RESIDUAL STRESS EVALUATION BY THE HOLE-DRILLING METHOD WITH ECCENTRIC HOLE

Karel Švaříček, Miloš Vlk*

The hole drilling method is the commonest method for residual stresses measurement. This method usually assumes the centric hole within the strain gauge rosette. However, the hole is never exactly located centric with the centre of the strain gauge rosette. In such a case it is committing some inaccuracy at the evaluated residual stresses. This paper interprets the value of this inaccuracy and provides an extension of the hole drilling method to case with the eccentric hole by D. Vangi [1].

Key words : *residual stress, hole-drilling method, eccentricity, finite element method*

1. Introduction

The residual stresses are in almost all structures. These stresses can be useful or harmful. The useful effect is where the residual stresses act opposite to stresses from a loading. This effect is made by hardening, shot peening or cold rolling. The harmful effect occurs more often and we must measure this level of the residual stresses. The mostly used method is the hole-drilling method. This method measures released strains due to boring a small hole. This released strain is measured by a strain gauge rosette around the hole. But the hole is not drilled exactly centrically towards strain gauge rosette. For this reason we can use some correction for elimination of the deviation for example [2] and [3]. But these corrections are only for a thin plate with a through hole. New approach produces the residual stress evaluation by the hole- drilling method with off-center hole by D. Vangi [1]. This approach is used to the residual stress evaluation for HBM RY 61 S strain gauge rosette in this paper.

Procedure of the hole drilling method with eccentric hole For a general formulation (for the centered and eccentered hole) accordance with D. Vangi [1] can be written the relationship between measured relaxed strains and evaluated stresses :

$$\begin{bmatrix} \boldsymbol{\varepsilon}_\mathrm{A} \\ \boldsymbol{\varepsilon}_\mathrm{B} \\ \boldsymbol{\varepsilon}_\mathrm{C} \end{bmatrix} = \begin{bmatrix} \mathbf{H}\,\mathbf{a}_\mathrm{A}^\mathrm{M} \\ \mathbf{H}\,\mathbf{a}_\mathrm{B}^\mathrm{M} \\ \mathbf{H}\,\mathbf{a}_\mathrm{C}^\mathrm{M} \end{bmatrix} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \\ \mathbf{T} \end{bmatrix} \tag{1}$$

where :

$$\boldsymbol{\varepsilon}_\mathrm{A} = \begin{bmatrix} \varepsilon_\mathrm{A}(z_1) & \cdots & \varepsilon_\mathrm{A}(z_n) \end{bmatrix}^\mathrm{T} \ , \tag{2}$$

$$\boldsymbol{\varepsilon}_\mathrm{B} = \begin{bmatrix} \varepsilon_\mathrm{B}(z_1) & \cdots & \varepsilon_\mathrm{B}(z_n) \end{bmatrix}^\mathrm{T} \ , \tag{3}$$

$$\boldsymbol{\varepsilon}_\mathrm{C} = \begin{bmatrix} \varepsilon_\mathrm{C}(z_1) & \cdots & \varepsilon_\mathrm{C}(z_n) \end{bmatrix}^\mathrm{T} \ , \tag{4}$$

* Ing. K. Švaříček, doc. Ing. M. Vlk, CSc., Institute of Solid Mechanics, Mechatronics and Biomechanics, Faculty of mechanical Engineering, BUT, Technická 2, 616 69 Brno

$\varepsilon_A(z_i)$, $\varepsilon_B(z_i)$, $\varepsilon_C(z_i)$ are the relaxed strains measured by the strain gauge A, B and C at the hole depth z_i and n is number of a hole depth.

The matrix $\mathbf{H}$ is material properties matrix defined :

$$\mathbf{H} = \left[\frac{1}{E}\mathbf{I} \quad \frac{\mu}{E}\mathbf{I} \quad \mathbf{0} \right] , \tag{5}$$

E is modulus of elasticity and μ is Poisson's ratio.

The matrixes $\mathbf{a}_j^{\mathrm{M}}$ are the transformed coefficient matrixes :

$$a_j^{\mathrm{M}} = \mathbf{M}(-\alpha_j)\,\mathbf{a}_j\,\mathbf{M}(\zeta_j) , \tag{6}$$

$$j \in \{\mathrm{A},\mathrm{B},\mathrm{C}\} . \tag{7}$$

The coefficient matrixes have this form :

$$\mathbf{a}_j = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{0} \\ \mathbf{a}_3 & \mathbf{a}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{a}_5 \end{bmatrix}_j , \tag{8}$$

$$\mathbf{a}_k = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{12} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_k \tag{9}$$

$$k \in \{1,2,3,4,5\} . \tag{10}$$

Each coefficient matrix $\mathbf{a}_j$ (means $\mathbf{a}_\mathrm{A}$, $\mathbf{a}_\mathrm{B}$, $\mathbf{a}_\mathrm{C}$) must be interpolated to the mean radius of the strain gauge rosette $R_{\mathrm{m}j}$ (R_{mA}, R_{mB}, R_{mC}).

$$R_{\mathrm{m}j} = \sqrt{e^2 + R_\mathrm{m}^2 - 2\,e\,R_\mathrm{m}\,\cos(\varphi_j - \beta)} . \tag{11}$$

The angle φ_j is a turning of the strain gauges of the strain gauge rosette (Fig. 1). The angle $\varphi_\mathrm{A} = 0°$, $\varphi_\mathrm{B} = -135°$ and $\varphi_\mathrm{C} = 90°$ can be considered for the strain gauge rosette RY 61 S by the mean radius $R_\mathrm{m} = 2.55$. The value e is the eccentricity and β is its turning.

The transformation matrix $\mathbf{M}$ is defined :

$$\mathbf{M}(\delta) = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cos(2\delta)\,\mathbf{I} & \sin(2\delta)\,\mathbf{I} \\ \mathbf{0} & -\sin(2\delta)\,\mathbf{I} & \cos(2\delta)\,\mathbf{I} \end{bmatrix} , \tag{12}$$

$$\delta \in \{\alpha_j, \zeta_j\} . \tag{13}$$

The angles α_j and ζ_j are shown on Fig. 1 and can be determined from :

$$\sin\alpha_j = \frac{e\,\sin(\varphi_j - \beta)}{R_{\mathrm{m}j}} , \tag{14}$$

$$\zeta_j = \varphi_j - \alpha_j . \tag{15}$$

I is an unitary matrix of dimension n defined:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} , \tag{16}$$

$$\mathbf{P} = [\, P(z_1) \quad \cdots \quad P(z_n)\,]^{\mathrm{T}} , \tag{17}$$

$$\mathbf{Q} = [\, Q(z_1) \quad \cdots \quad Q(z_n)\,]^{\mathrm{T}} , \tag{18}$$

$$\mathbf{T} = [\, T(z_1) \quad \cdots \quad T(z_n)\,]^{\mathrm{T}} , \tag{19}$$

The required principal residual stresses σ_1, σ_2 and the anticlockwise rotation angle γ of the principal stresses from the strain gauge **A** is given from **P**, **Q** and **T**.

$$\sigma_{1,2}(z_i) = P(z_i) \pm \sqrt{Q(z_i)^2 + T(z_i)^2} , \tag{20}$$

$$\gamma(z_i) = \arctan\left(\frac{T(z_i)}{Q(z_i)}\right) . \tag{21}$$

The calibration coefficients $a_1, a_2, \ldots, a_5$ are given by the relationship between the relaxed stresses σ_{r}, σ_{t}, τ_{rt} and the pre-existent stresses σ_{x}, σ_{y}, τ_{xy}.

$$2\,\sigma_{\mathrm{r}} = a_1\,(\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}}) + a_2\,(\sigma_{\mathrm{x}} - \sigma_{\mathrm{y}}) , \tag{22}$$

$$2\,\sigma_{\mathrm{t}} = a_3\,(\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}}) + a_4\,(\sigma_{\mathrm{x}} - \sigma_{\mathrm{y}}) , \tag{23}$$

$$\tau_{\mathrm{rt}} = a_5\,\tau_{\mathrm{xy}} . \tag{24}$$

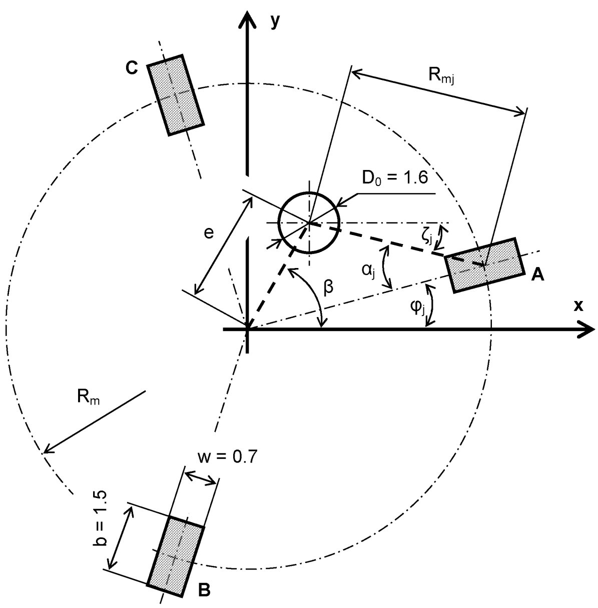

Fig.1: The strain gauge rosette RY 61 S; the eccentric hole

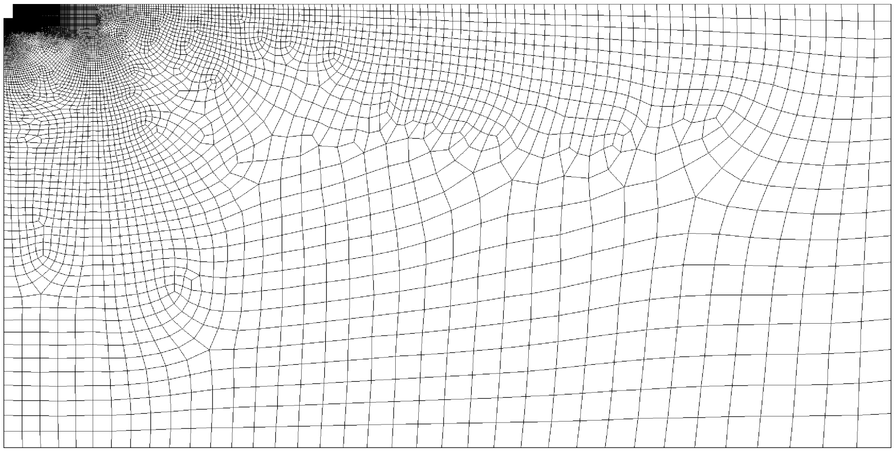

Fig.2: The finite element model

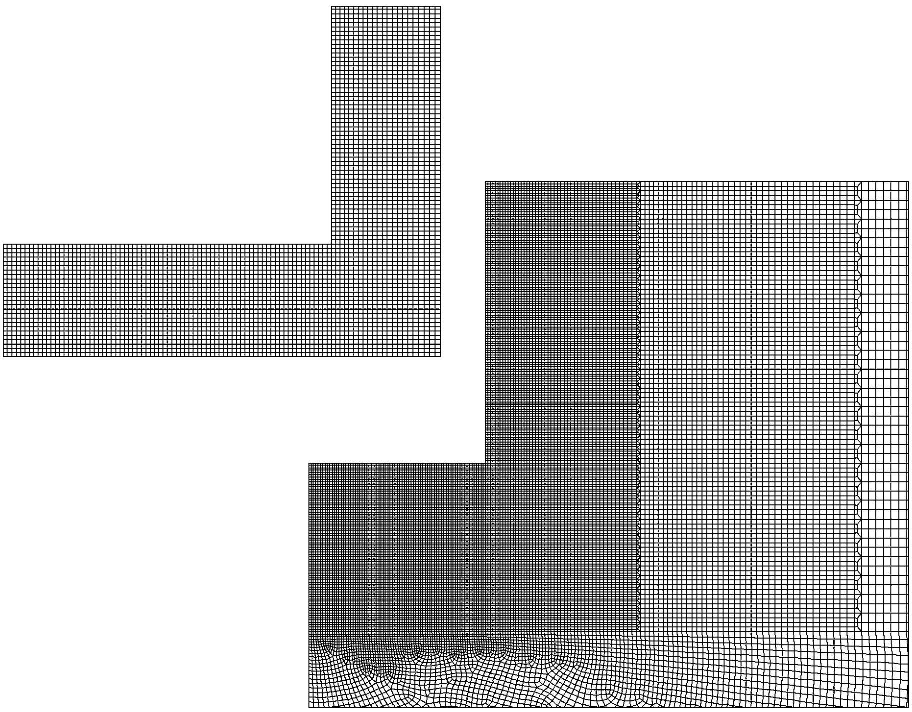

Fig.3: A detail of the FEM

2. The determination of the calibration coefficients

The calibration coefficients were determined by using the finite element method (FEM). Two dimensional axisymmetric model was created to the description of the real three dimensional body. The geometrical parameters of this model with strain gauge rosette RY 61 S are illustrated on Fig. 1. The hole depth is in the range from 0.255 to 1.275 mm in five increments. The mean radius of the strain gauge rosette R_{m} is changed from 2.25 mm to 2.85 mm with an increment 0.05 mm. The 2D model (Fig. 2 and 3) is acceptable when a non-axisymmetric load is considered. This non-axisymmetric load is done by Fourrier series. The material is linear, homogenous and isotropic. Modulus of elasticity $E = 210000\,\mathrm{MPa}$ and the Poisson's ratio $\mu = 0.3$. The model with these parameters is created in [4]. The total number 825 scalar coefficients were evaluated.

3. The residual stresses evaluation

A response of the strain gauge rosette was simulated. This response was simulated for some states of stress and some kind of eccentricity. The uniaxial stress, the equibiaxial stress and the shear stress were the considered states of stress (Fig. 4). The stress intensity was stated to 100 MPa in the solid without the hole. The eccentricity was 0.2 mm. The turning of the eccentricity – the angle β – was from 0 degrees to 315 degrees with diference 45 degrees. The angle between the first principal stress and the strain gauge A was 0 degrees.

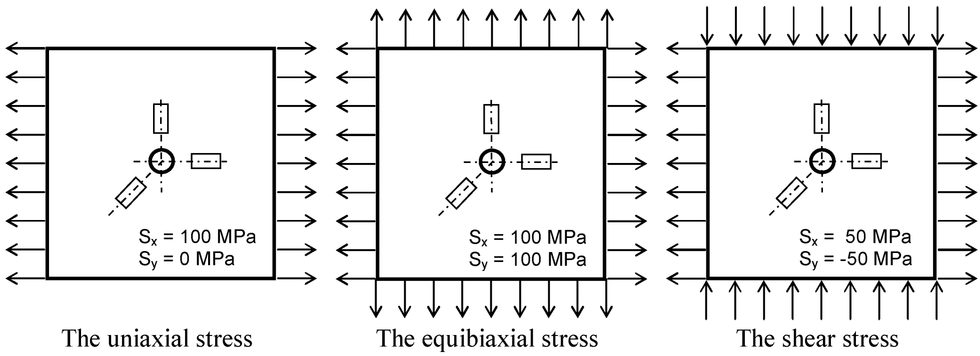

Fig.4: The considered states of stress

These responses – the simulated relaxed strains – were the input values to the equations above and the request residual stresses were determined. The determined residual stresses in the stress intensity form are listed in the table 1, 2 and 3. The true value of the residual stress intensity is 100 MPa for each case.

The hole depth [mm]	The turning of the eccentricity [deg]							
	0	45	90	135	180	225	270	315
0,255	100,2	99,2	98,6	99,2	100,3	99,3	98,5	98,9
0,510	100,3	99,2	98,6	99,1	100,5	99,3	98,4	98,8
0,765	100,7	99,1	98,6	99,1	100,9	99,3	98,4	98,7
1,020	101,4	99,1	98,6	99,2	101,6	99,2	98,2	98,4
1,275	104,2	99,0	98,7	99,4	103,5	99,2	97,9	97,8

Tab.1: The determined stress intensity for the uniaxial stress

The hole depth [mm]	The turning of the eccentricity [deg]							
	0	45	90	135	180	225	270	315
0,255	99,8	99,8	99,8	99,8	99,8	99,8	99,8	99,8
0,510	99,8	99,8	99,8	99,8	99,8	99,8	99,8	99,8
0,765	99,7	99,7	99,7	99,7	99,7	99,7	99,7	99,7
1,020	99,6	99,7	99,6	99,7	99,7	99,6	99,7	99,7
1,275	99,5	99,7	99,5	99,6	99,7	99,6	99,7	99,6

Tab.2: The determined stress intensity for the equibiaxial stress

The hole depth [mm]	The turning of the eccentricity [deg]							
	0	45	90	135	180	225	270	315
0,255	99,0	99,0	99,0	98,7	98,9	99,2	98,9	98,7
0,510	98,9	98,9	98,9	98,5	98,8	99,1	98,8	98,5
0,765	98,8	98,8	98,8	98,4	98,6	99,1	98,6	98,4
1,020	98,7	98,6	98,7	98,1	98,4	99,0	98,4	98,1
1,275	98,9	98,4	98,9	97,8	98,0	98,9	98,0	97,8

Tab.3: The determined stress intensity for the shear stress

The hole depth [mm]	The turning of the eccentricity [deg]							
	0	45	90	135	180	225	270	315
0,255	137,8	122,8	103,5	87,5	80,7	88,7	100,1	122,1
0,510	129,2	116,9	103,9	90,5	82,8	86,7	99,9	118,2
0,765	113,8	107,3	103,1	95,6	89,2	90,0	99,8	110,1
1,020	99,1	99,7	101,8	101,1	97,9	98,1	100,0	101,3
1,275	93,8	94,5	100,4	107,3	117,2	112,3	100,1	95,5

Tab.4: The determined stress intensity for the uniaxial stress, the eccentricity is not accounted

The hole depth [mm]	The turning of the eccentricity [deg]							
	0	45	90	135	180	225	270	315
0,255	142,3	151,2	142,3	114,9	110,5	116,3	110,5	114,9
0,510	129,1	136,8	129,1	109,4	105,3	107,9	105,3	109,4
0,765	108,0	112,2	108,0	101,2	99,1	98,4	99,1	101,2
1,020	96,6	95,1	96,6	98,8	104,7	108,6	104,7	98,8
1,275	85,4	79,1	85,4	101,0	127,2	140,6	127,2	101,0

Tab.5: The determined stress intensity for the equibiaxial stress, the eccentricity is not accounted

The hole depth [mm]	The turning of the eccentricity [deg]							
	0	45	90	135	180	225	270	315
0,255	114,8	119,2	114,8	100,4	87,1	81,6	87,1	100,4
0,510	111,0	115,0	111,0	99,5	88,5	83,5	88,5	99,5
0,765	104,7	107,1	104,7	98,7	92,1	88,3	92,1	98,7
1,020	99,7	99,6	99,7	99,2	96,4	93,7	96,4	99,2
1,275	96,4	93,5	96,4	101,0	100,9	98,8	100,9	101,0

Tab.6: The determined stress intensity for the shear stress, the eccentricity is not accounted

The table 4, 5 and 6 lists the determined residual stress intensities, for the same simulated relaxed strains above, by not accounted eccentricity according to [5] or [6].

4. Conclusion

This paper has described the Vangi's method for the residual stress determination with an eccentric hole. The necessary calibration coefficients were determined. Relaxed strains are simulated for eccentricity 0.2 mm for the uniaxial, the equibiaxial and the shear state stress and for the strain gauge rosette RY 61 S. The value of the simulated residual stress intensity is 100 MPa. The residual stress intensity was determined according to Vangi [1] and Schajer [6]. A maximum deviation with the respecting of the eccentricity for the uniaxial stress, the equibiaxial stress and the shear stress is 4.2 %, 0.5 % and 2.3 % respectively. The maximum deviation without the respecting of the eccentricity for the three stated stresses is 37.8 %, 51.2 % and 19.2 % respectively. These results give a good agreement to the true stresses when the eccentricity is respected. While the results without respecting of the eccentricity are unsatisfactory. The Vangi's method of the residual stress determining is suitable and necessary for the hole drilling method with the significantly eccentric hole.

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