METHOD FOR ANALYSING THE FUNCTIONALITY, RELIABILITY AND SENSITIVITY OF A MECHANISM BY COMBINED COMPUTER AIDED TOLERANCE ANALYSIS AND MULTI-BODY SYSTEMS SIMULATION – FUNCTIONALITY ANALYSIS WITH CAT AND MBS SIMULATION

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The dominating paradigm in mass production is the minimisation of loss units, sometimes referred as the Six Sigma philosophy. This requires the management of the geometrical variation of a product caused by the design, manufacturing and assembly processes. The geometrical variation is also a source of malfunction in a product; the specification of geometry and tolerance variation therefore affects its manufacturing, assembly and behaviour. Traditionally, feedback from the outcome of these processes has controlled the design. The short lead-time and other requirements of mass production, however, require design methods with faster feedback. Simulation tools such as Computer Aided Tolerance analysis and Multi-Body Systems analysis provide designers with models that can be utilized for this purpose. The objective of this paper is to study the similarities of modelling components in Computer Aided Tolerancing (CAT) and Multi-Body System analysis (MBS). Based on the study, a general process model for concurrent utilisation of the tools in the design process is presented. The dimensional variation analysed by CAT predicts the tolerance level of Six Sigma limits. The variation of Six Sigma limits is utilised in the MBS analysis to predict the functional variation of a product. Finally, the behaviour of mass-produced products is predicted as a function of their tolerance limits.

Key words : *computer aided tolerancing, multi-body system analysis*

1. Introduction

1.1. Background

The management of the mass production of electro-mechanical products requires the management of manufacturing tolerances at part and assembly levels. In recent years, a new manufacturing paradigm, Six Sigma quality management, has changed manufacturing strategy and minimized the quota of scrap units. In order to minimise time consumption and determine costs, the management of tolerance analysis has been transferred to the design phase to precede manufacturing. During this phase, the manufacturing process is not defined and the costs of process changes are less. The development of Computer Aided Engineering

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(CAE) has enabled two tools to simulate the product functions in early design: Multi-Body Systems (MBS) simulation and Computer Aided Tolerancing (CAT).

Computer Aided Tolerancing is a tool that simulates the effects of dimensional variation of the manufacturing of parts and their assembly as completed products. CAT models often consider the parts ideally rigid kinematically, with no elasticity or friction to compute the mutual position change between interacting parts and their clearances. The rigid and kinematic model is a strong idealisation; physically more realistic results are obtained by Multi-Body Systems simulation with elastic joints. The designer should ensure that the interacting parts have contacts only over bearing elements.

Tolerance analysis consists of tolerance specification, variation modelling, and sensitivity analysis. In tolerance specification, the allowed variation in shapes and configurations are defined for the parts of the system. The specification can be parametric, geometric [1, 5] or vector based [8]. Variation modelling produces mathematical models that map the tolerance specifications to assembly and function variations. Finally, with sensitivity analysis, the critical properties are studied with worst-case and statistical analyses.

1.2. Clearance effects

While, with CAT, the geometrical clearance can be analysed, MBS modelling is required to physically simulate the clearance function of a product. Slide bearings are often subjected to dynamic loading conditions, and therefore must be designed against fatigue failure. Accurate prediction of the dynamic behaviour of the bearing contact is therefore required. The clearance falls into three functional categories – negative clearance or compression, zero clearance and positive clearance – for every sliding bearing depending on the free distance between the shaft and the housing.

- In negative clearance, the shaft and the housing are compressed together so tightly that no free distance is available. The negative distance describes how much the parts would be compressed inside each other if the bodies were rigid rather than elastic. The bearing contact force is in the normal direction of the bearing functional plane, while its force magnitude is a function of the negative compression length.
- In zero-clearance, the parts are compressed against each other so that no free length exists between the shaft and housing. The compression force is very low or zero.
- In positive clearance, a loss of contact has occurred and there is a free gap between shaft and housing. When this happens, the bearing contact force is reduced to zero enabling the pin to move freely in the clearance gap. If this occurs, undesirable impact may result when contact is remade. To avoid such loss, a model, able to predict bearing forces accurately is required.

The objective of this paper is to present a method for the mutual utilisation of CAT and MBS simulation early in the product design phase to ensure the electro-mechanical products readiness for mass production. The method utilises the tolerance variation data from critical functional dimensions, i.e., the joint clearances. The variation of CAT clearances are utilised in MBS analysis to predict the behaviour of a slider mechanism as a function of clearance variation. This is the original contribution of the paper.

1.3. Review of the literature

Tolerance design involves tolerance specification, tolerance analysis and tolerance synthesis. First, the tolerance system is specified; in analysis, the product has given dimensions and tolerances, while, in synthesis, these are changed to obtain better functionality and less scrap. Statistical tolerance analysis has been reviewed by Turner and Nigam [4] and tolerance synthesis methods by Voelcker [5], Juster [6], and Chase and Parkinson [7]. According to [11], kinematic tolerance analysis falls into three general categories: static (small displacement) analysis, kinematic (large displacement) analysis and kinematic analysis with contact changes. The one-dimensional approach is simplest, but very applicable to many engineering problems and based on tolerance stack analysis. Simple min-max analysis gives the worst case (WC) analysis by adding the minimal and maximal dimensions. Another deterministic analysis type is DOE, Design of Experiments, where parameter sensitivity is studied by systematic variation. Many other deterministic methods exist, like integer programming, non-linear programming and heuristics, reviewed by Kusiak and Feng [2]. In statistical variation, the probability of worst limits decreases near zero, and the tolerance limits increases the manufacturing costs. Assumptions for the statistical distribution must therefore be applied. The RSS (Root Sum Square) approach computes the distribution usually assuming normal distribution. The synthesis of tolerance limits is then selected to follow standard deviation with $6\,\sigma$ ($\pm 3\,\sigma$) standard deviation limits leading to 2.7 scrapped units per thousand. The Monte Carlo method is a simple and popular method of statistical analysis. It is suitable for stack, but more applicable to planar and spatial tolerance analysis. Random values for variations are generated according to statistical distribution; the method is therefore also applicable to distributions other than normal. It can be easily applied to linear and non-linear response functions, since the function values are computed by simulation. The major drawback of the method is that, in contrast to deterministic methods, intensive simulation is required to get accurate estimates. On the other hand, if the number of points is insufficient, the Monte Carlo analysis becomes inaccurate. The number of evaluations is considerably reduced in Taguchi method. This reduces considerably the computational effort, apparently without compromising the reliability of the results.

In this paper, statistical tolerance analysis is used. Commercial CAT and MBS software are utilised. 3D solid geometry models are imported from CAD geometry. The modelling and solving of kinematic (small displacement) tolerance equations are performed with VisVSA and the dynamic (large displacement) motion analysis is performed with ADAMS.

2. Methods for CAT-MBS DOE-integration

2.1. Computer Aided Tolerance analysis

Computer Aided Tolerance analysis enables the computation of the mutual part clearance vector $\Delta\bar{\mathbf{q}}$. It depends on the geometry (dimension) variation vector $\bar{\mathbf{x}}$ and assembly variation vector $\bar{\mathbf{y}}$, which describes the position and orientation of all parts relative to the origin of the ground part. CAT analysis then solves the clearances in the mechanism system by computing the clearance vector $\Delta\bar{\mathbf{q}}$ several times, with appropriate variations on part and assembly parameters, the Monte Carlo method, for instance.

$$\Delta\bar{\mathbf{q}} = \Delta\bar{\mathbf{q}}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) \ . \tag{1}$$

It is possible by this method to obtain the statistical variation of the clearance vector.

The clearance vector $\Delta\bar{\mathbf{q}}$ describes the clearance between two mating parts in some certain designer selected points. To be re-utilised in multi-body analysis, the clearance points must be identical in CAT and MBS analysis. In MBS analysis, the joint clearances are described with position feedback force functions modelling the contact phenomena at joints (Figure 1 and 4).

In this paper, the numerical analysis of CAD geometry clearance and multi-body dynamics is performed with commercial software. The mechanical assembly vector $\bar{\mathbf{y}}$ can also be computed using the coordinate transformation method. For each part in the assembly, its position relative to origin is assumed to depend on its location in the chain of mated parts. The position chain is dependent on the variations in the mating chain. Each mated part varies the chain by three coordinate system transformations [9] :

1. The large displacement of part i due to kinematic displacement of joint $\mathbf{T}(\mathbf{q}_{\mathrm{kin}i})$ part i's small internal deformation due to tolerance variation $\mathbf{T}(\mathbf{q}_{\mathrm{tol}i})$.
2. Small assembly variation due to changes in part i's assembly tolerance $\mathbf{T}(\mathbf{q}_{\mathrm{as}i})$.

The coordinate system transforms from kinematic to tolerance and assembly variation are shown in Figure 1.

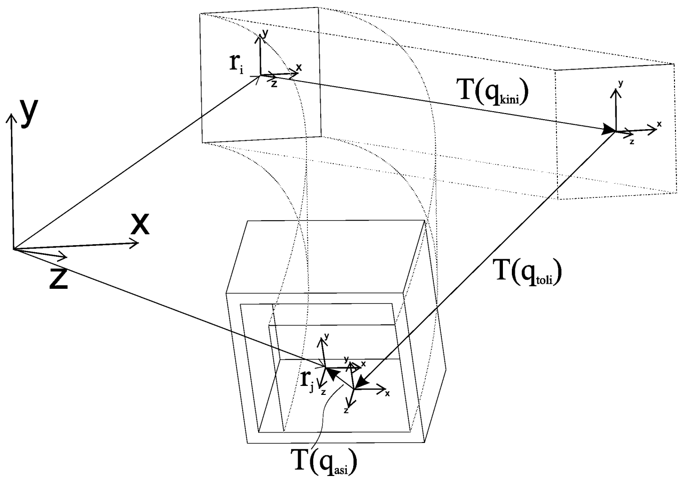

Fig.1: The coordinate transforms from part i to j with prismatic joint

The total chain of displacement for part n in the chain is thus

$$\mathbf{T}_{\mathrm{tot}} = \prod_{i=1}^{n} \mathbf{T}(\mathbf{q}_{\mathrm{kin}i})\,\mathbf{T}(\mathbf{q}_{\mathrm{tol}i})\,\mathbf{T}(\mathbf{q}_{\mathrm{as}i}) \ . \tag{2}$$

The total transformation with part geometry variation, kinematic movement, and assembly variation of the part mating chain is thus :

$$\mathbf{r}_{\mathrm{tot}} = \underset{4\times 4}{\mathbf{T}}{}_{\mathrm{tot}}\,\mathbf{r}_{\mathrm{br}} \tag{3}$$

where the dimension vectors depending on the geometry are expressed by the GD&T method (Geometric Dimensioning and Tolerancing). The variation chain method described by eq. (2)

is independent of the way tolerances are described; it can therefore be applied also to the Vectorial Dimensioning and Tolerance Concept originally described by Humienny [8]. The coordinate transformations are expressed by 4×4 homogenous transformation matrices (HTM-matrix), where the co-ordinate system is first rotated with a 3×3 submatrix by Euler angels or some other rotation specification method, and then translated with a 4×1 translation vector

$$\underset{4\times4}{\mathbf{T}}(\alpha,\beta,\gamma,r_\mathrm{x},r_\mathrm{y},r_\mathrm{z}) = \begin{bmatrix} \underset{3\times3}{\mathbf{R}(\alpha,\beta,\gamma)} & \begin{matrix} r_\mathrm{x} \\ r_\mathrm{y} \\ r_\mathrm{y} \end{matrix} \\ \underset{1\times3}{\mathbf{0}} & 1 \end{bmatrix} . \tag{4}$$

2.2. Multi-Body Systems analysis

The dimension of the assembly constraint vector is defined in MBS-analysis. The total degrees of freedom of the mechanism assembly is

$$n_\mathrm{tot} = 6\,(P-1)\ . \tag{5}$$

It is divided into two kinds of constraints, where the total number of constraints is n_cons: ideally kinematic constraints n_k and flexible tolerance modelling constraints n_cl

$$n_\mathrm{cons} = n_\mathrm{k} + n_\mathrm{cl} \tag{6}$$

where n_k is the number of constrained degrees of freedom by kinematic joints given by the equation

$$n_\mathrm{k} = \sum_{i=1}^{5} i\,c_i \tag{7}$$

where c_i is the number of joints constraining i degrees of freedom.

Thus, the large unconstrained displacement degree of freedom is n

$$n = n_\mathrm{tot} - n_\mathrm{cons} = n_\mathrm{tot} - n_\mathrm{k} - n_\mathrm{cl} = 6\,(P-1) - \sum_{i=1}^{5} i\,c_i - n_\mathrm{cl}\ . \tag{8}$$

The large displacement (dynamic) degree of freedom therefore depends on how many constraints there are (as in the Grübler equation). In kinematic systems, there are no elastic clearance joints, thus $n_\mathrm{cl} = 0$. However, in real mechanisms all constraints are flexible, thus $n_\mathrm{k} = 0$. There might also be redundant constraints in the systems; however, those must be considered as a special case and are not discussed in this paper.

The clearance gap variation of each component of the $\Delta\bar{\mathbf{q}}$ vector describes the clearance variation in flexible n_cl-restricted joint dimensions. In each dimension, the variation is computed with CAT Variational Systems Analysis software using a Monte Carlo analysis with a given tolerance specification. The variation results in a Gauss distributed variation of clearance. For each dimension j, the average value of clearance x_j and standard deviation σ_j is computed. These values are utilised in MBS-analysis, where the minimum clearance can be varied using an equation for each gap q_j. If $n_j = 3.0$, 99.87 % of products will have a greater clearance than the analysed mechanism conforming 1 300 scrapped units per million.

$$q_j = x_j - n_j\,\sigma_j\ . \tag{9}$$

The clearance gaps described by the clearance vector Δq are modelled by force restrictions in each dimension by position-dependent force functions in ADAMS, see Figure 2. These forces are acting in the normal direction of the plane. In the direction of the plane, Coulomb friction is assumed.

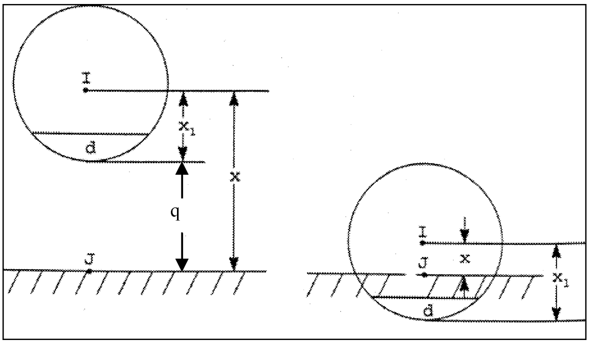

Fig.2: Impact-function's contact geometry

The contact force function is given by equation

$$F(x,\dot{x}) = \begin{cases} \max[0, k(x_1-x)^e - S(x, x_1-d, 1, x_1, 0)\, c\, \dot{x}] & \text{for } x < x_1 \ , \\ 0 & \text{for } x \geq x_1 \end{cases} \tag{10}$$

where S is a ADAMS STEP-function, mathematically a cubic spline as

$$S(x,x_0,h_0,x_1,h_1) = \begin{cases} x_1 & \text{for } x \leq x_0 \ , \\ h_0 + (h_1-h_0)\left[3\left(\dfrac{x-x_0}{x_1-x_0}\right)^2 - 2\left(\dfrac{x-x_0}{x_1-x_0}\right)^3\right] & \text{for } x_0 < x < x_1 \ , \\ h_1 & \text{for } x > x_1 \ , \end{cases}$$

k – contact stiffness, x_1 – free length of x (if x is less then x_1, the contact is established), e – force exponent of deformation characteristic, for stiffening contact $e > 1.0$, c – maximum damping coefficient, d – positive variable defining the penetration depth when maximum damping is applied.

The free clearance distance $q = x - x_1$ varies according the free clearance Δq_i for each dimension in each contact. ADAMS/Solver has a function similar to the above one, IMPACT and BISTOP. The Bistop-function is similar to the Impact-function, but here the clearance is limited from both sides from the maximum and minimum dimension, thus there are two equations (9) for each dimension q_i.

3. Example: CD-ROM Drive

3.1. Tolerance analysis

A CD-ROM drive slide frictional behaviour variation was analysed with three programs. The geometry was modelled with Pro/Engineer, the tolerance with VisVSA tolerance analysis software and the function was simulated with ADAMS Multi-Body Systems analysis program. The tolerance analysis was utilised to compute the clearance mean and standard deviation variation of the slider joints.

Mutual joint width	124 mm
Mutual joint distance	36 mm
Slider mass	81.8 g
Nominal height clearance	0.4 mm
Nominal width clearance	0.4 mm
Joint stiffness	100 N/mm
Joint damping	1 Ns/mm
Maximum damping depth	0.1 mm
Joint progressiveness exponent	1.5
Coulomb coefficient of friction	0.3

Tab.1: Mechanism data

The CD-ROM slider mechanism contained four linear glide joints, allowing the translational movement to have one degree of freedom. The mechanism had the following parameters (see Table 1).

The model of the CD-ROM drive consisted of 29 plane- and 18 point-features. Since there was no statistical data for tolerance measures for the demonstration example of this method, tolerances according to the ISO 2768H for flatness were chosen as the default.

Tolerances in the frame

For the clearance analysis between the mating elements of the slide and the frame, only features having tolerance dimensions can be tolerated. The size of the surface is not tolerated and therefore has no effect in the analysis process in VisVSA.

The following tolerances are included in the tolerance library:

- Connection between *frame/pintops* contains a profile tolerance for the top planes of all the pins on the ground plane of the frame.
- *Frame/wallpins* contains a profile tolerance for the lower plane of the pins on the side walls of the frame.
- *Frame/pinsides* contains parallelism tolerances between the left and the right plane of each pin on the ground plane of the frame.
- *Frame/large planes* contains a flatness tolerance for the ground and the side walls of the frame.

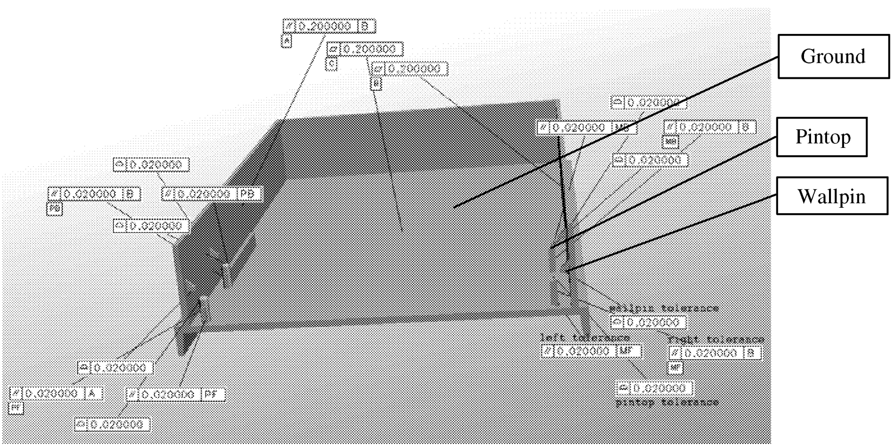

Fig.3: Defining tolerances (ISO) and references in the frame

Tolerances in the slide

The slide contains the rails for the guided motion of the mechanism. Important tolerances for the clearance analysis are all in the tolerance library:

– *Slide/flat* contains a flatness tolerance for rail planes, that serve as datum references,
– *Slide/parallel* contains a parallelism tolerance for the rail planes.

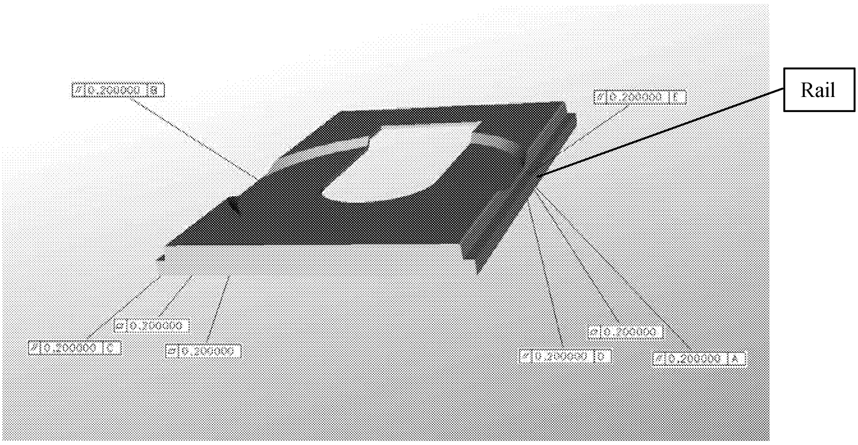

Fig.4: Defining tolerances (ISO) and references in the slide

Measures and Clearance analysis

In the model of the CD-ROM-drive, clearances are critical dimensions in the mechanism. They are computed as *measures*, *critical dimensions as function of defining tolerances*. Each pin-rail-wallpin-setup needs four dimensions to be fully described, see Figure 5. Also, two measurements are necessary to determine the distance of the two rails in the front and in the back of the slide. The model contains 18 measurement operations in total.

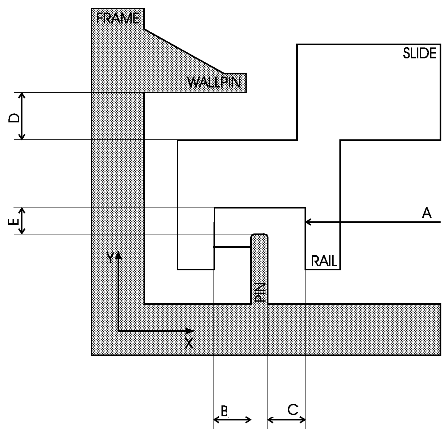

Fig.5: Clearances according to the measurement operations

All the measurements are distances from a plane. The result, calculated by VisVSA, is always the shortest distance between the plane and the definite point on the other surface.

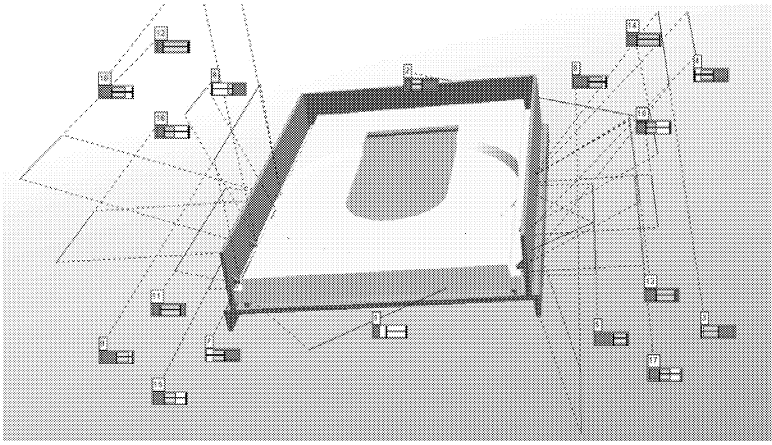

Fig.6: Measuring points to be computed in result-screen in VisVSA

LEFT BACK PIN				n	3.43
clearance	Nr.	mean value	std dev		
left of pin B	10	0.0064	0.0404	Mean	0.2368
right of pin C	8	0.4327	0.0266	Clearance	0.2093
from pin to rail E	16	−0.0003	0.0145	Mean	0.2331
from wallpin to slide D	12	0.5757	0.0465	Clearance	0.3662
RIGHT BACK PIN					
clearance	Nr.	mean value	std dev		
left of pin B	6	0.2049	0.0573	Mean	0.3018
right of pin C	4	0.7760	0.0478	Clearance	0.6204
from pin to rail E	18	0.0002	0.0148	Mean	0.2302
from wallpin to slide D	14	0.5779	0.0490	Clearance	0.3593
LEFT FRONT PIN					
clearance	Nr.	mean value	std dev		
left of pin B	9	−0.0223	0.0624	Mean	0.2363
right of pin C	7	0.4037	0.0488	Clearance	0.0000
from pin to rail E	15	−0.0003	0.0176	Mean	0.2184
from wallpin to slide D	11	0.5754	0.0581	Clearance	0.3154
RIGHT FRONT PIN					
clearance	Nr.	mean value	std dev		
left of pin B	5	0.2040	0.0767	Mean	0.3098
right of pin C	3	0.7776	0.0633	Clearance	0.5014
from pin to rail E	17	0.0002	0.0177	Mean	0.2199
from wallpin to slide D	13	0.5778	0.0579	Clearance	0.3187

Tab.2: Results from tolerance analysis, all dimensions in [mm]

When running the analysis, VisVSA calculates the parameters of each measurement operation. Since the tolerances are normally distributed, the results are Gaussian-curves. Table 2 shows the parameters of all the measurements. Fig. 6 shows the measurement positions of the model in VisVSA.

The results are summarised in Table 2. The table describes the clearance variation in the four clearance joint pins. In each pin, four clearances are measured as shown in Figure 5. In each analysis, the mean value and standard deviation are computed according

5000 Monte Carlo simulations using the ISO 2768H tolerances. According to these values, the relative position of each pin relative to the journal is computed in two directions: left-right position and bottom-top direction (intermediate results not shown). The results are shown in column n, computed according to eq. (9). The free clearance and relative mean value position in the clearance gap is computed using the relative variation $n = 3.43$, which means that 0.031 % of the production might have tighter clearances than this example. The mean and clearance values are the minimum values for lateral and top clearances and mean clearance with $n = 3.43$ in eq. (9). The letters B, C, E and D refer to Fig. 5 in respective pins.

3.2. Multi-Body simulation

A spatial 3D six degree of freedom model was created of the slide with ADAMS 12.0 software. The clearances in each joint were modelled having parametrically variable clearances. The total clearance in each gap was a function of the relative variance n, changing the minimum clearance gap in each joint as a function of relative variance. Since the minimum clearance is crucial for the clearance behaviour, the method enables to simulate the function of the products that have clearances smaller than the minimum clearance according to eq. (9). The slide carrier opening operation was simulated from the first 200 ms of the opening. The relative clearance variation was from $n = 0 \ldots 2.5$ $(50 - 0.62\,\%)$ of lost production because of too-tight tolerances. The carrier speed opening was velocity force controlled according to the ADAMS force function, see [10]

$$F_{\mathrm{z}} = 0.04\,(S(t, 0.05, 0, 1, 100) - v_{\mathrm{z}})\;. \tag{11}$$

The STEP-function is defined in the ADAMS/Solver Handbook [10]. The force function accelerates the velocity to a maximum of 100 mm/s according to the Step-function, with P-control gain 0.04 Ns/mm. v_{z} is the actual momentary speed of the slide. In Figure 7, the carrier velocity is shown during the start up-process with two small hang-ups caused by non-central driving. In side part the speed can even be very short time negative, since the slide turns when force is applied. The velocity variation is shown as a function of tolerance tightness variation according eq. (9).

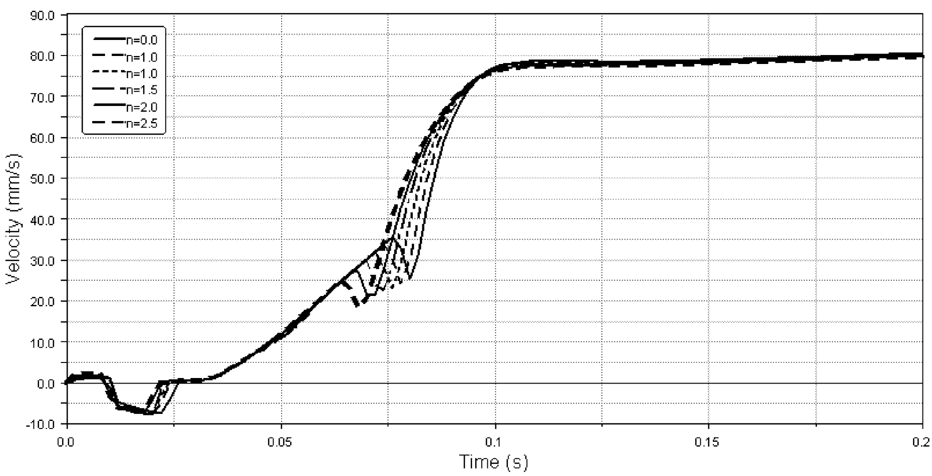

Fig. 7: Carrier velocity as a function of relative variance

The force required to move the carrier is shown in Figure 8 as a function of time and relative variance n in eq. (9).

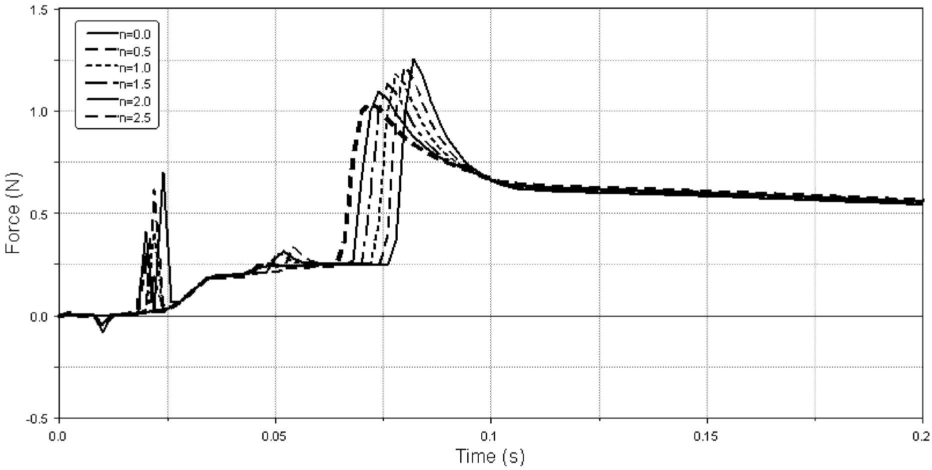

Fig.8: The required driving force, as a function of clearance variance n

4. Discussion

The Six Sigma method has become a dominating paradigm in mass production industries. This requires a valid design synthesis method for ensuring the production remains inside tolerance limits. Tolerance synthesis is impossible without a valid tolerance analysis method. Geometry-based tolerance analysis does not consider the functional effects of tolerance variation. Therefore, it is essential to couple multi-body simulation to tolerance analysis if functional variation is to be analysed. Tolerance analysis in this study is based on the Monte Carlo analysis assuming Gaussian distribution. In large production series, this assumption is valid. The multi-body analysis is based on the variation of clearances. Conventionally, the increase of clearance usually results in a smoother operation due to the elimination of compressive fits in bearings. This can be seen in Figure 8, where larger clearance results in a diminishing actuator force requirement. Variation between compressive and non-compressive fits in glide bearings results in a very variable friction coefficient, which is undesirable.

This paper demonstrates the methodology of subsequent tolerance and function simulations; it is probably the first to consider the parallel use of CAT and MBS.

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