FUZZY-RANDOM ANALYSIS OF STEEL STRUCTURES

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The uncertainty of stochastic computations for which input random quantities are introduced in the subjective manner is presented in this paper. To obtain a correct stochastic computation, it is necessary to have exclusively objective statistical information based on experiments including the correlation matrix at the disposal. If this information is not available, the vague (fuzzy) uncertainty is to be quantified by applying the so-called fuzzy analysis. In the present paper, these problems are demonstrated on the example of an analysis of fuzzy uncertainty of probability assessment of the reliability of a steel member under compression. The output is a fuzzy set of failure probability. The single crisp control output of failure probability was obtained with use of the defuzzification centre of gravity method. The value obtained was by 200 percent higher than that obtained by the classical stochastic computation. In conclusion, the applicability possibilities of the complex fuzzy-random uncertainty analysis at the probability assessment of the reliability of structures by the SBRA method.

Key words: *fuzzy, random, stochastic, steel, imperfection, buckling*

1. Introduction

Recently, we have frequently met the methods which assess the structure reliability by applying the probability computation. A correct probability computation requires exclusively objective statistical information based on very numerous experiments. All the input random quantities including the correlation matrix are to be exactly determined in the stochastic analysis.

Of course, it is not possible to guarantee the exclusively objective statistical information because each building structure is (apart from exceptions) unique.

In difficult operation conditions, the extensive and exacting measurements are either totally impossible or their information quality is very low so that they are inapplicable although the necessary robustness has been ensured [14].

If an expert finds an input random quantity in a subjective manner, a stochastic computation becomes a vaguely (fuzzy) uncertain one. The results of the stochastic computation are devaluated due to inaccurate information on input random quantities. The stochastic computation model becomes so a source of fuzzy uncertainty which, in complex systems, can predominate over the stochastic uncertainty significantly. The aim of the present paper is an analysis of the influence of fuzzy uncertainties on the fuzzy uncertainty of the output random quantity.

The vagueness as a concomitant phenomenon of all complex, hard to describe systems and eventually processes in which the human factor figures are most commonly formalized by

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means of the apparatus of fuzzy sets, founded by professor Lotfi A. Zadeh of the University of Berkeley. For the first time, the term 'fuzzy' was used by Lofti Zadeh in 1962 [22]. In 1965, L. Zadeh published his pioneer, today still classic paper entitled 'Fuzzy sets' [23]. The Zadeh's theory of fuzzy sets generalizes the theory of crisp sets, which was founded by the German mathematician George Cantor (1845–1918). Within the theory of crisp sets, an element belongs, or does not, into a set. If the valuation set is allowed to be the interval $\langle 0, 1\rangle$, we used term fuzzy set [23], see Fig. 1.

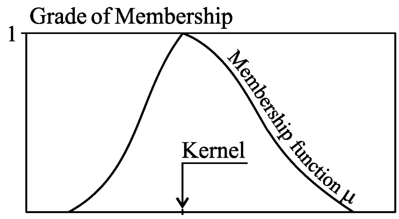

Fig.1: Fuzzy number a_0

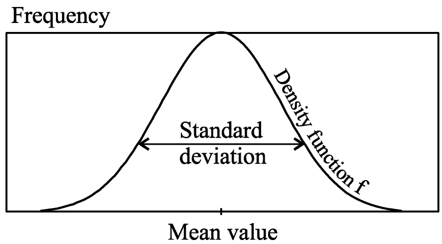

Fig.2: Random number

In the Zadeh's theory of fuzzy sets a fuzzy set is defined as a class attributing the uncertainty to elements by means of their partial membership in the form of so-called *degree of membership*. The number plotted on the horizontal axis expresses the value which is inaccurate, i.e., 'maybe a_0'.

The fuzzy number a_0 in Fig. 1 is a convex normalized fuzzy set of the real line R. The degree of membership to a set is presented on the vertical axis: 1 means the full pertinence to the set, 0 means the non-pertinence to a set at all. The transition area within an interval $(1, 0)$ determines the area of partial membership of an element in a set. The degree of membership is often characterized as a possibility of the phenomenon that element x belongs to the set X.

A fuzzy number expresses the uncertainty whatever the cause may be. Qualitatively different information is expressed by the probability function determining the occurrence frequency of a mass phenomenon, see Fig. 2.

The many typical problems of structural design are characterized both by fuzzy and random uncertainty [1]. Using fuzzy-random variables and fuzzy random functions, it is possible to mathematically describe the uncertainty characterized by fuzzy randomness [11]. Basic terms and definitions related to fuzzy randomness have been introduced, inter alia, by [15, 21].

The topic of the present paper is an analysis of fuzzy-random uncertainties of the load-bearing capacity of a member under compression. The initial beam curvature is considered to be a fuzzy-random quantity. The other quantities for which the input random quantities are known from experiments are considered to be random quantities.

2. Fuzzy-random initial imperfections

The curvature shape of the axis of hot-rolled steel beam approximates, according to the experimental research results [2] to one half-wave of the sine function. The variability of this initial imperfection is represented by the variability of the maximum amplitude e of the sine function, see Fig. 3. Bilaterally simply supported members with cross-section IPE180, with length $L = 1.93$ m manufactured of the steel grade S235, under axial compression were solved. The results of sensitivity analyses [5] have shown that the load-carrying capacity of a strut with IPE180 cross-section is the most sensitive to the initial imperfection e due to nondimensional slenderness $\bar{\lambda} = 1.0$.

However, the opinions of experts on the adequate density function imperfection e differ considerably. The published results of experimental research are often incomplete of evaluated, based on a little number of samples [2]. The most frequently met functions are the Gauss [13] and the lognormal [16] ones.

The Gaussian density function supposes the imperfections e to be both positive and negative. The lognormal density function supposes only positive values. Consideration of rectangular density and/or histograms within the framework of the SBRA method [8] is another variant. A standard assumption is the fact that 95 % of realizations of amplitude e lie within the tolerance limits of the standard [25]. Nevertheless, the experimental research results [2, 10] show that neither this binding condition is always to be fulfilled.

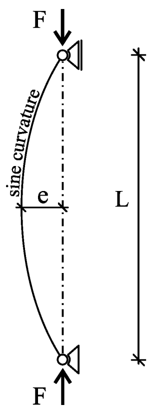

Fig.3: Initial imperfection

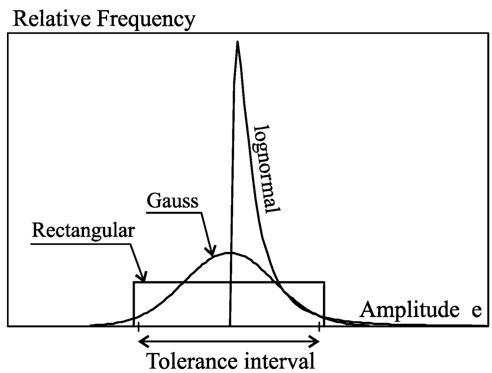

Fig.4: Used models of amplitude e

The pieces of information on initial imperfection being necessary for the failure probability analysis are exacting both by the method of their objective obtaining and requirements of their quality.

Let us imagine that we would measure a large quantity of data on initial imperfection experimentally. Let be the curvature shape approximated by one half-wave of the sine function. Thus, it can be supposed for statistical characteristics of the random quantity e:

1) *Mean value* and *skewness* are equal to zero, i.e., positive and negative values of e occur in the same frequency.
2) *Standard deviation* is determined, based on the consideration that 95 % of realizations lie within the tolerance limits of the standard [25].
3) *Kurtosis* is a fuzzy number.

Kurtosis is a typical fuzzy characteristic, the signification of which is highly underestimated. Most frequently, the values of skewness and kurtosis are determined by the choice of density function. The kurtosis of the lognormal density function in Fig. 4 is equal to $k_\mathrm{e} = 45.2$. The Gaussian density function has the value of kurtosis $k_\mathrm{e} = 3.0$. Kurtosis of the rectangular density function is $k_\mathrm{e} = 1.8$.

When assessing the experimental data, *kurtosis* is the fourth statistical characteristic, and therefore it is loaded by a statistical error larger than mean value, standard deviation and skewness. It is rather a pity that the values of kurtosis are published less than the other three quantities.

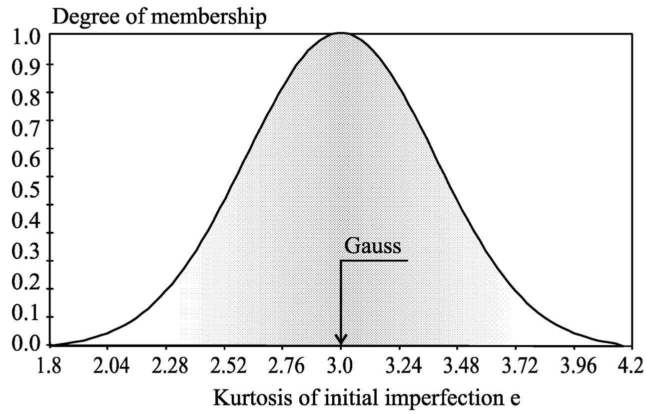

Fig.5: Fuzzy number of kurtosis

For the imperfection e the Hermite four-parametric density function has been assumed which enables to take also the influence of skewness and kurtosis into consideration. The Hermite density function is available in the programme Statrel 3.10. The fuzzy number of kurtosis is presented in Fig. 5. A symmetrical membership function has been chosen which is identical in shape with the Gaussian distribution. The kurtosis $k_e = 3.0$ (Gauss density function) is attributed to the maximum degree of membership (truth 100 %). The determination of the membership function in Fig. 5 was carried out in a subjective manner, based on consultations with experts who were co-authors of published experimental research results [9, 18].

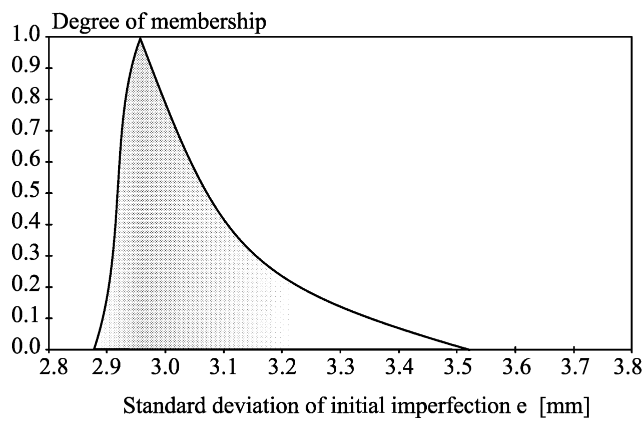

Fig.6: Fuzzy number of standard deviation

Further on, the fuzzy number of standard deviation was assessed, see Fig. 6. Fuzzy standard deviation was assessed, based on the extension principle for ten so-called α-cuts [1, 12, 3, 19]. The general dependence between kurtosis and standard deviation of initial imperfection is shown in Fig. 7. The standard deviation and kurtosis values of the Hermit density fulfil the condition that 95 % of imperfection e realization lie within the tolerance limits of the standard [25]. These are determined by ± 0.3 %, i.e., for the strut $L = 1.93$ m, $\Delta = \pm 5.792$ mm.

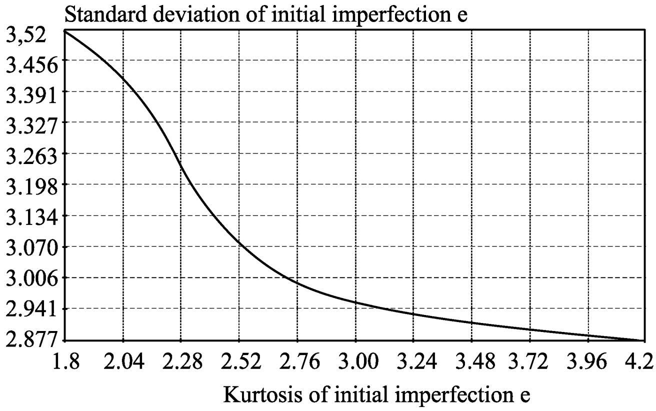

Fig.7: The dependence between the standard deviation and kurtosis of the Hermite density function

The yield point f_y can be mentioned among further imperfections of the IPE180 profile geometrical characteristics. The histogram and Gauss density functions with statistical characteristics in compliance with [9] were considered. For the Young's modulus E, the mean value 210 GPa and the standard deviation 12.6 GPa were supposed, based on two independent research works [2, 17]. The tenth random quantity is the load action applied for the fuzzy-random failure probability analysis. The clear arrangement survey of input random quantities is presented in Tab. 1.

	Symbol	Distribution	Mean value	Standard deviation	Kurtosis
1.	h	Gauss	180 mm	0.8 mm	3
2.	b	Gauss	91 mm	0.9 mm	3
3.	t_1	Gauss	5.3 mm	0.22 mm	3
4.	t_2	Gauss	8 mm	0.37 mm	3
5.	E	Gauss	210 GPa	12.6 GPa	3
6.	f_y	Histogram	297.3 MPa	16.8 MPa	3
9.	e	Hermite	0	Fuzzy	Fuzzy
10	F	Gauss	190 kN	19 kN	3

Tab.1: Input quantities

The amplitude e of the initial sinusoidal curvature is a fuzzy-random quantity. The fuzzy number of Hermite density function is generally sketched in Fig. 8. The full line represents the density functions the degrees of membership of which are 0, 1 and 0. The limit density function for kurtosis $k_e = 1.8$ is the rectangular density function. All the density functions in Fig. 8 comply with the conditions that 95 % of imperfection e is within the tolerance limits $\langle -5.792, 5.792 \rangle$ mm.

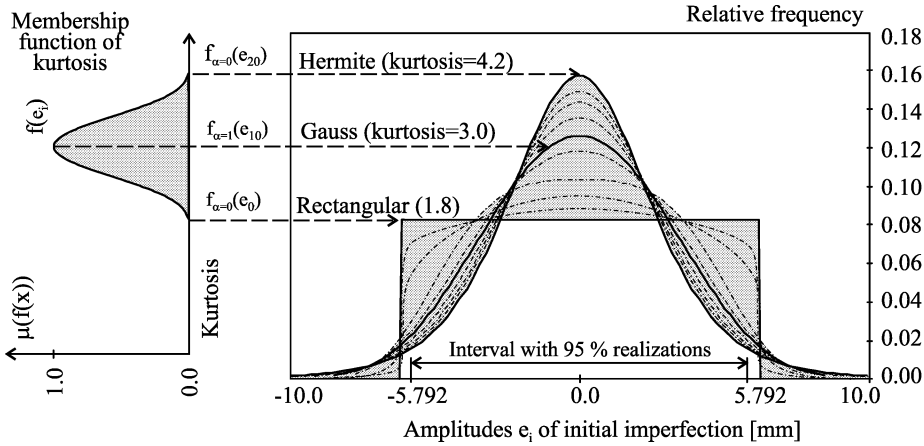

Fig.8: Fuzzy-random initial imperfection e

3. Computational model

The load-carrying capacity of the axially compressed member was designated as the force F at which the axial stress, in the most stressed section, is equal to the yield strength. Initial curvature and buckling were assumed in the direction perpendicular to the web. Initial imperfection in the form of the sine function was introduced. Based on these assumptions, the stress σ_x of the strut can be defined acc. to the relation:

$$\sigma_{\rm x} = \frac{F}{A} + \frac{F\,e}{W_{\rm z}\,(1-\frac{F}{F_{\rm cr}})} = f_{\rm y} \Rightarrow F \tag{1}$$

where A is the cross-sectional area, F is axial member force, W_z is the section modulus to the z-axis. F_{cr} is the Euler critical force, which, for a bilaterally hinged strut, is defined as:

$$F_{\rm cr} = \frac{\pi^2\,E\,I_{\rm z}}{L_{\rm cr}^2}\ . \tag{2}$$

A strut of non-dimensional slenderness ratio defined acc. to EUROCODE 3 [24] was analysed. The critical length for a bilaterally hinged strut is $L_{cr} = 1.93\,\mathrm{m}$. The evaluation of the load-carrying capacity according to (1) does not take into consideration the influence of residual stresses. Integration would require the utilization of FEM. The limit state is then defined upon attaining the design relative strain [7], which can then be consequently generalized even for the analysis of the uncertainty of the behaviour of welded joints.

4. Fuzzy-random analysis

The fuzzy analysis was evaluated according to the general extension principle [1, 12, 3]. Let $\bullet$ be an arithmetic operation (e.g. addition, division) and Z_1, $Z_2 \subseteq R$ be fuzzy numbers. The extension principle then allows the extension of operation $\bullet$ to the operation $\odot$ with fuzzy numbers in the following manner:

$$(Z_1 \odot Z_2) = \bigvee_{z=x\bullet y} \big(Z_1(x) \wedge Z_2(y)\big)\ . \tag{3}$$

The result of operation $\odot$ is a fuzzy number $Z_1 \odot Z_2$, consisting of elements $z = x \bullet y$ with a membership function that is given by the minimum of membership values of operators x into fuzzy number Z_1 and y into fuzzy number Z_2. The extension principle in the form of α-cuts was used for the analysis.

4.1. Fuzzy analysis of random load-carrying capacity

In Fig. 9 there are presented the density functions of load-carrying capacity computed for the kurtosis from the interval $k_e \in \{1.8, 4.2\}$. The density functions present the approximations of relative frequency histograms. Each density function was determined for 200 thousand runs of the Monte Carlo method. Analogously as in Fig. 8, significant density functions the degrees of membership of which are 0, 3 and 4.2 are set off by full lines.

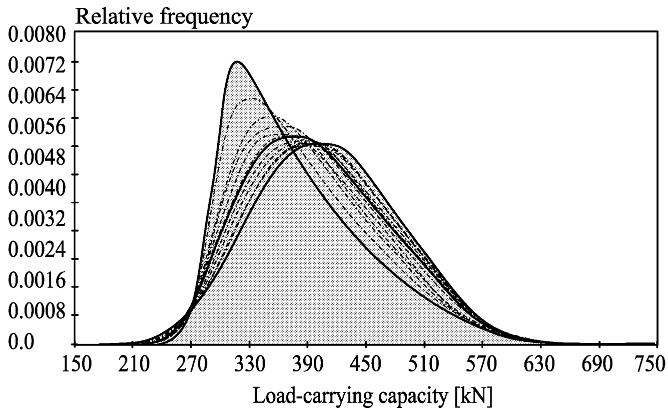

Fig.9: Set of load-carrying capacity density functions

A degree of membership is attributed to each density function from Fig. 9. Graphically, it is transparently feasible to set off only the fuzzy number of statistical characteristics. Fig. 10 shows the fuzzy number of mean value of load-carrying capacity. The fuzzy number of standard deviation of the load-carrying capacity is presented in Fig. 11. The skewness or kurtosis fuzzy numbers could be represented in a similar way.

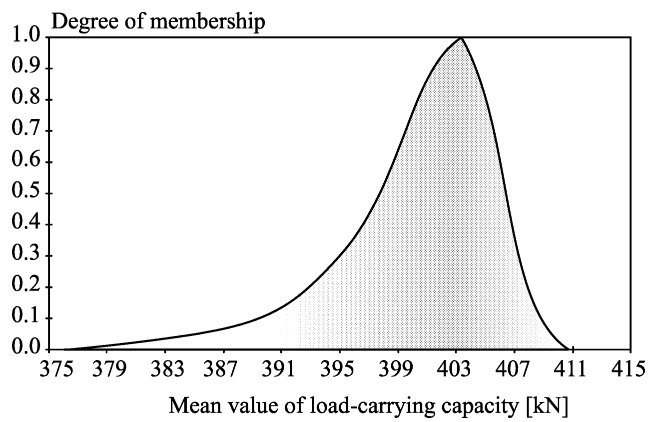

Fig.10: Fuzzy number of mean value of load-carrying capacity

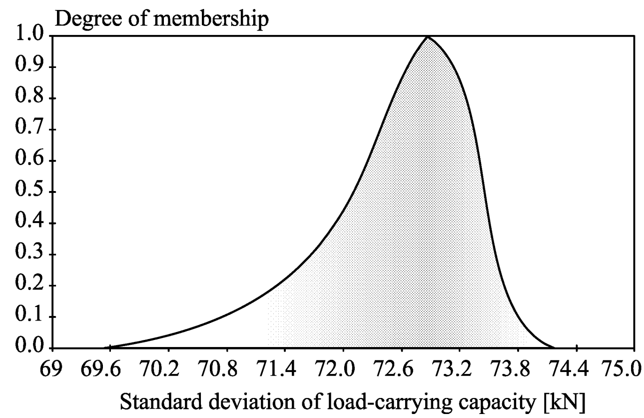

Fig.11: Fuzzy number of standard deviation of load-carrying capacity

4.2. Fuzzy analysis of failure probability

The fuzzy number of failure probability is presented in Fig. 12. The failure probability is defined as a probability saying that the loading is greater than the load-carrying capacity. The probability analysis was assessed by using the Monte Carlo method. So many runs of the method were applied that the failure would appear for 100 times minimum.

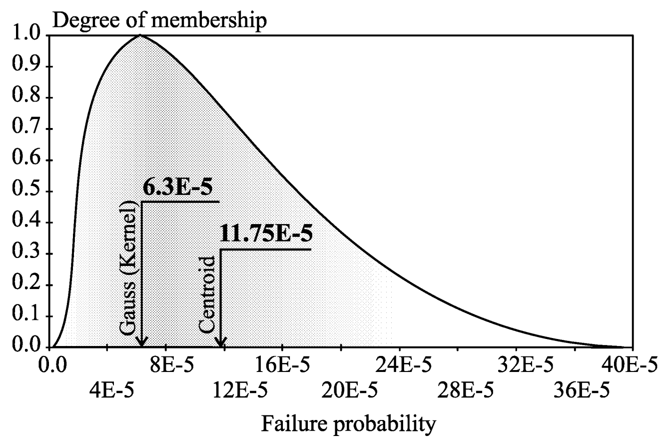

Fig.12: Fuzzy failure probability

The single crisp control output can be obtained by applying the so-called *defuzzification methods*. We have developed several methods which defuzzify fuzzy rules into a crisp control output. The most widespread and physically appealing of all the *defuzzification methods* is the centre of area or centre of gravity method, see Fig. 12. It is given by the algebraic expression:

$$P_{\mathrm{f}} = \frac{\int \mu(x)\, x \,\mathrm{d}x}{\int \mu(x)\,\mathrm{d}x} \tag{4}$$

where $\mu(x)$ denotes the membership function of failure probability. Quantity x means failure probability for all $x \in (0, \infty)$.

In Fig. 12 there is also presented the value 6.3E−5 which would be obtained by the stochastic analysis if the kurtosis were considered to be a singleton by value 3 (Gauss density function). Provided that the reference value of failure probability is 7.2E−5, the failure probability 11.75E−5 is then higher, and therefore the structure is not satisfactory. Compared with this, the purely stochastic value 6.3E−5 is satisfactory.

5. Fuzzy SBRA as a strong tool for complex uncertainty analysis

In recent years, the frequency of discussions about possibilities of application of probability computation in design activity increases. The method SBRA [8] belongs among the very effective and elaborated methods for evaluating the engineering reliability of structural systems. Some histograms of input random quantities are determined in a subjective manner, and can cause the vague uncertainty of solution.

If the same 'stochastic' computation is set to more experts, the fuzzy number of subjective solutions coming into consideration is the output. The uncertainty of histograms of system imperfections is a typical example [6]. The stochastic computation model becomes so the source of vague (fuzzy) uncertainty which, in complex systems, can significantly predominate over the stochastic uncertainty.

The specialists have been made familiar with this method satisfactorily. Now it is necessary to attempt at decreasing the vague (fuzzy) uncertainties of the SBRA method.

The histograms of random quantities for which there is not satisfactorily information based on experiments can be substituted by a fuzzy set of histograms. With the knowledge of fuzzy and stochastic uncertainty, it is possible to assess the *complex fuzzy-random uncertainty analysis*, e.g., how it has been presented in foregoing paragraphs. Let the fuzzy random modification of the SBRA method be named by the abbreviation SBRA. The fuzzy set of histograms is to be determined in a subjective manner according to the opinions of a large number of experts. The degree of membership is attributed according to the frequency of opinions appearing, see Fig. 13.

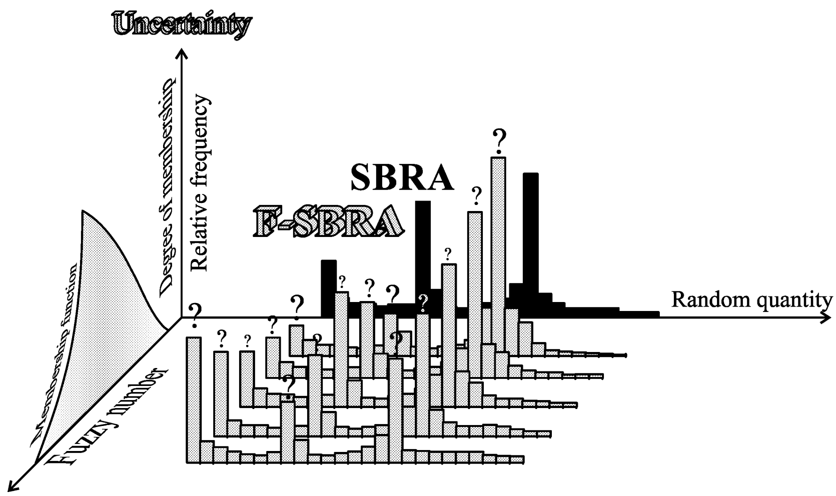

Fig.13: Fuzzy set of histogram of long term load action

An example of a fuzzy set of long term load actions can be seen in Fig. 13. Another application of the fuzzy analysis can take place in assessing the existing structures, as far as their residual service life is concerned [20]. The fuzzification process is rather very complicated in all the cases mentioned and it should be object of future analyses. The problems can be expected at the fuzzification of uncertainties of the correlation matrix of input random quantities. However, larger attention will have to be paid to this problem. As it has been shown in [6], the correlation negligence can lead to an error as high as 300 percent.

It can be mentioned for the completeness' sake that besides the uncertainty of the stochasticity type, also other uncertainty types can be distinguished when solving the complex systems [14]. They are: fuzzitivity (vagueness), non-specificity (bad limitation), and conflict which are examined within the framework of five theories, within which there has been created a method for their quantification (classical sets theory, fuzzy sets theory, probability theory, possibility theory, and Dempter-Shafer theory).

6. Conclusion

The asymmetric distribution of membership function of the failure probability is the cardinal information following from the fuzzy computation, see Fig. 12. We have obtained an asymmetric output distribution although the input membership function of kurtosis is symmetric, see Fig. 5. In Fig. 12 there is a large quantity of precious information at our disposal.

– Single crisp control output is 11.75E−5,
– Support of fuzzy failure probability is the interval (0.16E−5, 38.7E−5),
– Kernel of failure probability is equal to 6.3E−5.

Stochastic solution: Supposing that the amplitude e has a Gauss density function, all the quantities given in Tab. 1 are random ones; therefore a classical stochastic computation is concerned. The failure probability 6.3E−5 (singleton) is the output of this computation. The Gauss density function is probably the most logical choice, provided that we were forced to decide only for one (crisp) density function.

Fuzzy-random solution: The fuzzy number of initial random imperfection e is the input characteristic. The lack of exclusively objective statistical information on the kurtosis is the reason of introduction of a fuzzy number (see Fig. 5). The output is represented by a fuzzy number again. Provided that we want to obtain the single crisp control output the output fuzzy set is to be defuzzified. The value 11.75E−5 was obtained by the defuzzification of fuzzy failure probability.

Comparison of stochastic and fuzzy random solution: When comparing the defuzzified value 11.75E−5 with the value of classical stochastic analysis 6.3E−5, the difference is 200 percent!

The possibility of vague (fuzzy) uncertainty must be admitted in stochastic analysis SBRA. The computation model is always the source of vague (fuzzy) uncertainty which, in complex systems, can predominate over the stochastic uncertainty significantly. Provided that there is the exclusively objective statistical information not at the disposal, the fuzzy set of histograms – the so-called F-SBRA – can be defined, based on large quantity of experiments.

The probabilistic computation cannot be applied in design practice without creating generally valid standards including related instruments for probability design. The fuzzy random analysis can be a strong instrument which will help to create these standards.

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References

[1] Dubois D. Prade H.: Fuzzy Sets and Systems: Theory and Applications, Mathematics in Science and Engineering, Vol. 144, ISBN 0-12-222750-6.

[2] Fukumoto Y., Kajita N., Aoki T.: Evaluation of Column Curves Based on Probabilistic Concept, In Proc. of Int. Conference on Stability, Prelim. Rep., publ. by Gakujutsu Bunken Fukyu-Kai, Tokyo, 1976, pp. 1–37

[3] Jura P.: Basics of Fuzzy Logic for Control and Modeling , VUTIUM, 2003, p. 132, ISBN 80-214-2261-0 (in Czech)

[4] Kala Z.: Fuzzy Sets Theory in Comparison with Stochastic Methods to Analyze Non-linear Behaviour of a Steel Member under Compression, Journal Non-linear Analysis: Modeling and Control, Vol. 10, No. 1, 2005, pp. 65–75, ISSN 1392–5113

[5] Kala Z.: Sensitivity Analysis of the Stability Problems of Thin-walled Structures, Journal of Constructional Steel Research 61 (2005), pp. 415–422

[6] Kala Z.: From the Partial Safety Factor Methods to the Probabilistic Concept of the Assessment of Reliability of Systems, In Proc. Of VII. Conference on Reliability of Structures, Topic: Extension of Probabilistic Assessment from Elements and Members of Structures to Systems, Prague, 2006, pp. 21–32, ISBN 80-02-01770-6, (in Czech)

[7] Kmeť S., Tomko M., Brda J.: Probability Assessment of System Composed of Suspension Flexural-rigid Members, In: Proc. of VII Conf. on Reliability of Structures, Prague, pp. 67–78, ISBN 80-02-01770-6 (in Slovak)

[8] Marek P., Guštar M., Anagnos T.: Simulation-Based Reliability Assessment for Structural Engineering, CRC Press, Inc., Boca Raton, Florida, 1995

[9] Melcher J., Kala Z., Holický M., Fajkus M., Rozlívka L.: Design Characteristics of Structural Steels Based on Statistical Analysis of Metallurgical Products, Journal of Constructional Steel Research 60 (2004), pp. 795–808, ISSN 0143-974X

[10] Melcher J.: Thin-walled Metallic Beam in a Supporting Structural System, Thesis to obtain the title DrSc., University of Technology in Brno, Facul.ty of Civil Engineering, 1990), Brno, 1990 (in Czech)

[11] Möller B., Beer M. Reuter U.: Theoretical Basics of Fuzzy Randomness – Application to Time Series with Fuzzy Data, In CD Proc. of. 9th Int. Conf. On Structural Safety and Reliability ICOSSAR'05, Rome (Italy), 2005, ISBN 90-5966-040-4

[12] Novák V.: Fuzzy Sets and Their Applications, SNTL 1986 (in Czech)

[13] Páleš D.: Resistivity and Verification of Reliability of a Centrically Compressed Beam, Taking Buckling into Consideration, PhD Thesis, Bratislava: SAV, 1996 (in Slovak)

[14] Pokorný M.: Artificial Intelligence in Modeling and Control, BEN, Praha, 1996, ISBN 80-901984-4-9 (in Czech)

[15] Puri M.L., Ralescu D.: Fuzzy Random Variables, Journal Math. Anal. Appl. Vol. 114: 409–422, 1986

[16] Sadovský Z., Páleš D.: Third-moment Identification of Structural Element Resistance by Form, Building Research Journal, 1999, Vol. 47, No. 3, pp. 197–213

[17] Guedes Soares C.: Uncertainty Modelling in Plate Buckling, Structural Safety, 1988, (5), pp. 17–34

[18] Strauss A., Kala Z., Bergmeister K., Hoffmann S., Novák D.: Technologische Eigenschaften von Stählen im europäischen Vergleich, Stahlbau, 75 (2006), Januar 2006, Heft 1, ISSN 0038-9145

[19] Štemberk P. Kruis J.: Fuzzy Distribution of Internal Forces in Dynamically Loaded Structures, Journal Engineering Mechanics (in print)

[20] Vičan J., Koteš P.: Bridge Management System of Slovak Railways, In Proc. of the III. International Scientific Conference Quality and Reliability of Building Industry, Levoča (SR), 2003, pp. 539–542, ISBN 80-7099-746-X

[21] Wang G., Zang Y.: The Theory of Fuzzy Stochastic Processes, Fuzzy Sets Syst., Vol. 51: 161–178, 1992
[22] Zadeh L.A.: From Circuit Theory to System Theory, In Proc. of Institute of Ratio Eng., 50:pp. 856–865, 1962
[23] Zadeh L.A.: Fuzzy Sets, Information and Control, 8 (1965), 3, pp. 338–353
[24] EN 1993-1-1:2005(E): Eurocode 3: Design of Steel Structures – Part 1-1: General Rules and Rules for Buildings, CEN, 2005
[25] ENV 1090-2: 2005(E), Execution of Steel Structures and Aluminium Structures – Part 2: Technical Requirements for the Execution of Steel Structures, Stage 34, B-1050 Brussels

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