USAGE OF THE GENERALIZED MODAL SYNTHESIS METHOD IN DYNAMICS OF MACHINES

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Classical approach to complex dynamical systems modelling using modal synthesis method is based on the generalized coordinates transformation to the new configuration space by means of chosen master eigenmodes of vibration. The paper introduces generalization of the modal synthesis method with quasistatic consideration of slave eigenmodes of vibration. This improvement brings better approximation of the system behaviour while the number of degrees of freedom of the reduced model is the same as in the case of classical approach. The results of modal and acoustic analysis of the test-gearbox, made a gain on a classical and new reduced model, are compared.

Key words : *modal synthesis method, condensed model, master and slave eigenmodes of vibration, large rotating systems*

1. Introduction

Many real mechanical systems are composed from flexible bodies mutually joined by discrete couplings. The mathematical model of these systems after discretization of each body by the finite element method has a large number of degrees of freedom (DOF number). That is why the standard numerical methods of dynamic analysis, tuning and optimization of these complex systems are very hardly applicable. The best-known reduction DOF number methods based on the transformation of the coordinates by modal or other reduction matrices of the whole system [2], whose complex model is not in entrance known, cannot be applied. A suitable method for the modelling of dynamic behaviour of large multi-body systems is a modal synthesis method [9], [14]. The modal approach has a long tradition in structure engineering [1] and at the workplace of authors it was used e.g. for a vibration analysis of the screw compressors [8] and car gearbox [10], [13], a sensitivity analysis and spectral tuning of a centrifugal fan [12], a modelling and seismic analysis of a primary circuit of NPP with the VVER reactor type [5], [6] and for other complex systems.

A modal synthesis method is used for modelling of dynamical systems that can be decomposed into subsystems joined by various discrete couplings. Classical approach, applied in the above-cited publications, is based on the generalized coordinates transformation to the new configuration space by means of chosen master eigenmodes of vibration that originate from modal analysis of mutually uncoupled, undamped and nonrotating subsystems. This paper introduces generalization of the modal synthesis method with quasistatic consideration of slave eigenmodes of vibration. This improvement brings better approximation of the system behaviour while the DOF number of the reduced model is the same as in the case of a classical approach. Also the reduced model can have less degrees of freedom considering

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a certain number of slave eigenmodes of vibration and can be used in computations with long computational time where the time depends on the level of model reduction (e.g. for numerical simulations or optimizations). The level of improvement is verified by numerical experiments with simple test-gearbox. The results of modal and acoustic analysis of the test-gearbox are compared for modelling by the classical and new approaches.

2. Modelling

Rotating mechanical systems can be decomposed into stator and rotor (shafts with gears) subsystems j joined by gear and bearing couplings. Model of the decomposed system is of the form [11]

$$\mathbf{M}_j\,\ddot{\mathbf{q}}_j(t) + (\mathbf{B}_j + \omega_j\,\mathbf{G}_j)\,\dot{\mathbf{q}}_j(t) + \mathbf{K}_j\,\mathbf{q}_j(t) = \mathbf{f}_j^{\mathrm{E}}(t) + \mathbf{f}_j^{\mathrm{C}}\ , \quad j = 1, 2, \ldots, S\ , \tag{1}$$

where $\mathbf{M}_j$, $\mathbf{B}_j$ and $\mathbf{K}_j$ are symmetrical mass, damping and stiffness matrices, $\mathbf{G}_j$ is an antisymmetrical gyroscopic matrix and ω_j is an angular shaft velocity of the mutually uncoupled subsystem j. All matrices are square matrices of the n_j-th order. Vector $\mathbf{f}_j^{\mathrm{E}}(t)$ is a force vector of external dynamical loads. Generalized coordinates in vectors $\mathbf{q}_j$ express dynamical displacements from a static equilibrium position. The linearized global coupling force vector in the configuration space

$$\mathbf{q} = [\,\mathbf{q}_j\,] = \left[\,\mathbf{q}_1^{\mathrm{T}}\ \mathbf{q}_2^{\mathrm{T}}\ \ldots\ \mathbf{q}_S^{\mathrm{T}}\,\right]^{\mathrm{T}} \tag{2}$$

can be written as

$$\mathbf{f}_{\mathrm{C}} = [\,\mathbf{f}_j^{\mathrm{C}}\,] = -\,(\mathbf{B}_{\mathrm{G}} + \mathbf{B}_{\mathrm{B}})\,\dot{\mathbf{q}}(t) - (\mathbf{K}_{\mathrm{G}} + \mathbf{K}_{\mathrm{B}})\,\mathbf{q}(t) + \mathbf{f}_{\mathrm{I}}(t)\ , \tag{3}$$

where $\mathbf{K}_{\mathrm{G}}$ and $\mathbf{K}_{\mathrm{B}}$ are stiffness matrices of linearized gear and bearing couplings and $\mathbf{f}_{\mathrm{I}}(t)$ is the vector of internal excitation in the couplings. Damping coupling matrices $\mathbf{B}_{\mathrm{G}}$ and $\mathbf{B}_{\mathrm{B}}$ are of the same structure as corresponding stiffness matrices. After the modal analyses of the uncoupled, undamped and nonrotating subsystems we choose for each subsystem j a set of m_j master eigenmodes of vibration, which will be ordered in matrix ${}^{\mathrm{m}}\mathbf{V}_j \in \mathbb{R}^{n_j, m_j}$, and a set of s_j slave eigenmodes of vibration, which will be ordered in matrix ${}^{\mathrm{s}}\mathbf{V}_j \in \mathbb{R}^{n_j, s_j}$ as is schematically shown in the left part of Figure 1. A set of other eigenmodes of vibration will be neglected. Diagonal square spectral matrices ${}^{\mathrm{m}}\boldsymbol{\Lambda}_j \in \mathbb{R}^{m_j, m_j}$ and ${}^{\mathrm{s}}\boldsymbol{\Lambda}_j \in \mathbb{R}^{s_j, s_j}$ composed of eigenfrequencies squares on their diagonals correspond with the sets of master and slaves modes of vibration. We suppose, that conditions of orthonormality [7]

$$\begin{aligned} {}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}}\,\mathbf{M}_j\,{}^{\mathrm{m}}\mathbf{V}_j = \mathbf{E}\ , \qquad {}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}}\,\mathbf{K}_j\,{}^{\mathrm{m}}\mathbf{V}_j = {}^{\mathrm{m}}\boldsymbol{\Lambda}_j\ , \\ {}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}}\,\mathbf{M}_j\,{}^{\mathrm{s}}\mathbf{V}_j = \mathbf{0}\ , \qquad {}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}}\,\mathbf{K}_j\,{}^{\mathrm{s}}\mathbf{V}_j = \mathbf{0}\ , \end{aligned} \tag{4}$$

are satisfied.

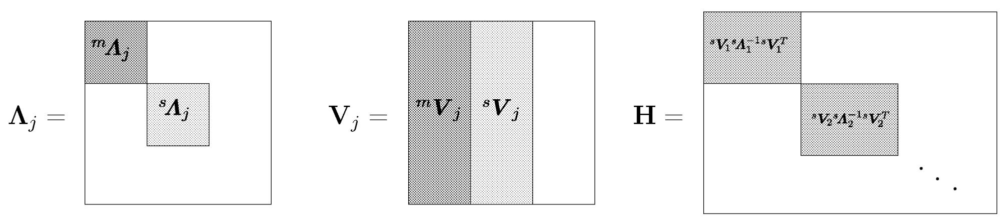

Fig.1: Structure of matrices $\boldsymbol{\Lambda}_j$, $\mathbf{V}_j$, $\mathbf{H}$

We introduce the transformation of the subsystems generalized coordinates vectors

$$\mathbf{q}_j = {}^{\mathrm{m}}\mathbf{V}_j\, {}^{\mathrm{m}}\mathbf{x}_j + {}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\mathbf{x}_j\ , \qquad j = 1, 2, \dots, S\ , \tag{5}$$

and express it in the model (1)

$$\begin{aligned}\mathbf{M}_j\, ({}^{\mathrm{m}}\mathbf{V}_j\, {}^{\mathrm{m}}\ddot{\mathbf{x}}_j + {}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\ddot{\mathbf{x}}_j) + (\mathbf{B}_j + \omega_j\, \mathbf{G}_j)\, ({}^{\mathrm{m}}\mathbf{V}_j\, {}^{\mathrm{m}}\dot{\mathbf{x}}_j + {}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\dot{\mathbf{x}}_j) +\\ + \mathbf{K}_j\, ({}^{\mathrm{m}}\mathbf{V}_j {}^{\mathrm{m}}\mathbf{x}_j + {}^{\mathrm{s}}\mathbf{V}_j {}^{\mathrm{s}}\mathbf{x}_j) = \mathbf{f}_j^{\mathrm{E}}(t) + \mathbf{f}_j^{\mathrm{C}}\ .\end{aligned} \tag{6}$$

After premultiplying of this equation by matrix ${}^{\mathrm{m}}\mathbf{V}_j^T$ with consideration of the orthonormality conditions (4) we have

$$\begin{aligned}{}^{\mathrm{m}}\ddot{\mathbf{x}}_j(t) + {}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}}\, (\mathbf{B}_j + \omega_j\, \mathbf{G}_j)\, {}^{\mathrm{m}}\mathbf{V}_j\, {}^{\mathrm{m}}\dot{\mathbf{x}}_j + \underbrace{{}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}}\, (\mathbf{B}_j + \omega_j\, \mathbf{G}_j)\, {}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\dot{\mathbf{x}}_j}_{\doteq 0} +\\ + {}^{\mathrm{m}}\mathbf{\Lambda}_j\, {}^{\mathrm{m}}\mathbf{x}_j = {}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}} \left(\mathbf{f}_j^{\mathrm{E}}(t) + \mathbf{f}_j^{\mathrm{C}}\right)\ ,\end{aligned} \tag{7}$$

where contribution of the slave mode shapes of vibration to damping and gyroscopic forces was neglected and therefore term ${}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}}\, (\mathbf{B}_j + \omega_j\, \mathbf{G}_j)\, {}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\dot{\mathbf{x}}_j$ is equal to zero. If we premultiply equation (6) by matrix ${}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}}$, it holds

$${}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}}\, \mathbf{K}_j\, ({}^{\mathrm{m}}\mathbf{V}_j\, {}^{\mathrm{m}}\mathbf{x}_j + {}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\mathbf{x}_j) = {}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}} \left(\mathbf{f}_j^{\mathrm{E}}(t) + \mathbf{f}_j^{\mathrm{C}}\right) \tag{8}$$

for the quasistatic solution. Then we get the relation

$${}^{\mathrm{s}}\mathbf{x}_j = {}^{\mathrm{s}}\mathbf{\Lambda}_j^{-1}\, {}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}} \left(\mathbf{f}_j^{\mathrm{E}}(t) + \mathbf{f}_j^{\mathrm{C}}\right)\ , \tag{9}$$

which can be substituted to (5) and we have expression

$$\mathbf{q}_j = {}^{\mathrm{m}}\mathbf{V}_j\, {}^{\mathrm{m}}\mathbf{x}_j + {}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\mathbf{\Lambda}_j^{-1}\, {}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}} \left(\mathbf{f}_j^{\mathrm{E}}(t) + \mathbf{f}_j^{\mathrm{C}}\right)\ , \qquad j = 1, 2, \dots, S\ , \tag{10}$$

for the transformation of the original subsystems generalized coordinates vector to the space of master modal coordinates of uncoupled, undamped and nonrotating subsystems. Equation (7) and transformation (10) can be rewritten in the global forms

$${}^{\mathrm{m}}\ddot{\mathbf{x}}(t) + (\mathbf{D} + \omega_0\, \mathbf{G})\ {}^{\mathrm{m}}\dot{\mathbf{x}}(t) + {}^{\mathrm{m}}\mathbf{\Lambda}\, {}^{\mathrm{m}}\mathbf{x}(t) = {}^{\mathrm{m}}\mathbf{V}^{\mathrm{T}}\, (\mathbf{f}_{\mathrm{E}}(t) + \mathbf{f}_{\mathrm{C}})\ , \tag{11}$$

$$\mathbf{q}(t) = {}^{\mathrm{m}}\mathbf{V}\, {}^{\mathrm{m}}\mathbf{x}(t) + \mathbf{H}\, (\mathbf{f}_{\mathrm{E}}(t) + \mathbf{f}_{\mathrm{C}})\ , \tag{12}$$

where block diagonal residual compliance matrix (see the right part of Figure 1)

$$\mathbf{H} = \mathrm{diag}\left({}^{\mathrm{s}}\mathbf{V}_j\, {}^{\mathrm{s}}\mathbf{\Lambda}_j^{-1}\, {}^{\mathrm{s}}\mathbf{V}_j^{\mathrm{T}}\right) \in \mathbb{R}^{n,n} \tag{13}$$

was introduced. Other block diagonal matrices in global forms are

$$\mathbf{D} = \mathrm{diag}({}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}}\, \mathbf{B}_j\, {}^{\mathrm{m}}\mathbf{V}_j) = \mathrm{diag}(2\, D_\nu^{(j)}\, \Omega_\nu^{(j)})\ , \qquad \mathbf{G} = \frac{1}{\omega_0}\, \mathrm{diag}(\omega_j\, {}^{\mathrm{m}}\mathbf{V}_j^{\mathrm{T}}\, \mathbf{G}_j\, {}^{\mathrm{m}}\mathbf{V}_j)\ ,$$

$${}^{\mathrm{m}}\mathbf{\Lambda} = \mathrm{diag}({}^{\mathrm{m}}\mathbf{\Lambda}_j) = \mathrm{diag}(\Omega_\nu^{(j)2})\ , \qquad {}^{\mathrm{m}}\mathbf{V} = \mathrm{diag}(\,{}^{\mathrm{m}}\mathbf{V}_j\,)\ ,$$

and ω_0 is a reference angular velocity, e.g. the angular velocity of the first shaft subsystem ($j = 1$). Then ω_1 is the reference velocity ($\omega_0 = \omega_1$).

To complete the model, it is necessary to express the vector of coupling forces $\mathbf{f}_\mathrm{C}$. After substituting the transformation (12) in equation (3), we have

$$\begin{aligned}\mathbf{f}_\mathrm{C} = &-(\mathbf{B}_\mathrm{G}+\mathbf{B}_\mathrm{B})\left({}^\mathrm{m}\mathbf{V}\,{}^\mathrm{m}\dot{\mathbf{x}}(t)+\underbrace{\mathbf{H}\,\dot{\mathbf{f}}_\mathrm{E}(t)+\mathbf{H}\,\dot{\mathbf{f}}_\mathrm{C}}_{\doteq 0}\right)-\\ &-(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\left({}^\mathrm{m}\mathbf{V}\,{}^\mathrm{m}\mathbf{x}(t)+\mathbf{H}\,\mathbf{f}_\mathrm{E}(t)+\mathbf{H}\,\mathbf{f}_\mathrm{C}\right)+\mathbf{f}_\mathrm{I}(t)\ ,\end{aligned} \tag{14}$$

where the contributions of the slave mode shapes of vibration to expression of damping forces transmitted by couplings are not respected. We rewrite expression (14)

$$\begin{aligned}\left[\mathbf{E}+(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\,\mathbf{H}\right]\mathbf{f}_\mathrm{C} = &-(\mathbf{B}_\mathrm{G}+\mathbf{B}_\mathrm{B})\,{}^\mathrm{m}\mathbf{V}\,{}^\mathrm{m}\dot{\mathbf{x}}(t)-\\ &-(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\left({}^\mathrm{m}\mathbf{V}\,{}^\mathrm{m}\mathbf{x}(t)+\mathbf{H}\,\mathbf{f}_\mathrm{E}(t)\right)+\mathbf{f}_\mathrm{I}(t)\end{aligned} \tag{15}$$

and introduce matrix

$$\mathbf{C}=\mathbf{E}+(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\,\mathbf{H}\in\mathbb{R}^{n,n}\ . \tag{16}$$

Then it holds for the coupling force vector

$$\mathbf{f}_\mathrm{C}=-\mathbf{C}^{-1}\left[(\mathbf{B}_\mathrm{G}+\mathbf{B}_\mathrm{B})\,{}^\mathrm{m}\mathbf{V}\,{}^\mathrm{m}\dot{\mathbf{x}}(t)+(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\left({}^\mathrm{m}\mathbf{V}\,{}^\mathrm{m}\mathbf{x}(t)+\mathbf{H}\,\mathbf{f}_\mathrm{E}(t)\right)-\mathbf{f}_\mathrm{I}(t)\right]\ . \tag{17}$$

This expression can be substituted in equation (11) and we get the condensed model of a large rotating system of order $m=\sum m_j$ created by means of modal synthesis method with quasistatic consideration of slave eigenmodes of vibration in the form

$$\begin{aligned}&{}^\mathrm{m}\ddot{\mathbf{x}}(t)+\left[\mathbf{D}+\omega_0\,\mathbf{G}+{}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,(\mathbf{B}_\mathrm{G}+\mathbf{B}_\mathrm{B})\,{}^\mathrm{m}\mathbf{V}\right]{}^\mathrm{m}\dot{\mathbf{x}}(t)+\\ &\quad+\left[{}^\mathrm{m}\mathbf{\Lambda}+{}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\,{}^\mathrm{m}\mathbf{V}\right]{}^\mathrm{m}\mathbf{x}(t)=\\ &={}^\mathrm{m}\mathbf{V}^\mathrm{T}\left[\mathbf{E}-\mathbf{C}^{-1}\,(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\,\mathbf{H}\right]\mathbf{f}_\mathrm{E}(t)+{}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,\mathbf{f}_\mathrm{I}(t)\ .\end{aligned} \tag{18}$$

In the case, that we don't consider any contribution of slave eigenmodes of vibration, the residual compliance matrix $\mathbf{H}$ is a null matrix, $\mathbf{C}^{-1}=\mathbf{E}$ is a unit matrix, and the condensed model of the system is of the classical form presented e.g. in [11].

3. Dynamical analysis

Different dynamical analyses of the large nonconservative mechanical system can be performed on the basis of model (18). Usage of the new approach to the calculation of the system dynamical response with quasistatic consideration of the slave eigenmodes of vibration can be in some cases more advantageous than the original approach. Consideration of a certain number of the slave eigenmodes can make better approximation of the system behaviour, while the DOF number is the same as for the classical approach. Contrary on the classical approach only based on chosen master mode shapes of vibration it is necessary to compute $\mathbf{C}^{-1}$ and $\mathbf{H}$ matrices. Because the order of these square matrices is equal to the DOF number of the original nonreduced model, there can be some problems with practical realization for very large mechanical systems mainly from the reason of not enough memory. But it can be used a specific sparse structure of matrices $\mathbf{C}$ a $\mathbf{H}$ and the computation can be realized with less memory requirements [4] by means of the Householder identity.

Eigenfrequencies $\Omega_\nu=2\pi f_\nu$ and eigenmodes ${}^\mathrm{m}\mathbf{x}_\nu$ can be obtained for $\mathbf{f}_\mathrm{E}(t)=\mathbf{f}_\mathrm{I}(t)=\mathbf{0}$ and undamped couplings and subsystems from the modal analysis of the conservative model

$${}^\mathrm{m}\ddot{\mathbf{x}}(t)+\left[{}^\mathrm{m}\mathbf{\Lambda}+{}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,(\mathbf{K}_\mathrm{G}+\mathbf{K}_\mathrm{B})\,{}^\mathrm{m}\mathbf{V}\right]{}^\mathrm{m}\mathbf{x}(t)=\mathbf{0}\ . \tag{19}$$

Eigenmodes of vibration can be transformed according to (12) and (17) from the space of master modal coordinates of the uncoupled, undamped and nonrotating subsystems to the original configuration space of the generalized coordinates of subsystems by

$$\mathbf{q}_\nu = \left[\mathbf{E} - \mathbf{H}\,\mathbf{C}^{-1}\,(\mathbf{K}_\mathrm{G} + \mathbf{K}_\mathrm{B})\right]{}^\mathrm{m}\mathbf{V}\,{}^\mathrm{m}\mathbf{x}_\nu\ , \quad \nu = 1, 2, \ldots, m\ . \tag{20}$$

Another important problem in dynamics of machines is analysis of the steady state dynamic response to the various types of polyharmonic excitations. The common type of excitation in the rotating systems with gears is the internal excitation of each gear meshing by periodic gear kinematic errors. These errors can be approximated by Fourier series with several harmonic components $k\,\omega_z$, where ω_z is the meshing frequency. Let the meshing stiffness and coefficient of viscous damping of gearing z on the gear mesh line be indicate k_z and b_z. The vector of the internal excitation can be, in this case, written in the complex form [11]

$$\mathbf{f}_\mathrm{I}(t) = \sum_{z=1}^{Z}\sum_{k=1}^{K}(k_z\,\Delta_{z,k} + \mathrm{i}\,k\,\omega_z\,b_z\,\Delta_{z,k})\,\mathbf{c}_z\,\mathrm{e}^{\mathrm{i}\,k\,\omega_z\,t}\ , \qquad \Delta_{z,k} = \Delta^\mathrm{C}_{z,k} - \mathrm{i}\,\Delta^\mathrm{S}_{z,k}\ , \tag{21}$$

where $\Delta_{z,k}$ is the complex amplitude of the k-th harmonic component of the error measured on gear mesh line of the gear meshing z and the vector $\mathbf{c}_z$ is determined by geometrical parameters of gears in a mesh [3]. We assume that the vector of external dynamical loads $\mathbf{f}_\mathrm{E}(t)$ is zero. Particular solution of the condensed model (18) can be estimated also in the complex form as

$${}^\mathrm{m}\tilde{\mathbf{x}}(t) = \sum_{z=1}^{Z}\sum_{k=1}^{K}{}^\mathrm{m}\mathbf{x}_{z,k}\,\mathrm{e}^{\mathrm{i}\,k\,\omega_z\,t}\ . \tag{22}$$

After substituting one harmonic component of the excitation vector and the supposed solution in model (18) and after an arrangement we have for chosen z and k

$$\begin{aligned}-k^2\,\omega_z^2\,{}^\mathrm{m}\mathbf{x}_{z,k} + \left[\mathbf{D} + \omega_0\,\mathbf{G} + {}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,(\mathbf{B}_\mathrm{B} + \mathbf{B}_\mathrm{G})\,{}^\mathrm{m}\mathbf{V}\right]\mathrm{i}\,k\,\omega_z\,{}^\mathrm{m}\mathbf{x}_{z,k} + \\ + \left[{}^\mathrm{m}\mathbf{\Lambda} + {}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,(\mathbf{K}_\mathrm{B} + \mathbf{K}_\mathrm{G})\,{}^\mathrm{m}\mathbf{V}\right]{}^\mathrm{m}\mathbf{x}_{z,k} = {}^\mathrm{m}\mathbf{V}^\mathrm{T}\mathbf{C}^{-1}\,\mathbf{c}_z\,(k_z + \mathrm{i}\,k\,\omega_z\,b_z)\,\Delta_{z,k}\ .\end{aligned} \tag{23}$$

The expression for calculation of the steady state dynamic response amplitudes in dependence on the reference revolutions $n = 30\,\omega_0/\pi$ is then

$${}^\mathrm{m}\mathbf{x}_{z,k}(n) = \mathbf{Z}^{-1}_{z,k}(n)\,\mathbf{f}_{z,k}(n)\ , \tag{24}$$

where

$$\begin{aligned}\mathbf{Z}_{z,k} = -k^2\,\omega_z^2\,\mathbf{E} + \mathrm{i}\,k\,\omega_z\left[\mathbf{D} + \omega_0\,\mathbf{G} + {}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,(\mathbf{B}_\mathrm{B} + \mathbf{B}_\mathrm{G})\,{}^\mathrm{m}\mathbf{V}\right] + \\ + {}^\mathrm{m}\mathbf{\Lambda} + {}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,(\mathbf{K}_\mathrm{B} + \mathbf{K}_\mathrm{G})\,{}^\mathrm{m}\mathbf{V}\end{aligned} \tag{25}$$

and

$$\mathbf{f}_{z,k} = {}^\mathrm{m}\mathbf{V}^\mathrm{T}\,\mathbf{C}^{-1}\,\mathbf{c}_z\,(k_z + \mathrm{i}\,k\,\omega_z\,b_z)\,\Delta_{z,k}\ . \tag{26}$$

The transformation of complex amplitudes ${}^{\mathrm{m}}\mathbf{x}_{z,k}$ into the original configuration space of generalized subsystems coordinates is according to (12), (17) and (21) of the form

$$\mathbf{q}_{z,k} = {}^{\mathrm{m}}\mathbf{V}\,{}^{\mathrm{m}}\mathbf{x}_{z,k} - \mathbf{H}\,\mathbf{C}^{-1}\Big[\mathrm{i}\,k\,\omega_z\,(\mathbf{B}_{\mathrm{G}} + \mathbf{B}_{\mathrm{B}})\,{}^{\mathrm{m}}\mathbf{V}\,{}^{\mathrm{m}}\mathbf{x}_{z,k} + (\mathbf{K}_{\mathrm{G}} + \mathbf{K}_{\mathrm{B}})\,{}^{\mathrm{m}}\mathbf{V}\,{}^{\mathrm{m}}\mathbf{x}_{z,k} - \mathbf{c}_z\,(k_z + \mathrm{i}\,k\,\omega_z\,b_z)\,\Delta_{z,k}\Big]\,. \tag{27}$$

4. Numerical experiments

The presented methodology was verified using the simple test-gearbox (Figure 2). The gearbox was decomposed into two rotating shafts with helical spur gears ($j = 1, 2$) and into the housing ($j = 3$). The shafts were discretized using shaft finite elements [7] and the gears were modelled using their discrete parameters (mass and moments of inertia). The shaft subsystem models were created and their modal analyses were performed in MATLAB code. The housing was modelled as 3D continuum using FEM in ANSYS system. The necessary housing modal values (eigenfrequencies and chosen eigenvectors) were exported from ANSYS to MATLAB. The condensed model of the whole system was assembled in MATLAB code on the basis of the presented methodology. The MATLAB system was also used for the computation of eigenvalues and steady state dynamic response. The original nonreduced models of subsystems had together over 15 000 DOF. The shaft system is included by means of flexible torsional couplings into a drive system. They are supposed constant angular speeds ω_1 and ω_2 of the driving and driven parts of the system. The static external loading was defined by initial static torsional preloading $\Delta\varphi_1$ and $\Delta\varphi_2$ on both sides of the drive system (see Figure 2). We assume that the static preloading is sufficient for the constant gear mesh and that is why the gear coupling can be linearized.

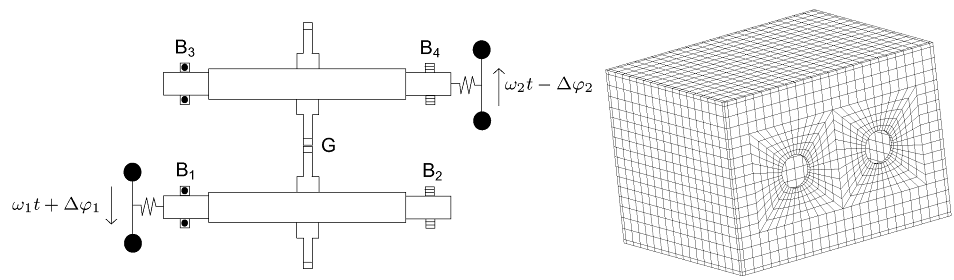

Fig.2: Scheme of the test-gearbox

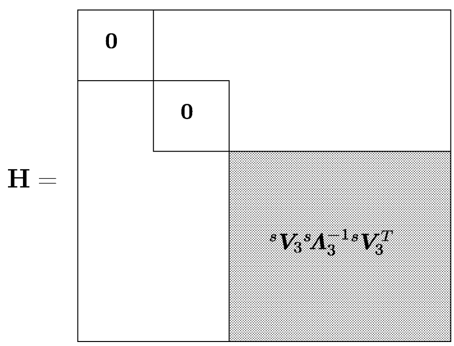

Fig.3: Structure of the residual compliance matrix of the test-gearbox

The improvement caused by the contribution of a certain number of the slave eigenmodes of vibration was at first tested for the modal analysis of the conservative model (19). The referential condensed model ($m = 580$) was composed of 90 master eigenmodes of each shaft subsystem ($m_1 = m_2 = 90$, $s_1 = s_2 = 0$) and 400 master eigenmodes of the stator subsystem ($m_3 = 400$, $s_3 = 0$). Further modal analyses were performed for additional types of condensed models characterized by the same number of master mode shapes of the shaft subsystems ($m_1 = m_2 = 90$, $s_1 = s_2 = 0$), but different number of stator master and slave mode shapes of vibration. The corresponding residual compliance matrix $\mathbf{H}$ is of the form schematically shown in Figure 3.

The selected numbers of master and slave eigenmodes of the stator were

$$m_3 = \{\,50, 100, 150, 200\,\} \qquad \text{and} \qquad s_3 = \{\,0, 50, 100\,\}\ .$$

Altogether twelve modal analyses of various types of condensed models were performed. The difference of the obtained eigenfrequencies $f_\nu^{(\mathrm{N})}$ and eigenfrequencies $f_\nu^{(\mathrm{R})}$ of referential condensed model was evaluated by the cumulative relative error

$$\varepsilon(f_\nu^{(\mathrm{N})}, f_\nu^{(\mathrm{R})}) = \sum_{\nu=1}^{50} \frac{|f_\nu^{(\mathrm{N})} - f_\nu^{(\mathrm{R})}|}{f_\nu^{(\mathrm{R})}}\ . \tag{28}$$

The sums were constructed for 50 lower eigenfrequencies. The comparison of the cumulative relative error of eigenfrequencies for different numbers of chosen master and slave mode shapes of the stator is shown in Figure 4. It can be noted, that consideration of slave eigenmodes of vibration brings improvement of modal analysis results.

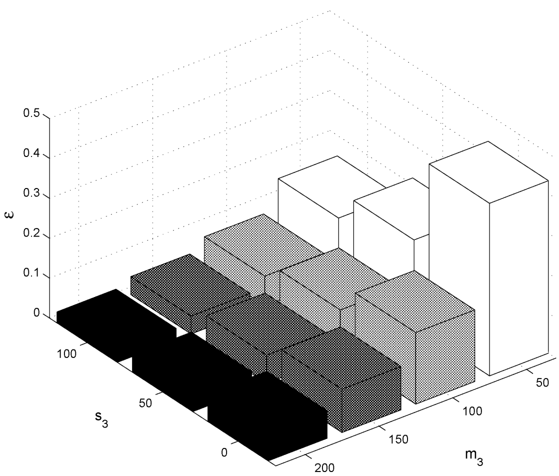

Fig.4: Comparison of the cumulative relative error for different levels of reduction and different numbers of slave modes of vibration

A suitable criterion for comparison of the computed eigenvectors $\mathbf{q}_i^{(\mathrm{N})}$ and referential eigenvectors $\mathbf{q}_j^{(\mathrm{R})}$ of vibration defined in expression (20) is the normalized cross orthogonality (NCO) matrix

$$\mathbf{N}\left(\mathbf{q}_i^{(\mathrm{N})}, \mathbf{q}_j^{(\mathrm{R})}\right) = \frac{\left(\mathbf{q}_i^{(\mathrm{N})\mathrm{T}}\,\mathbf{M}\,\mathbf{q}_j^{(\mathrm{R})}\right)^2}{\left(\mathbf{q}_i^{(\mathrm{N})\mathrm{T}}\,\mathbf{M}\,\mathbf{q}_i^{(\mathrm{N})}\right)\left(\mathbf{q}_j^{(\mathrm{R})\mathrm{T}}\,\mathbf{M}\,\mathbf{q}_j^{(\mathrm{R})}\right)}\ , \qquad i, j = 1, 2, \ldots, 50\ , \tag{29}$$

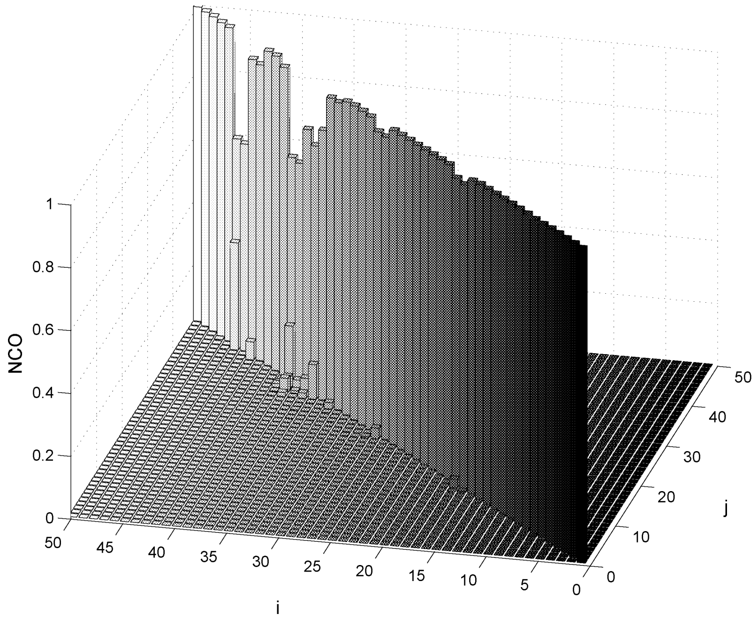

Fig.5: Normalized cross orthogonality (NCO) matrix for the model composed of 400 master mode shapes and any slave mode shapes of the stator and for the model composed of 150 master mode shapes and any slave mode shapes of the stator

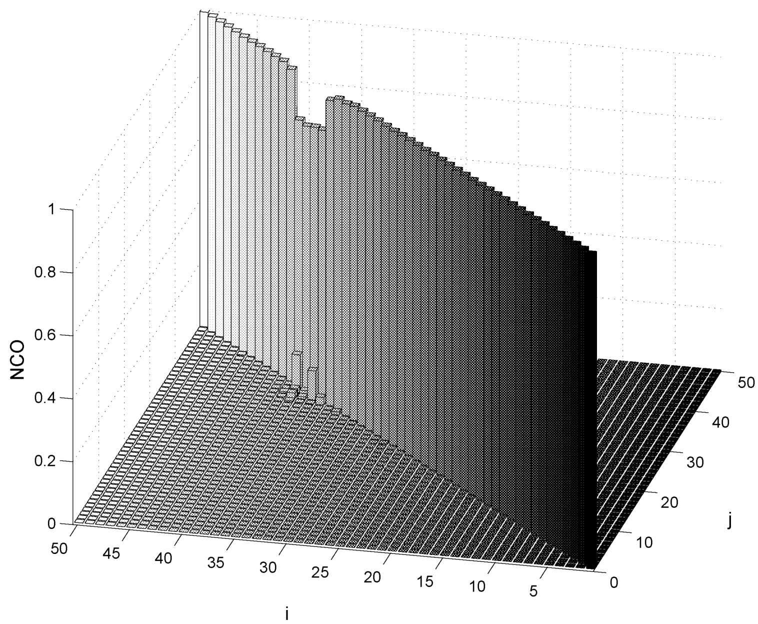

Fig.6: Normalized cross orthogonality (NCO) matrix for the model composed of 400 master mode shapes and any slave mode shapes of the stator and for the model composed of 150 master mode shapes and 100 slave mode shapes of the stator

where $\mathbf{M} \in \mathbb{R}^{n,n}$ is the mass matrix of the whole system. The normalized cross orthogonality matrix is a unit matrix in the idealized case, when the compared sets of eigenvectors are identical. The illustration of NCO matrix calculated for the eigenvectors of the referential model and of the model composed of 150 stator master mode shapes and any stator slave mode shapes is in Figure 5. The NCO matrix calculated for the eigenvectors of the referential model and of the model composed of 150 stator master mode shapes and 100 stator slave mode shapes is shown in Figure 6. The improvement caused by consideration of slave mode shapes of vibration is obvious.

The improvement of the steady state dynamic response was studied using overall acoustic power radiated by the stator surface. Acoustic power was computed for upper effective estimates of the gearbox housing nodal velocities. The corresponding theory is described in [3]. Figure 7 shows the comparison of the acoustic power computed for the referential model and for the models characterized by 200 stator master mode shapes and 100 or zero stator slave mode shapes.

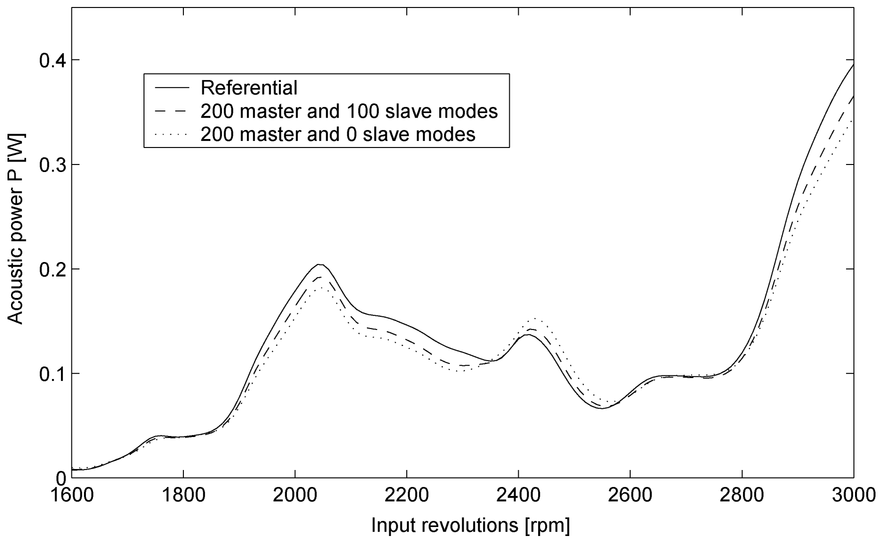

Fig.7: Comparison of the gearbox acoustic power in dependence on input revolutions for the referential model ($m_1 = m_2 = 90$, $m_3 = 400$, $s_1 = s_2 = s_3 = 0$) and for the models with 200 stator master mode shapes and 100 or zero stator slave mode shapes

5. Conclusion

The modal synthesis method with quasistatic consideration of slave eigenmodes of vibration usable for modelling dynamical mechanical systems was presented in this paper. Contrary on the original approach characterized by generalized coordinates transformation using only master mode shapes of vibration [11], the new approach is moreover based on the usage of a certain number of slave eigenmodes of vibration. Consideration of the chosen slave eigenmodes can improve approximation of the system behaviour, while the DOF number is the same as for the original approach. This fact was verified by means of numerical experiments with the simple test-gearbox. The improvement of the modal and acoustic analysis results by considering the slave mode shapes of vibration is documented.

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