ADDITIONAL CONTRIBUTION TO THE PROBLEM OF SELF-EXCITED SYSTEMS HAVING SEVERAL UNSTABLE VIBRATION MODES

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This contribution is based on paper [1] in which a three-mass chain system, excited by van der Pol type excitation, has been investigated for the case when two different unstable vibration modes can occur. Here, the efficiency of tuning the system using a single-frequency parametric excitation by unifying the conditions for suppressing both vibration modes is investigated. The suppressing efficiency is not high, especially when the system is close to be tuned into a so-called 'internal resonance'. It seems that the best way for vibration suppression is the combination of passive and active means (using parametric excitation).

Key words: *self-excitation of van der Pol type, parametric excitation due to spring stiffness variation, self-excited vibration with several modes*

1. Introduction

In paper [1] a self-excited three-mass chain system has been analysed. Two upper masses (m_1, m_2) are self-excited by flow, which is expressed by a van der Pol model. The motion of the foundation mass (m_3) is damped and the elastic mounting exhibits a parametric excitation due to a harmonically changing component of the spring stiffness (see Figure 1). The mass deflections are denoted by y_j ($j = 1, 2, 3$) and the spring stiffnesses by k_1, k_2, $k_3 = k_0\,(1 + \varepsilon\,\cos\omega\,t)$. This system is governed by the equations:

$$
\begin{gathered}
m_1\,\ddot{y}_1 - (b_1 - d_1\,y_1^2)\,\dot{y}_1 + k_1\,(y_1 - y_2) = 0\ , \\
m_2\,\ddot{y}_2 - (b_2 - d_2\,y_2^2)\,\dot{y}_2 - k_1\,(y_1 - y_2) + k_2\,(y_2 - y_3) = 0\ , \\
m_3\,\ddot{y}_3 + b_3\,\dot{y}_3 - k_2\,(y_2 - y_3) + k_3\,y_3 = 0\ .
\end{gathered}
\tag{1}
$$

In Ref. [1] the following parameters were considered: $k_1 = k_2 = k_0 = k$, $m_1 = m_2 = m$, $m_3 = m/2$.

The aim of the previous analysis [1] was to investigate the effect of parametric excitation, because in case of systems where only a single vibration mode is unstable, there exists the possibility of even fully suppressing self-excited vibrations of the van der Pol type (see [2]–[8]). The results of the investigation show, that for systems where several vibration modes can occur, only a single vibration mode can be suppressed with a single-frequency parametric excitation.

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For several alternative parameter values, the numerical analysis has shown that a single frequency vibration prevails, i.e., the system vibrates with one mode although this vibration is not the only possible one. When changing the parametric excitation frequency, the vibration with a given mode can be fully suppressed for a certain driving frequency but, in this case, another vibration mode is simultaneously initiated. Furthermore, a double-frequency parametric excitation was considered, with frequencies corresponding to natural frequencies, where, under certain conditions, the corresponding mode could be suppressed. This study wasn't a full success, either.

Further investigations were suggested for tuning the normal mode frequencies of the system such that $\Omega_2 - \Omega_1 = \Omega_3 - \Omega_2$, with the intention to achieve the suppressing effect of both vibration modes with a single-frequency parametric excitation [9]. An additional contribution to the analysis of such a system is here presented.

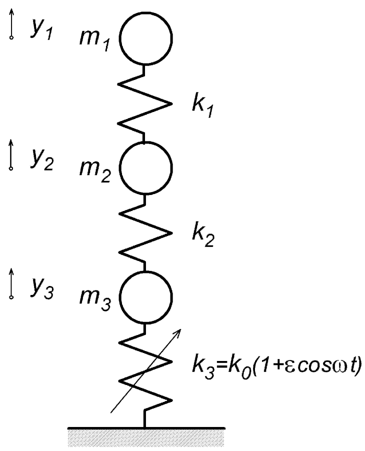

Fig.1: Schematic representation of the system with parametric excitation at the base stiffness element

2. Enhanced system

In order to unify the conditions for suppressing both vibration modes, i.e. $\Omega_2 - \Omega_1 = \Omega_3 - \Omega_2$, only some changes in the mass ratio are required to fulfil the condition for the natural frequencies of the abbreviated system. Let $m_1 = m$, $m_2 = m/\mu$, $m_3 = 2m$, and for $\mu = m_2/m_1$ such a value will be found in order to meet the above condition.

Using time transformation $\omega_0\, t = \tau$ $(\omega_0 = \sqrt{k/m})$ the following equations are obtained:

$$\begin{aligned} y_1'' - (\beta_1 - \delta_1\, y_1^2)\, y_1' + y_1 - y_2 = 0\ , \\ y_2'' - (\beta_2 - \delta_2\, y_2^2)\, y_2' + 2\,\mu\, y_2 - \mu\,(y_1 + y_3) = 0\ , \\ y_3'' + \kappa\, y_3' - \frac{1}{2}\, y_2 + \left(1 + \frac{1}{2}\,\varepsilon\, \cos\nu\,\tau\right) y_3 = 0\ , \end{aligned} \tag{2}$$

where $\beta_k = b_k/m_k\,\omega_0$, $\delta_k = d_k/m_k\,\omega_0$, for $k = 1,2$ and $\kappa = b_3/m_3\,\omega_0$, $\nu = \omega/\omega_0$.

System (2) can be transformed into the quasi-normal form:

$$x_s'' + \Omega_s^2\, x_s + \sum_{k=1}^{3} (\Theta_{sk}\, x_s' + Q_{sk}\, x_s\, \cos\nu\,\tau) = 0\ , \quad (s = 1,2,3) \tag{3}$$

using the transformation:

$$
\begin{aligned}
y_1 &= a_{11}\, x_1 + a_{12}\, x_2 + a_{13}\, x_3 \ , \\
y_2 &= a_{21}\, x_1 + a_{22}\, x_2 + a_{23}\, x_3 \ , \\
y_3 &= a_{31}\, x_1 + a_{32}\, x_2 + a_{33}\, x_3 \ ,
\end{aligned}
\tag{4}
$$

where:

$$
\begin{aligned}
a_{1k} &= 1 \ , \\
a_{2k} &= \frac{\mu\,(1-\Omega_k^2)}{(1-\Omega_k^2)\,(2\,\mu-\Omega_k^2)-\frac{1}{2}\mu} \ , \\
a_{3k} &= \frac{\frac{1}{2}\mu}{(1-\Omega_k^2)\,(2\,\mu-\Omega_k^2)-\frac{1}{2}\mu} \ ,
\end{aligned}
\tag{5}
$$

The natural frequencies of the abbreviated system are given by the equation:

$$
\begin{vmatrix}
1-\Omega^2 & -1 & 0 \\
-\mu & 2\,\mu-\Omega_2 & -\mu \\
0 & -\frac{1}{2} & 1-\Omega_2
\end{vmatrix}
= (1-\Omega^2)\left[\Omega^4-(1+2\,\mu)\,\Omega^2+\frac{1}{2}\,\mu\right] = 0 \ ,
\tag{6}
$$

from which it follows:

$$
\begin{aligned}
\Omega_1^2 &= \mu + \frac{1}{2} - \sqrt{\mu^2+\frac{1}{2}\,\mu+\frac{1}{4}} \ , \\
\Omega_2^2 &= 1 \ , \\
\Omega_3^2 &= \mu + \frac{1}{2} + \sqrt{\mu^2+\frac{1}{2}\,\mu+\frac{1}{4}} \ .
\end{aligned}
\tag{7}
$$

To determine μ such that the condition $\Omega_2-\Omega_1=\Omega_3-\Omega_2$ holds, the following relations are used:

$$
\begin{aligned}
\Omega_1+\Omega_3 &= 2 \ , \\
\Omega_1^2+\Omega_3^2 &= 1+2\,\mu \ , \\
\Omega_1^2\,\Omega_3^2 &= \frac{1}{2}\,\mu \ .
\end{aligned}
\tag{8}
$$

From above we obtain $\mu = 0.8486122$, $\Omega_1 = 0.4095661$, $\Omega_2 = 1$, $\Omega_3 = 1.5904339$, $\nu = 0.590434$.

Parameter values of β_1, β_2 and κ were chosen such that the system is unstable in two vibration modes, which are the first and third mode in this case. In this respect, the aim of the previous analysis [1] was to investigate the effect of parametric excitation in case of systems, where only a single vibration mode is unstable. In this case, there exists even a possibility of full suppression of the self-excited vibration (see [2], [3], ...). The results of investigation in [1] show that, for systems where more vibration modes can occur, only a single vibration mode can be suppressed by means of a single-frequency parametric excitation.

3. Results of numerical solution

Equations (2) have been solved numerically, but the results are presented in quasi-normal coordinate using the inverse relations of (4). The parameter values of the examples considered are shown in Table 1.

Figure #	κ	β_1	β_2	δ_1	δ_2	ε	ν
2	0.15	0.05	0.05	0.50	0.50	0.15	variable
3	0.15	0.05	0.05	0.50	0.50	0.25	variable
4	0.15	0.05	0.05	0.50	0.50	variable	0.590434
5	0.10	0.04	0.02	0.50	0.50	0.15	variable
6	0.10	0.04	0.02	0.50	0.50	0.25	variable
7	0.10	0.04	0.02	0.50	0.50	variable	0.590434
8	0.15	0.05	0.05	0.50	0.50	0.15	0.590434
9	0.10	0.04	0.02	0.50	0.50	0.15	0.590434

Tab.1: Values of system parameters for numerical simulation

For constant parameter values of κ, β_1, β_2, δ_1, δ_2, results are shown for extreme values of quasi-normal coordinates (denoted as $[x_k]$, $k = 1, 2, 3$) as a function of the parametric excitation frequency ν, for $\nu = \Omega_2 - \Omega_1 = \Omega_3 - \Omega_2 = 0.590434$ and for varying ε.

Figure 2 shows the extreme deflections of quasi-normal coordinates as a function of ν, with parameter values as shown in Table 1. We can see that only a slight reduction occurs at $\nu = \Omega_2 - \Omega_1 = \Omega_3 - \Omega_2 = 0.590434$ for $[x_2]$, $[x_3]$, but $[x_1]$ increases. A similar situation is observed at $\nu = \Omega_3 - \Omega_1 = 1.180868$. The strong parametric resonance at $\nu = 2\,\Omega_1 = 0.819132$ for $[x_1]$ results in full suppression of $[x_3]$. A similar phenomenon happens at $\nu = 2\,\Omega_2 = \Omega_1 + \Omega_3 = 2.000000$, but only a minor decrease of $[x_3]$ occurs. Moreover, $[x_2]$ is small only at $\nu = \Omega_2 - \Omega_1 = 0.590434$ while it becomes larger at $\nu = \Omega_2 + \Omega_3 = 2.590434$. From the condition $\Omega_2 - \Omega_1 = \Omega_3 - \Omega_2$ it follows, that $\Omega_1 + \Omega_3 = 2\,\Omega_2$, which means, that the parametric resonance of the second kind (combination resonance) merges with the parametric resonance of the first kind. There is a certain similarity with the case of a system tuned to the so-called 'internal resonance', which can influence the behaviour of the system.

Figure 3 presents results for a higher value of the parametric excitation amplitude ε, compared to the previous system. The results obtained are very similar, but the parametric resonance for $[x_1]$ and the suppression effect on $[x_3]$ are more pronounced. This is also valid for $[x_2]$. Figure 4 shows the effect of increasing/decreasing the intensity of parametric excitation for $\nu = \Omega_2 - \Omega_1 = \Omega_3 - \Omega_2$. We can see that by increasing ε the extreme deflection $[x_3]$ decreases, but $[x_1]$ increases. When ε exceeds the threshold value of 0.4, the vibration intensity increases at all vibration modes, especially for the first and the second mode.

Figures 5 and 6 show the results for smaller values of β_1, β_2 and κ. Both cases differ only by the parametric excitation amplitude ε. The results are very similar to those presented in Figures 2 and 3. Figure 7 shows the effect of ε. When comparing it with Figure 4 we can see a very similar behaviour; only the boundary of the substantial change in vibration intensity and character lies at a slightly lower value of parametric excitation amplitude.

Figures 8 and 9 show the time histories for $[x_1]$, $[x_2]$ and $[x_3]$ at $\nu = \Omega_2 - \Omega_1 = \Omega_3 - \Omega_2 = 0.590434$. We can see that the system in both cases vibrates with dominant components corresponding to the first and third vibration modes, i.e. the double-frequency vibration occurs.

4. Conclusions

The efficiency of self-excited vibration quenching by using parametric excitation is smaller for the case when the equilibrium state is unstable to more than one vibration mode, in comparison with the case when only one vibration mode can be initiated. Although the

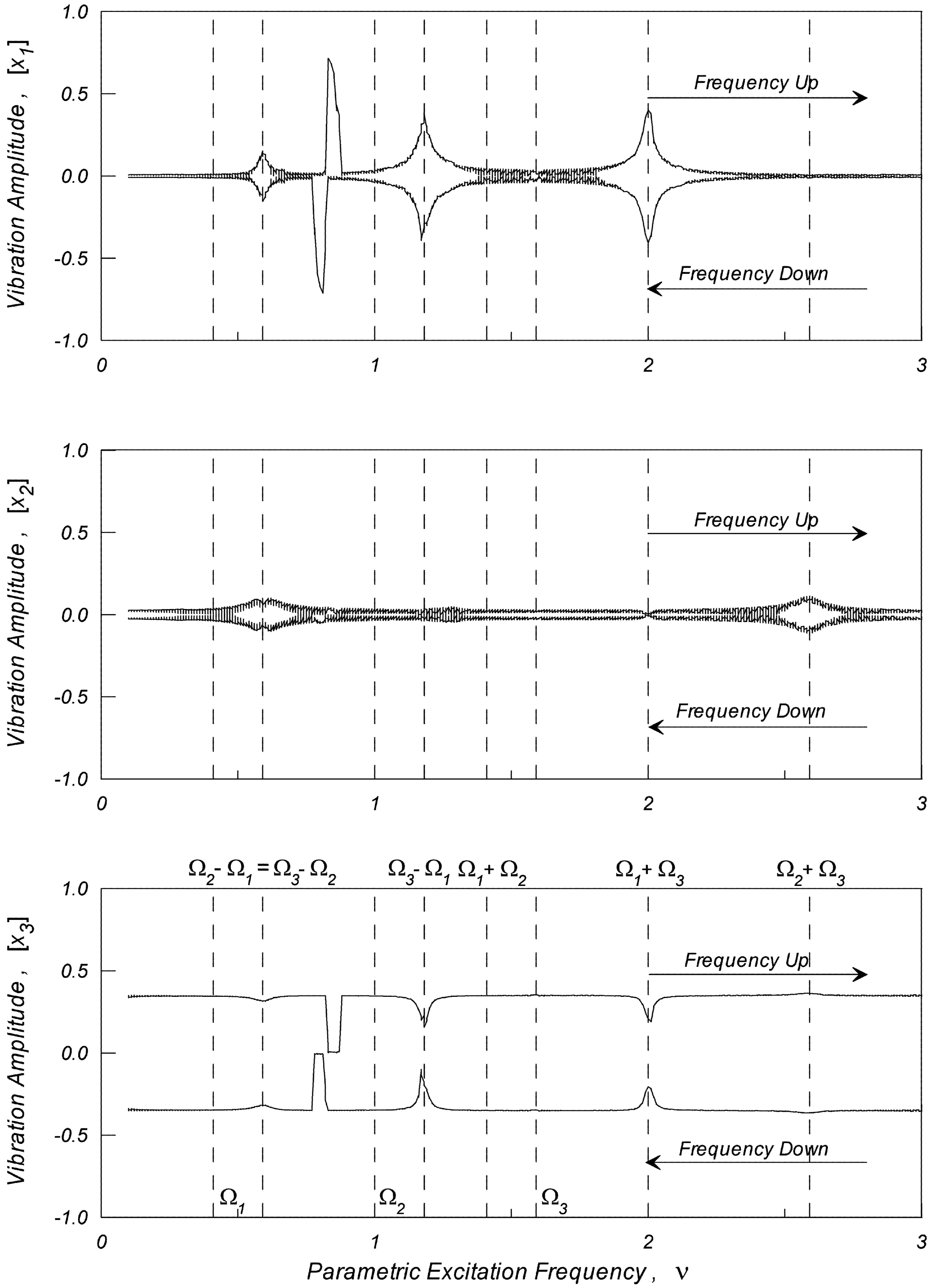

Fig.2: Extreme values $[x_1]$, $[x_2]$ and $[x_3]$ of normal mode vibration amplitudes x_1, x_2 and x_3 versus applied parametric excitation frequency ν for $\kappa = 0.15$, $\beta_1 = 0.05$, $\beta_2 = 0.05$, $\delta_1 = 0.50$, $\delta_2 = 0.50$ and $\varepsilon = 0.15$

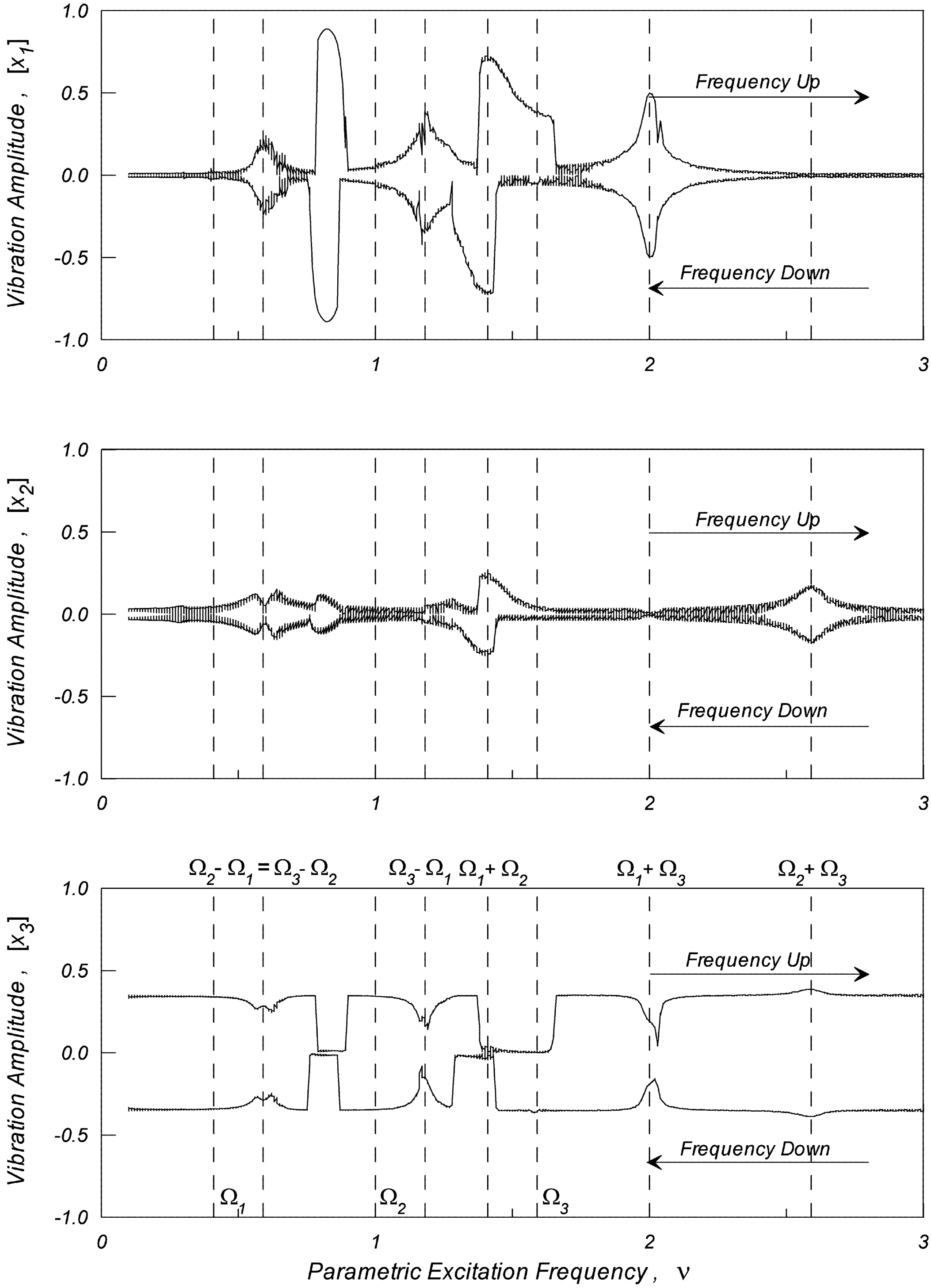

Fig.3: Extreme values $[x_1]$, $[x_2]$ and $[x_3]$ of normal mode vibration amplitudes x_1, x_2 and x_3 versus applied parametric excitation frequency ν for $\kappa = 0.15$, $\beta_1 = 0.05$, $\beta_2 = 0.05$, $\delta_1 = 0.50$, $\delta_2 = 0.50$ and $\varepsilon = 0.25$

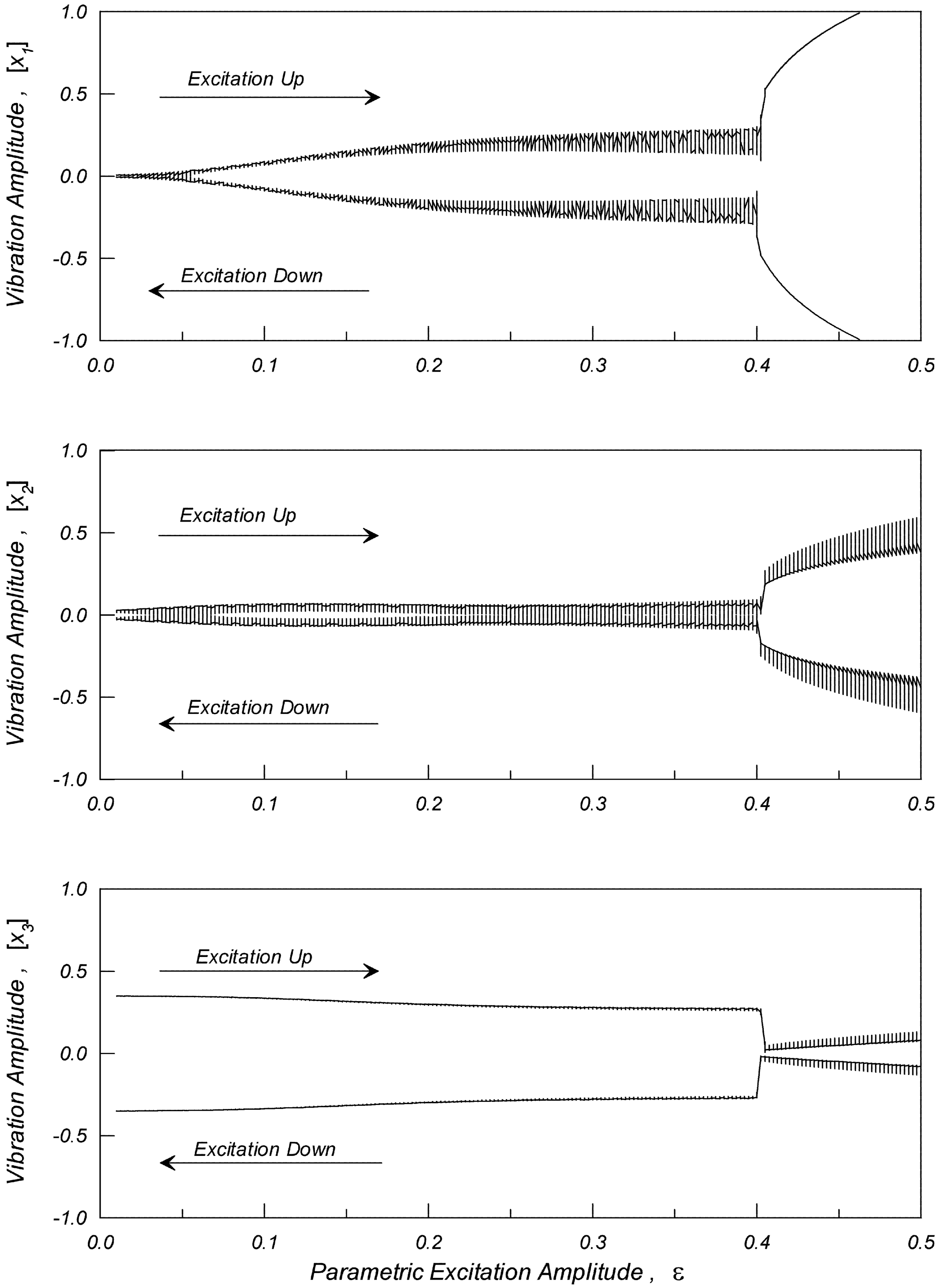

Fig.4: Extreme values $[x_1]$, $[x_2]$ and $[x_3]$ of normal mode vibration amplitudes x_1, x_2 and x_3 versus parametric excitation amplitude ε for $\kappa = 0.15$, $\beta_1 = 0.05$, $\beta_2 = 0.05$, $\delta_1 = 0.50$, $\delta_2 = 0.50$ and $\nu = 0.590434$

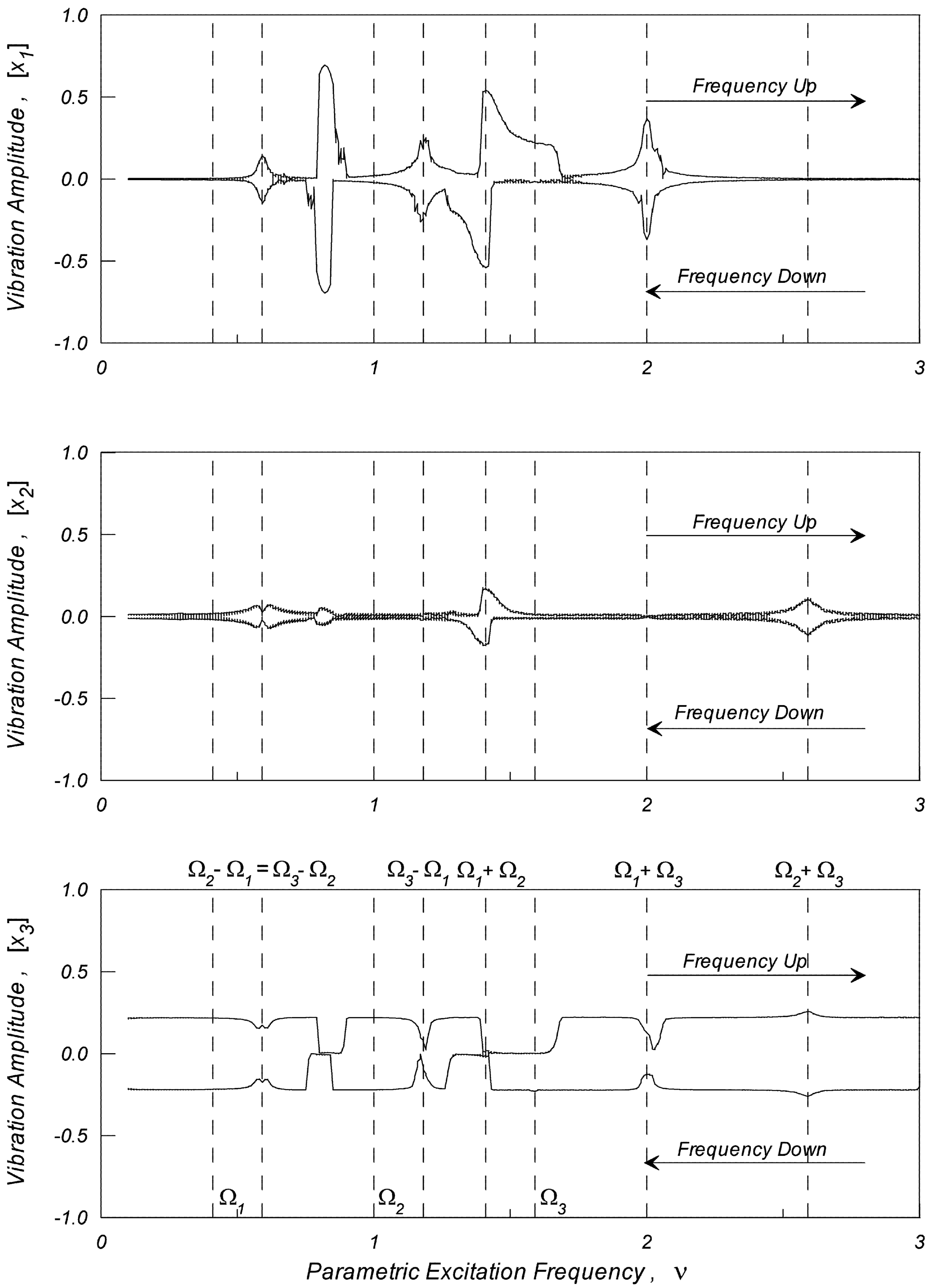

Fig.5: Extreme values $[x_1]$, $[x_2]$ and $[x_3]$ of normal mode vibration amplitudes x_1, x_2 and x_3 versus applied parametric excitation frequency ν for $\kappa = 0.10$, $\beta_1 = 0.04$, $\beta_2 = 0.02$, $\delta_1 = 0.50$, $\delta_2 = 0.50$ and $\varepsilon = 0.15$

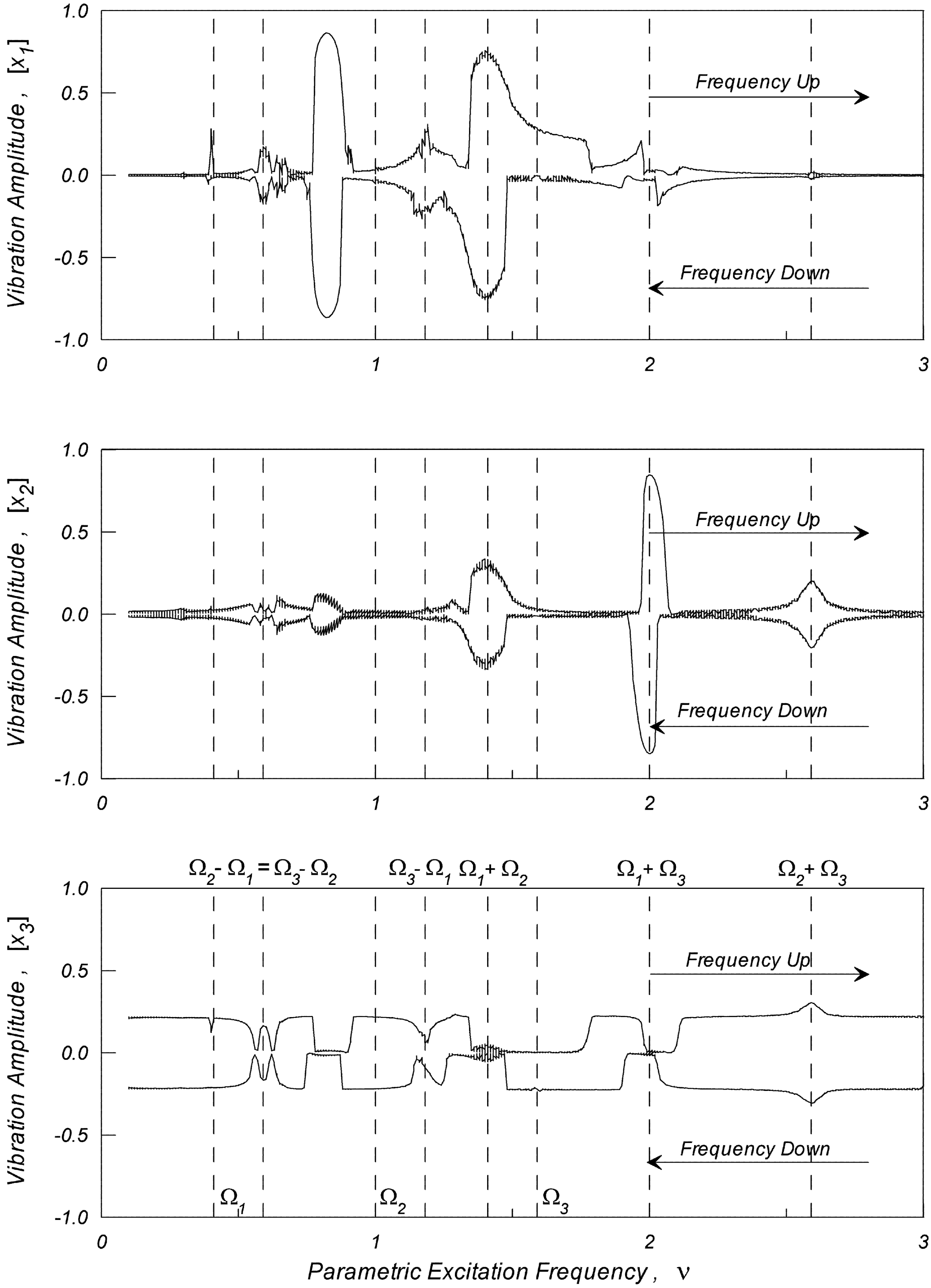

Fig.6: Extreme values $[x_1]$, $[x_2]$ and $[x_3]$ of normal mode vibration amplitudes x_1, x_2 and x_3 versus applied parametric excitation frequency ν for $\kappa = 0.10$, $\beta_1 = 0.04$, $\beta_2 = 0.02$, $\delta_1 = 0.50$, $\delta_2 = 0.50$ and $\varepsilon = 0.25$

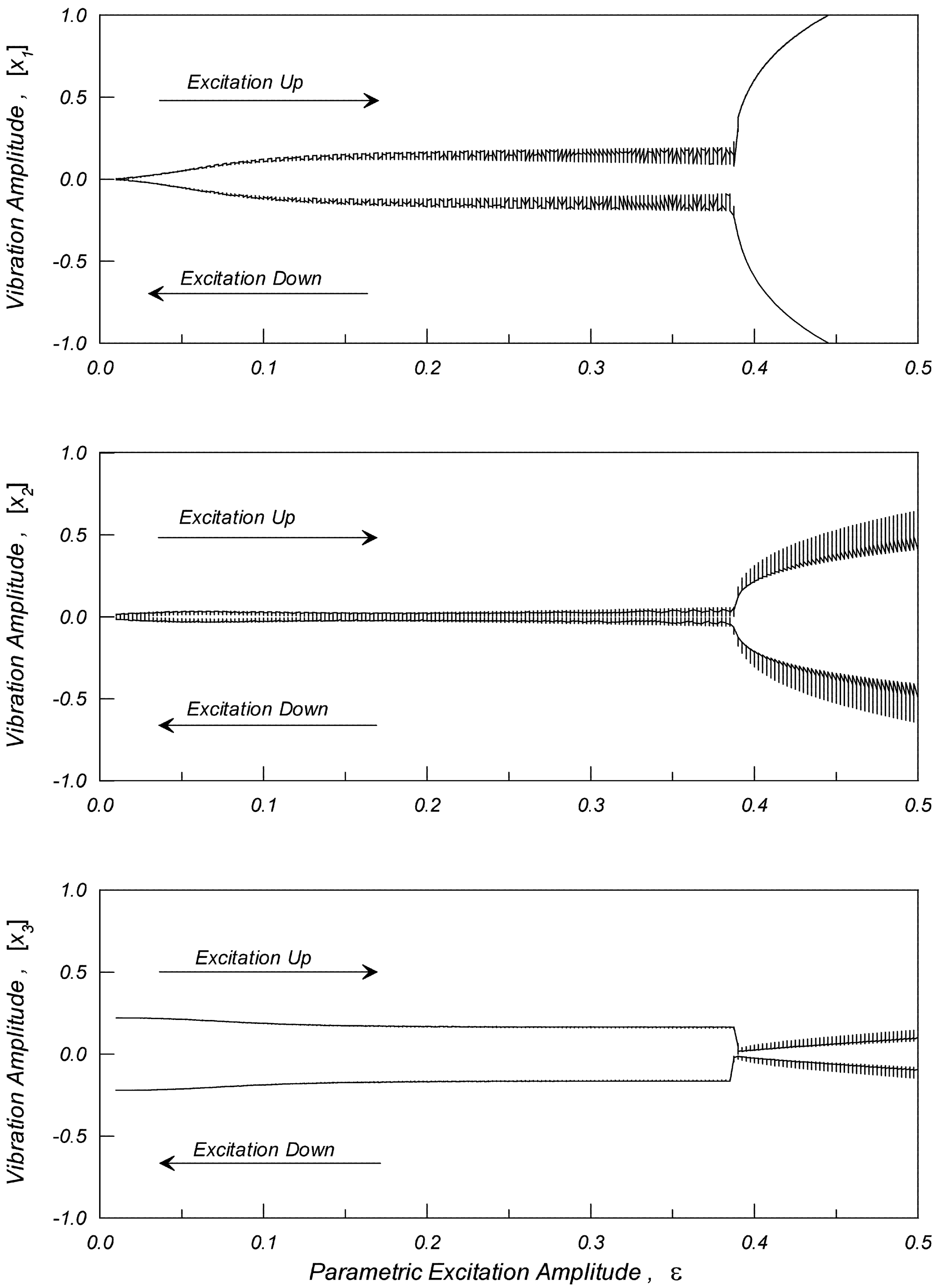

Fig.7: Extreme values $[x_1]$, $[x_2]$ and $[x_3]$ of normal mode vibration amplitudes x_1, x_2 and x_3 versus parametric excitation amplitude ε for $\kappa = 0.10$, $\beta_1 = 0.04$, $\beta_2 = 0.02$, $\delta_1 = 0.50$, $\delta_2 = 0.50$ and $\nu = 0.590434$

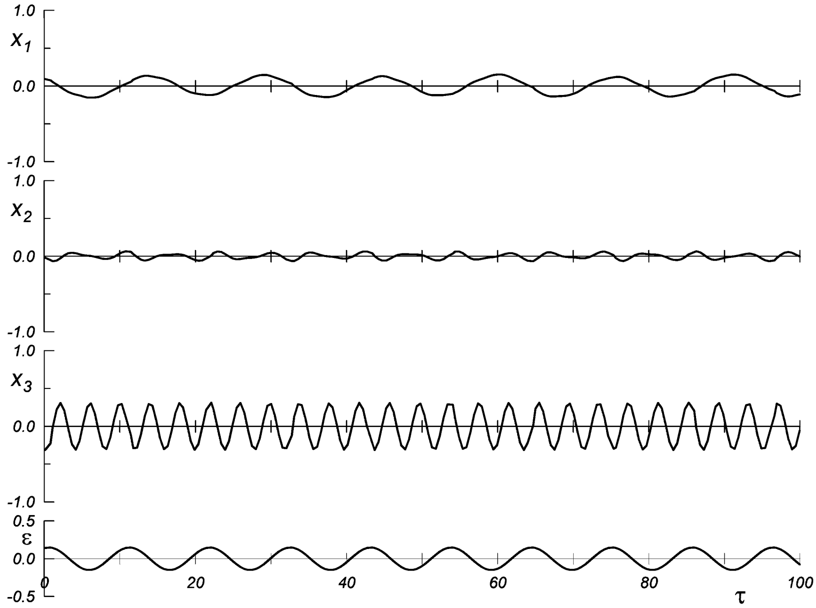

Fig.8: Time series of normal mode vibration amplitudes x_1, x_2 and x_3 for $\kappa = 0.15$, $\beta_1 = 0.05$, $\beta_2 = 0.05$, $\delta_1 = 0.50$, $\delta_2 = 0.50$, $\varepsilon = 0.15$ and $\nu = 0.590434$

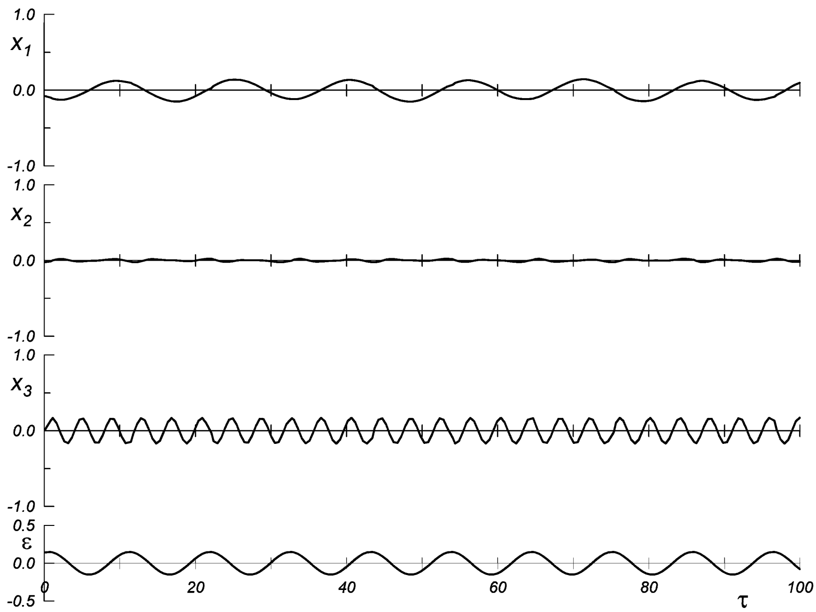

Fig.9: Time series of normal mode vibration amplitudes x_1, x_2 and x_3 for $\kappa = 0.10$, $\beta_1 = 0.04$, $\beta_2 = 0.02$, $\delta_1 = 0.50$, $\delta_2 = 0.50$, $\varepsilon = 0.15$ and $\nu = 0.590434$

single-frequency vibration prevails, a double-frequency vibration occurs at parametric excitation frequency $\nu = \Omega_2 - \Omega_1 = \Omega_3 - \Omega_2$. The extreme deflections of individual vibration modes are smaller than those of corresponding single-frequency vibrations but this does not lead to a substantial reduction of vibration intensity.

The goal of unifying the conditions for suppressing two different vibration modes has not been successful, since it led to a certain tuning of the system into so-called ‘internal resonances’. The best way for vibration quenching seems to be the simultaneous combination of two different means: the passive reduction of substantial vibrations for one possible mode, combined with the parametric excitation of the stiffness variation.

References

[1] Tondl A., Nabergoj R., Ecker H.: Quenching of self-excited vibrations in a system with two unstable vibration modes, Proc. 7th Int. Conference on Vibration Problems – ICOVP (2005)

[2] Tondl A.: To the problem of quenching self-excited vibrations, Acta Technica ČSAV, 43 (1998), 109–116

[3] Ecker H.,Tondl A.: Suppression of flow-induced vibrations by a dynamic absorber with parametric excitation, Proc. 7th Int. Conference on Flow-Induced Vibrations – FIV (2000)

[4] Tondl A.: Suppressing self-excited vibration by means of parametric excitation, Proc. Colloquium Dynamics of Machines (2000), 225–230

[5] Nabergoj R., Tondl A.: Self-excited vibration quenching by means of parametric excitation, Acta Technica ČSAV, 46 (2001), 107–118

[6] Tondl A.: Self-excited vibration quenching in a rotor system by means of parametric excitation, Acta Technica ČSAV, 45 (2000) 199–211

[7] Tondl A.: Three-mass self-excited systems with parametric excitation, Acta Technica ČSAV, 47 (2002), 165–176

[8] Tondl A., Ecker H.: On the problem of self-excited vibration quenching by means of parametric excitation, Archive of Applied Mechanics, 72 (2003), 923–932

[9] Nabergoj R., Tondl A., Ecker H.: To the problem of self-excited systems having several unstable vibration modes, Proc. Colloquium Dynamics of Machines (2006), 71–78

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