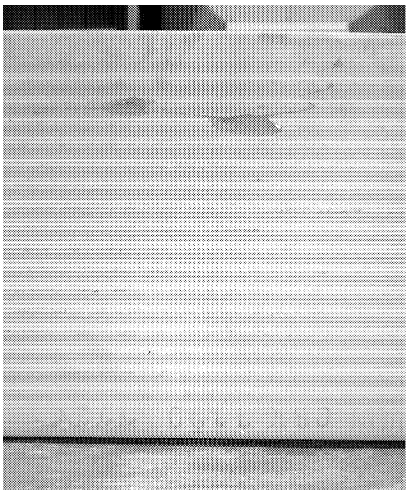

Fig.3: Possible defective product

The text to follow will only deal with the diagram of the time-frequency characteristics obtained by means of different mathematical procedures. The Short-Time Fourier transform will be considered as a basic transform. The time-frequency curve for the defect-free product (Fig. 4) clearly shows two frequency components on frequencies 2.5 kHz and 3.3 kHz. The component on frequency 2.5 kHz is damped by 20 dB approximately in 30 ms, while the frequency component 3.3 kHz is damped in 50 ms. These two frequency components compared with other frequency components are remarkable in their spectra. The change of the time-frequency spectrum of a defective specimen (Fig. 5) can easily be distinguished. The distinctive frequency region, approximately from 700 Hz to 3.2 kHz contains more than two important frequency components – e.g. it is possible to choose the values on 700 Hz, 1.1 kHz, 1.8 kHz, 2.2 kHz, 2.5 kHz, 2.8 kHz and 3.2 kHz – their values are less than one order lower than the value of the maximum component. The time periods for damping of these frequency components by 20 dB are usually shorter than the periods for selected components in defect-free specimens. These periods reach the values of damping approximately between 15 ms and 30 ms.

Let us mention that the Short-Time Fourier transform is one of the basic and also fast procedures for the time-frequency analysis of signals. However, the accuracy and appropriateness of this method depends upon the choice of the window function, its size and on the

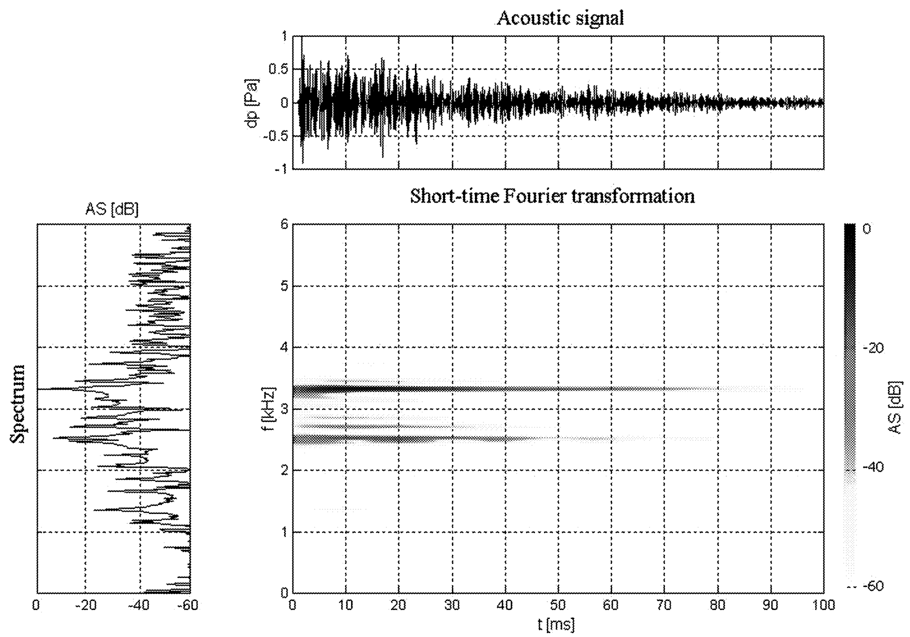

Fig.4: Defect-free product – time behaviour signal from microphone, frequency spectrum and time – frequency analysis by the Short-Time Fourier transform

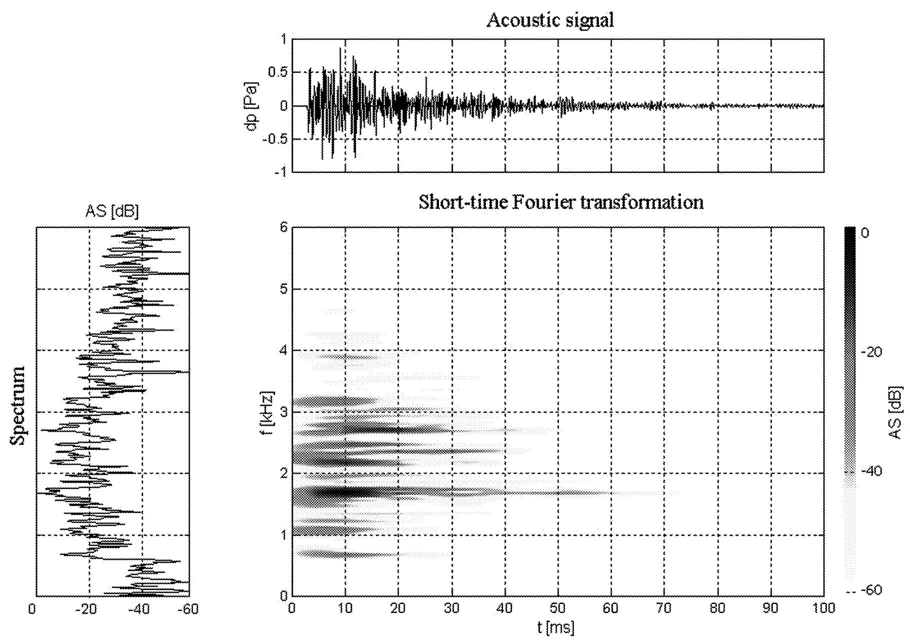

Fig.5: Defective product – time behaviour signal from the microphone, frequency spectrum and time – frequency analysis by the Short-Time Fourier transform

overlap of particular segments. The application of the method requires a certain experience gained for the 'rational' definition of input parameters and also in the interpretation of its spectrum.

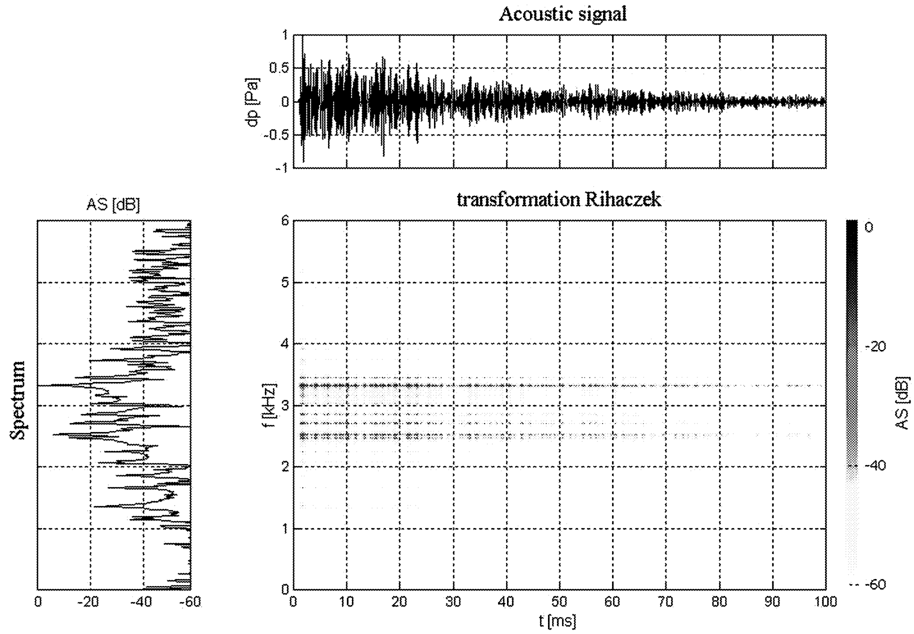

Fig.6: Defect-free product – time course signal from the microphone, frequency spectrum and time – frequency analysis by the Rihaczek transform

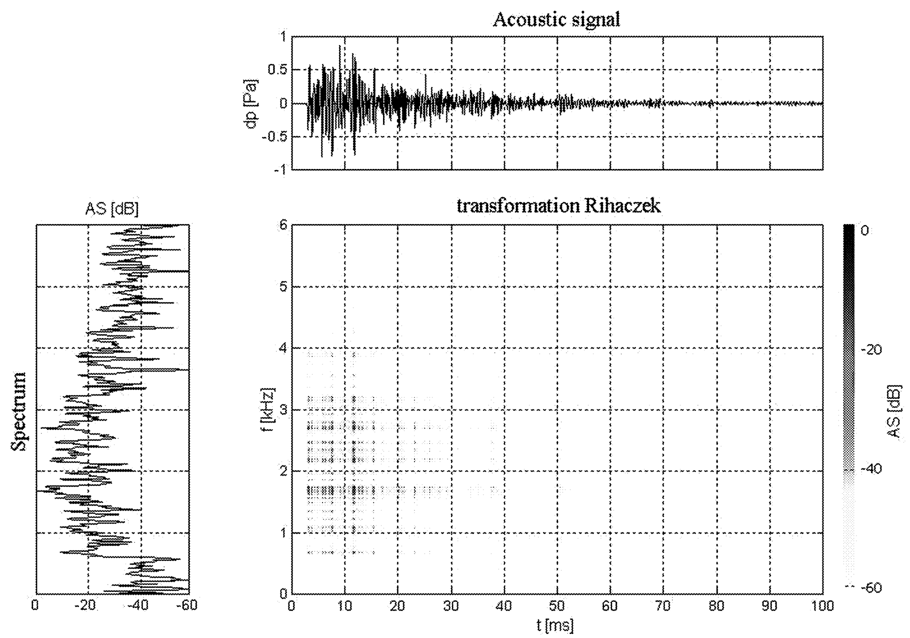

Fig.7: Defective product – time behaviour signal from the microphone, frequency spectrum and time – frequency analysis by the Rihaczek transform

Therefore it is often more advantageous to use non-linear time frequency transformations for this analysis of the transient signals. The characteristic feature of non-linear transformations rests in their resultant resolution in time and frequency which is not limited by the Heisenberg uncertainty principle. This fact includes a high resolution capability in the time frequency level which demonstrates itself by the 'accurate' localisation of important frequency components in time.

The Rihaczek transforms applied to both signals (defective ones in Fig. 6 and defect-free ones in Fig. 7) show a similar jointed time-frequency spectrum of the standardised acoustic pressure level as in case of the Short-Time Fourier transform. The time-frequency spectrum (the Rihaczek transform) shows local frequency maximums of particular frequency components in time more distinctively (precisely) than it is with the spectrum calculated by the Short-Time Fourier transform. This is given especially by the type of transform where mainly the kernel and the local auto-correlation function participate in the phenomenon presented.

The time-frequency spectrum calculated by the Rihaczek transform shows sharper courses of particular distinguished frequency components ('slighter lines') than it is with the spectra calculated by the Short-Time Fourier transform.

As apparent from Figs. 4 to 7, the whole process of response may be roughly divided into three stages. In the first stage, a very fast growth of the amplitudes of the key frequency components to maximum values is apparent. In the second stage, the descent and extinguishing of higher frequency components occur. However, in both phases of the response the contents of the frequency cross-sections remain approximately the same. In the third stage, the fading of the response occurs. This is usually characterised by the existence of the lowest frequency component.

If some material cracks occur, the speed of the growth and the declination of amplitudes of the spectrum are higher than for a defect-free material. It should, however, be stated that the speed of the growth and declination of amplitudes of particular important elements are not the same. The frequency or the time-frequency spectrum of defective materials is generally much wider, and the so-called clusters of important frequency components occur there. This phenomenon is properly interpreted by particular diagrams in Figs. 4 to 7.

4. Conclusion

Based on the measurement and analyses made, it is possible to state that the experiment checked the possibilities of using the given methodology for the detection of structural defects of the measured products and materials from the homogeneity and the cracks formation points of view. The analysed parameters enable us to distinguish a good product from a defective one. The given methodology may be successfully applied to all ceramic and concrete products. It can also be established that modern means of signal analysis, especially the time-frequency transforms highly contributed to a quality processing of the measurement. These methods provide us with a time localisation of frequency components contained in the measured signal. In this way, these methods offer new possibilities in the experimental analyses of structural elements and materials. This possibility is included in both the linear and the non-linear time-frequency procedures. By comparing the obtained results we can clearly observe that when analysing the responses to the mechanical shock (heavily damped time signals) the procedures, namely those including into the class of the Cohen time-frequency transforms, show a very good time localisation of significant frequency components contained in the analysed signal.

In some particularly important cases the depicting of results of the computes and analyses by the time and frequency sections may be completed. Thus, these provide us with a profound support in the analysis of the time-frequency results. This procedure often appears as more suitable than e.g. a separate space arrangement. This makes it possible to locate very precisely the time behaviour of particular important frequency components or to show all important frequency components contained in the spectrum in a given time.

In conclusion it may be mentioned that the given methodology can also be incorporated in the process of the half-automated quality control of products under in the production line. When using the above methodology together with the methods of qualitative analysis or the artificial intelligence (fuzzy and rough sets, neurone networks, genetic algorithms etc.), then the process of evaluating the quality of products can fully be automated.

Acknowledgement

This outcome has been achieved with the financial support of the Ministry of Education, Youth and Sports of the Czech Republic, project No. 1M6840770001, within activities of the CIDEAS research centre and project GACR No. 103/05/0085.

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Received in editor's office: January 16, 2006
Approved for publishing: August 8, 2006