VIBRATION CONTROL OF PLATE STRUCTURES

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The paper describes two approaches to the problem of active damping of vibrations of plate structures. First one is based on the full state feedback designed by the pole placement or LQ (Linear Quadratic) optimization and state observer. The incomplete pole assignment method is used instead of the standard full assignment. The second one is based on experimental identification of the first mode shape and design dynamic compensator. Both methods are confronted in regard of quality and the robustness of the control law.

Key words : *active damping, vibration control, pole placement problem, linear quadratic controller, SC2FA*

1. Introduction

Vibration control of flexible structures is an important issue in many engineering applications. Balancing the stringent performance objectives of modern structures such as superior strength and minimal weight introduces a dynamic component that needs to be considered. Depending on the applications, low structural damping can lead to problems such as measurement inaccuracy of attached equipment, transmission of acoustic noise or structural failure. Various methods to suppress vibrations have been developed and these commonly include active, passive, semi-active and hybrid vibration control systems.

In recent years, the research topic of active control for flexible structures has received considerable attention. In order to reject vibrations an optimal LQ or state feedback based on pole assignment has been addressed in literature as a means used in solving active control problems ([8–11]).

This paper addresses the vibration control of rectangular plate structures by the methods of linear feedback control. It is concerned with state feedback designed by pole placement method (in our case modification of this method – incomplete pole assignment), linear quadratic optimization and by self tuning regulator. Applying these methods an optimal place is chosen for sensor and actuator by the method shown in [12] which is based on mode shapes – amplitudes and nodal points.

Mathematical model of finite rectangular plate is considered in Section 2. The resulted model is used in Section 3 where three possibilities of control law are described. Section 3.1 describes the incomplete pole assignment method, while Section 3.2 deals with linear quadratic controller which is often suggested in literature as a means for solving active control problems. The third control law – self tuning controller is described in Section 3.3. All these methods are compared in the regard of quality of control law and the robustness in the last part of Section 3. Finally, in Section 4, some overall conclusions are drawn.

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2. Mathematical model

The mathematical model of thin rectangular plate vibration – equation of motion has the following form [1], [7]

$$\Delta\Delta q = -\frac{g_{\mathrm{F}}}{D} \ , \tag{2.1}$$

where g_{F} contains all forces, inertia ones included, D is cylindrical stiffness of the plate and the operator is defined by

$$\Delta\Delta = \frac{\partial^4}{\partial x^4} + 2\,\frac{\partial^4}{\partial x^2\,\partial y^2} + \frac{\partial^4}{\partial y^4} \ . \tag{2.2}$$

Considering only one discrete force the equation of motion takes the form

$$\Delta\Delta q = \frac{1}{D}\left[\gamma(t)\,\delta(\mathbf{r}-\mathbf{z}) - \mu\,\ddot{q}\right] \ , \tag{2.3}$$

where $\mu = h\,\varrho$, ϱ is density, h is thickness, $\mathbf{r} = [x, y]$ is radius vector of arbitrary point, $\mathbf{z} = [x_{\mathrm{a}}, y_{\mathrm{a}}]$ is position of the actuator, $\gamma(t)$ is time dependence of the force and $\delta(\mathbf{r}-\mathbf{z})$ is Dirac impulse – spatial distribution of the force.

Free oscillations of the plate can be described by the equation

$$\overbrace{\Delta\Delta}^{\mathbf{K}}\, q = -\overbrace{\frac{\mu}{M}}^{\mathbf{M}}\ \ddot{q} \ . \tag{2.4}$$

Solving (2.4) by separation of variables yields

$$\overbrace{\Delta\Delta}^{\mathbf{K}}\, v - \Omega^2\ \overbrace{\frac{\mu}{M}}^{\mathbf{M}}\ v = 0 \ . \tag{2.5}$$

The solution is assumed in the form

$$v = \sum a_{ij}\,\varphi_{ij}(\mathbf{r}) = \sum_i \sum_j a_{ij}\ \sin\frac{i\,\pi\,x}{a}\ \sin\frac{j\,\pi\,y}{b} \ . \tag{2.6}$$

Substituting (2.6) to (2.5) we obtain the natural frequencies

$$\Omega_{ij} = \pi^2\sqrt{\frac{D}{\mu}}\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right) \tag{2.7}$$

and eigenfunctions

$$v_{ij} = 2\sqrt{\frac{D}{\mu\,a\,b}}\ \sin\frac{i\,\pi\,x}{a}\ \sin\frac{j\,\pi\,y}{b} \ , \tag{2.8}$$

corresponding to i and j subscripts. The eigenfunctions are orthonormalized by the M-norm. These functions (2.8) respect the boundary conditions (plate is simply supported on the boundary).

Forced vibrations of thin plate excited by the discrete force located in position $\mathbf{z}$ have a response

$$q(\mathbf{r},t) = \sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}\Bigg[\langle v_{ij}, M\,p\rangle\,\cos\Omega_{ij}t + \frac{1}{\Omega_{ij}}\,\langle v_{ij}, M\,r\rangle\,\sin\Omega_{ij}t + + \frac{1}{\Omega_{ij}}\int_0^t \langle v_{ij}, F\rangle\,\gamma(t)\,\sin\Omega_{ij}(t-\tau)\,\mathrm{d}\tau\Bigg]\;, \tag{2.9}$$

where p is initial displacement field, r is initial velocity field and $F = \delta(\mathbf{r}-\mathbf{z})$ is spatial distribution of the force (in our case p and r are zeroes) and scalar product $\langle v(x), w(x)\rangle$ is defined by

$$\langle v(x), w(x)\rangle = \int_{\overline{\Omega}} v(x)\,w(x)\,\mathrm{d}V\;, \tag{2.10}$$

where $\overline{\Omega}$ is occupied domain by continuum and $\mathrm{d}V$ is a volume infinitesimal element.

The equation (2.9) can be rewritten into form

$$q(\mathbf{r},t) = \sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}\left[\frac{1}{\Omega_{ij}}\,v_{ij}(\mathbf{z})\int_0^t \gamma(\tau)\,\sin\Omega_{ij}(t-\tau)\,\mathrm{d}\tau\right]\;. \tag{2.11}$$

The proportional damping is assumed.

Laplace transform of the response under (2.11) yields the formula

$$Q(\mathbf{z},\mathbf{r}_\mathrm{s},s) = \underbrace{\sum_{i,\,j} v_{ij}(\mathbf{r}_\mathrm{s})\,\frac{v_{ij}(\mathbf{z})}{\Omega_{ij}^2 + s^2 + s\,2\,D_{ij}\,\Omega_{ij}}}_{H(\mathbf{z},\mathbf{r}_\mathrm{s},s)}\,\Gamma(s)\;, \tag{2.12}$$

where $\mathbf{r}_\mathrm{s} = [x_\mathrm{s}, y_\mathrm{s}]$ is position of the sensor, $\mathbf{z} = [x_\mathrm{a}, y_\mathrm{a}]$ is position of the actuator, s is parameter of Laplace transformation, $H(\mathbf{z},\mathbf{r}_\mathrm{s},s)$ is transfer function and $\Gamma(s)$ is Laplace transformation of $\gamma(t)$.

Problem: Size: $250\times200\times1$ mm ($a\times b\times h$)
Material properties (steel): $E = 2.1\times10^{11}$ Pa
$\nu = 0.3$
$\varrho = 7800\,\mathrm{kg/m^3}$
$D_{ij} = 0.01$ (damping parameter)
Boundary conditions: simply supported on all edges

	Eigenfrequency [rad/s]
1	635.4
2	1379.2
3	2619.0
4	1797.7
5	2541.5
6	3781.3
7	3734.8
8	4478.7
9	5718.4

Tab.1: Eigenfrequencies of uncontrolled model

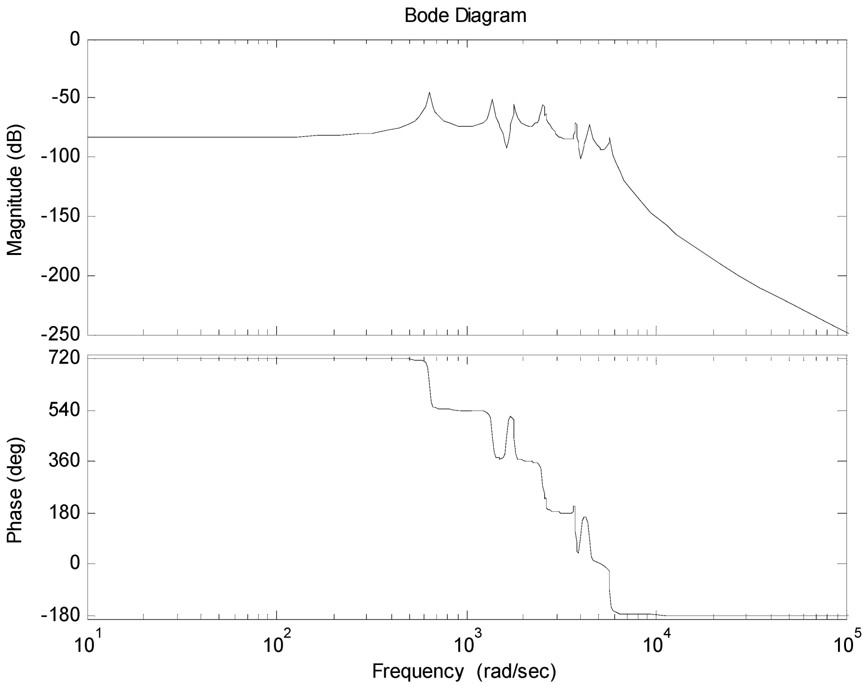

Fig.1: Bode diagram of uncontrolled model of thin rectangular plate

3. Vibration control

In this section, the truncated transfer function description (2.12) is substituted by the equivalent state space model

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}\;, \\ \mathbf{y} &= \mathbf{C}\,\mathbf{x}\;, \end{aligned} \tag{3.1}$$

where $\mathbf{x} \in \mathbf{R}^{2n}$ is the state of the system, n is the number of mode shapes considering, $\mathbf{u}$ is the input, $\mathbf{y}$ is the output, and $\mathbf{A} \in \mathbf{R}^{2n\times 2n}$, $\mathbf{B} \in \mathbf{R}^{2n\times 1}$, $\mathbf{C} \in \mathbf{R}^{1\times 2n}$ are system matrices given by where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & & & \\ -\beta_1 & -\alpha_1 & & & \\ & & \ddots & & \\ & & & 0 & 1 \\ & & & -\beta_n & -\alpha_n \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ K_1 \\ \vdots \\ 0 \\ K_n \end{bmatrix}, \quad \mathbf{C} = [\,1 \quad 0 \quad \cdots \quad 1 \quad 0\,]\;, \tag{3.2}$$

$$\left.\begin{aligned} \alpha_k &= 2\,D_{ij}\,\Omega_{ij}\;, \\ \beta_k &= \Omega_{ij}^2\;, \\ K_k &= \nu_{ij}(\mathbf{r}_\mathrm{s})\,\nu_{ij}(\mathbf{z}) \end{aligned}\right\} \quad \begin{aligned} & k = 3\,(i-1)+j\;, \\ & k \in \{1,2,\dots,n\}\;, \\ & i,j = 1,2,3\;. \end{aligned} \tag{3.3}$$

In the following examples, $n = 9$ is assumed.

Further, it is considered that the inputs for disturbances are the same as for the control actions.

3.1. Incomplete pole assignment

It is well known that the positions of the poles of the system (3.1) determine the damping of system responses. For this purpose the slightly damped eigenvalues of the matrix **A** should be properly changed. Especially, the eigenvalues with the minimal absolute values should be located to the suitable positions. Since the complete pole assignment by state feedback is unrealistic, because of ill conditioning of the corresponding problem, we focus on the incomplete assignment. Thus, only the m closed loop poles are required to the assignment to the properly chosen locations in the complex plane. For this purpose the standard control configuration with state observer is used (Fig. 2).

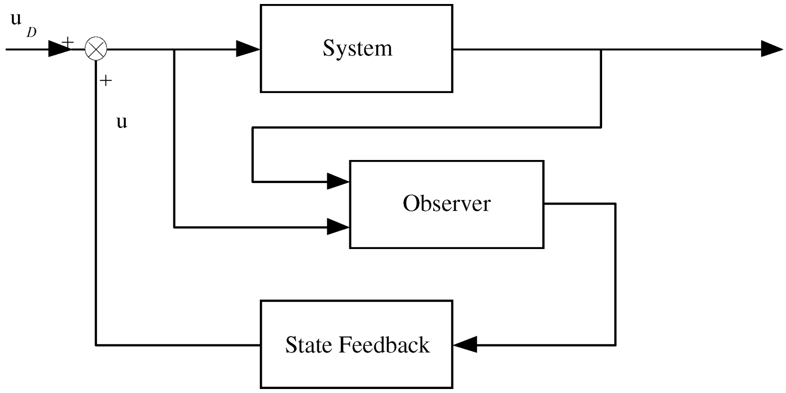

Fig.2: Standard configuration of the state feedback with the state observer

The state feedback

$$\mathbf{u} = \mathbf{F}\,\mathbf{x}\ , \tag{3.4}$$

where $\mathbf{F} \in \mathbf{R}^{1\times 2n}$, gives the closed loop system

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{F}\,\mathbf{u} = (\mathbf{A} + \mathbf{B}\,\mathbf{F})\,\mathbf{x}\ . \tag{3.5}$$

The incomplete assignment problem considered above yields the requirement

$$\mathbf{A} + \mathbf{B}\,\mathbf{F} \approx \begin{bmatrix} \mathbf{L} & * \\ \mathbf{0} & * \end{bmatrix}\ , \tag{3.6}$$

where the symbol $\approx$ denotes the relation of matrix similarity and $\mathbf{L} \in \mathbf{R}^{m\times m}$ is given matrix with required eigenvalues of the closed loop. In [2] and [4], it was shown that any matrix **F** satisfying (3.6) can be expressed in the form

$$\mathbf{F}(\mathbf{H}, \hat{\mathbf{F}}) = \mathbf{H}[\mathbf{X}^{\mathrm{T}}(\mathbf{H})\,\mathbf{X}(\mathbf{H})]^{-1}\,\mathbf{X}^{\mathrm{T}}(\mathbf{H}) + \hat{\mathbf{F}}\ , \tag{3.7}$$

where $\mathbf{X}(\mathbf{H})$ is the solution of the matrix equation

$$\mathbf{A}\,\mathbf{X} - \mathbf{X}\,\mathbf{L} + \mathbf{B}\,\mathbf{H} = 0 \tag{3.8}$$

where $\mathbf{H} \in \mathbf{R}^{1\times m}$ and $\hat{\mathbf{F}} \in \mathbf{R}^{1\times 2n}$ is an arbitrary matrix satisfying the condition

$$\hat{\mathbf{F}}\,\mathbf{X}(\mathbf{H}) = 0\ . \tag{3.9}$$

Moreover in [2] it is proved that $\mathbf{X}(\mathbf{H})$ has fuel rank for almost any $\mathbf{H}$ and $\mathbf{F}(\mathbf{H},\hat{\mathbf{F}})$ given by (3.7) satisfies (3.6). Thus (3.7) can be used for computing of the state feedback assigning the eigenvalues of $\mathbf{L}$ to the matrix $\mathbf{A}+\mathbf{B}\,\mathbf{F}$. The freedom in this procedure caused by the free choosing of the matrix $\mathbf{H}$ can be used for obtaining the most robust solution by Monte Carlo method.

The observer is described by

$$\begin{aligned}\dot{\hat{\mathbf{x}}} &= \mathbf{A}\,\hat{\mathbf{x}}+\mathbf{B}\,\mathbf{u}+\mathbf{K}\,(\hat{\mathbf{y}}-\mathbf{y})\ ,\\ \hat{\mathbf{y}} &= \mathbf{C}\,\hat{\mathbf{x}}\ ,\end{aligned} \tag{3.10}$$

where $\hat{\mathbf{x}}$ is the estimation of the state $\mathbf{x}$, $\hat{\mathbf{y}}$ is the estimation of the output $\mathbf{y}$ and $\mathbf{K}\in\mathbf{R}^{2n\times1}$ is the appropriate gain matrix obtained from the eigenvalues assignment problem for the observer matrix $\mathbf{A}+\mathbf{K}\,\mathbf{C}$ (for details see [3]).

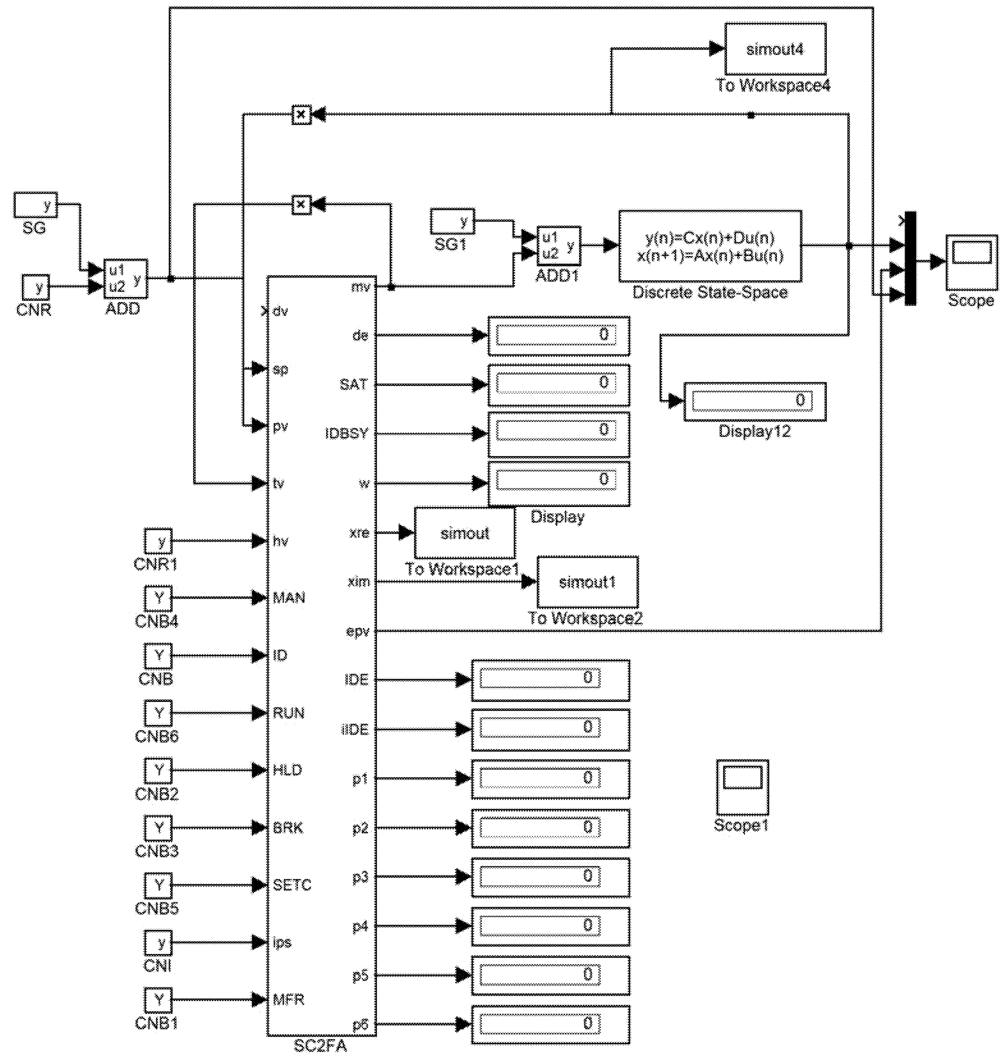

Fig.3: Model of SC2FA in Matlab – Simulink

3.2. Linear quadratic optimization [3]

In this case, the performance index is given by

$$I = \int_0^\infty (\mathbf{x}^\mathrm{T}\,\mathbf{Q}\,\mathbf{x} + \mathbf{u}^\mathrm{T}\,\mathbf{R}\,\mathbf{u})\,\mathrm{d}t \rightarrow \min \ , \tag{3.11}$$

where $\mathbf{Q}$ is a positive semi-definite matrix and $\mathbf{R}$ is a positive definite matrix.

The optimal control has the form

$$\begin{aligned} \mathbf{u} &= \mathbf{F}\,\mathbf{x} \ , \\ \mathbf{F} &= -\mathbf{R}^{-1}\,\mathbf{B}^\mathrm{T}\,\mathbf{P} \ , \\ I &= \mathbf{x}_0^\mathrm{T}\,\mathbf{P}\,\mathbf{x}_0 \ , \end{aligned} \tag{3.12}$$

where $\mathbf{P}$ is the solution of the algebraic Riccati equation

$$\mathbf{A}^\mathrm{T}\,\mathbf{P} + \mathbf{P}\,\mathbf{A} - \mathbf{P}\,\mathbf{B}\,\mathbf{R}^{-1}\,\mathbf{B}^\mathrm{T}\,\mathbf{P} + \mathbf{Q} = \mathbf{0} \ . \tag{3.13}$$

The design freedom of the matrices $\mathbf{Q}$ and $\mathbf{R}$ can be used again to obtain the good performance and the robustness by Monte Carlo method.

3.3. Self tuning controller SC2FA

The self tuning controller SC2FA is a special controller for vibration damping from the control system REX [5], [6]. This function block provides all necessary steps to design the control law for suppressing the first mode shape. Particularly, it provides the automatic identification of the first mode dynamics, design of the state feedback and state observer according to the user design specification. Also, the corresponding control law is implemented within this block – Fig. 3.

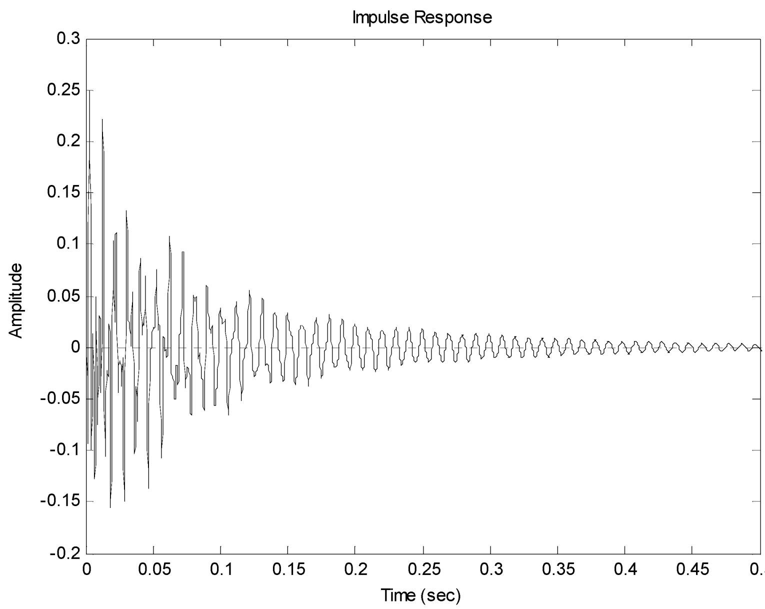

Fig.4: Impulse response of uncontrolled model of thin rectangular plate

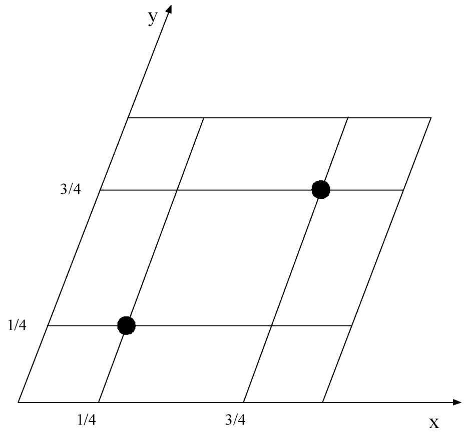

Fig.5: Positions of sensor and actuator

3.4. Results

The positions of the sensor and actuator for design of the state feedback are chosen as

$$\text{Sensor:} \quad \left[\frac{3\,a}{4}, \frac{3\,b}{4}\right]$$

$$\text{Actuator:} \quad \left[\frac{a}{4}, \frac{b}{4}\right]$$

These locations are depicted in Fig. 5.

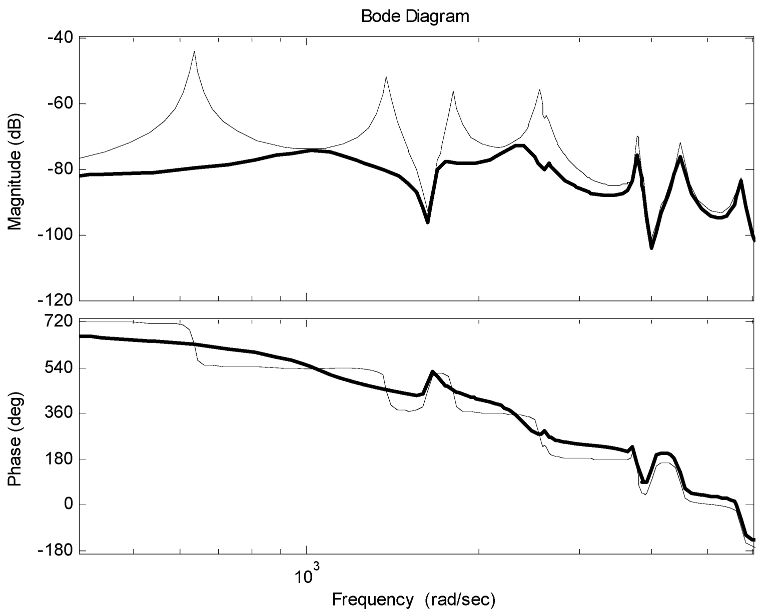

Fig.6: Comparison of Bode diagram of uncontrolled model and model with controller designed by incomplete pole assignment method (heavy line is controlled model)

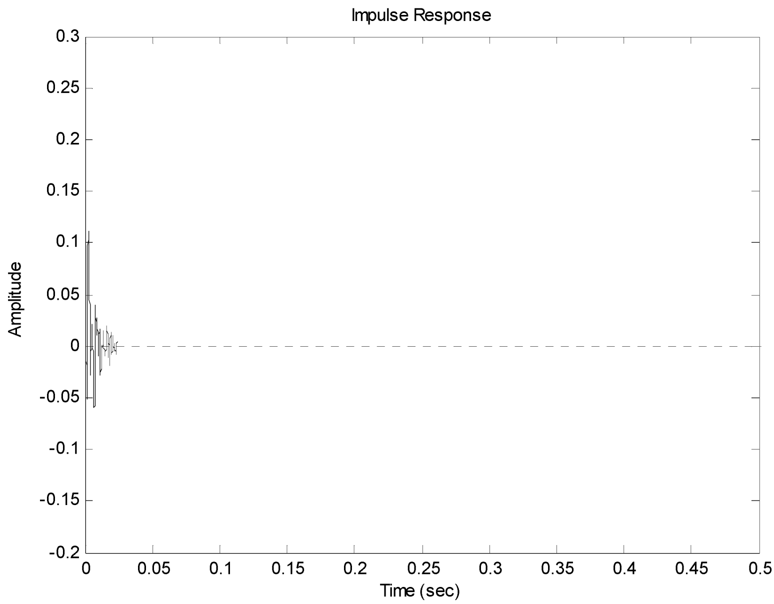

Fig.7: Impulse response of controlled system, state feedback controlled designed by incomplete pole assignment method

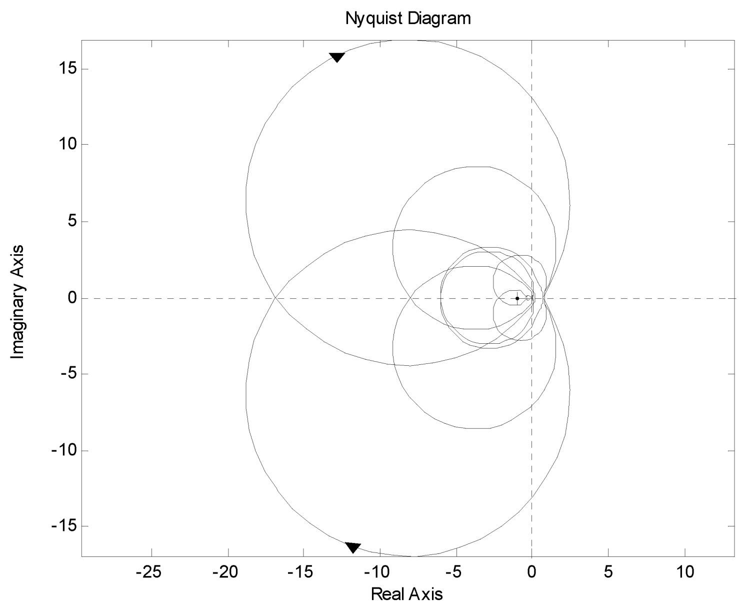

Fig.8: Nyquist diagram of open loop of the system with controller designed by incomplete pole assignment method

The grater distance of the Nyquist plot from the point $[-1, 0]$ the more robust system is achieved.

From Fig. 10 it follows that LQ controller damps the impulse response well, but from Fig. 7 it is clear that the incomplete pole assignment control law yields the quicker and more damped response.

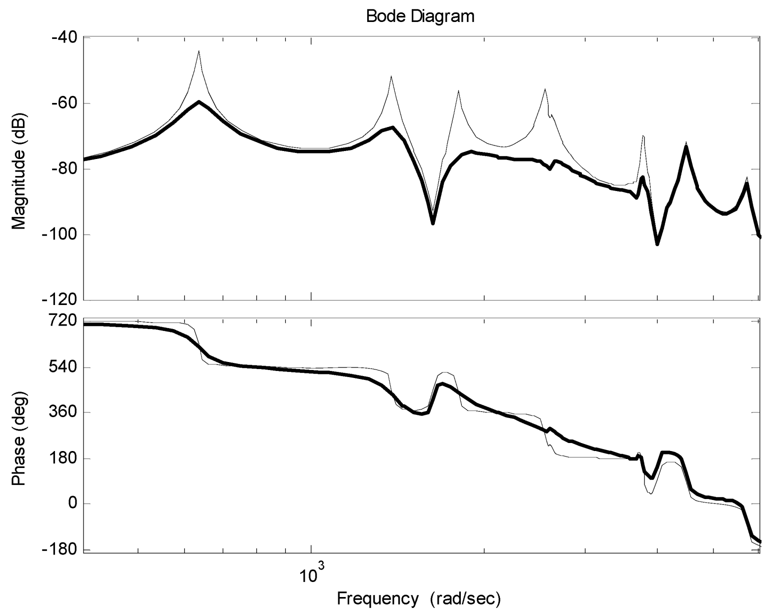

Fig.9: Comparison of Bode diagram of uncontrolled model (thin line) and model with controller designed by linear quadratic optimization method (heavy line)

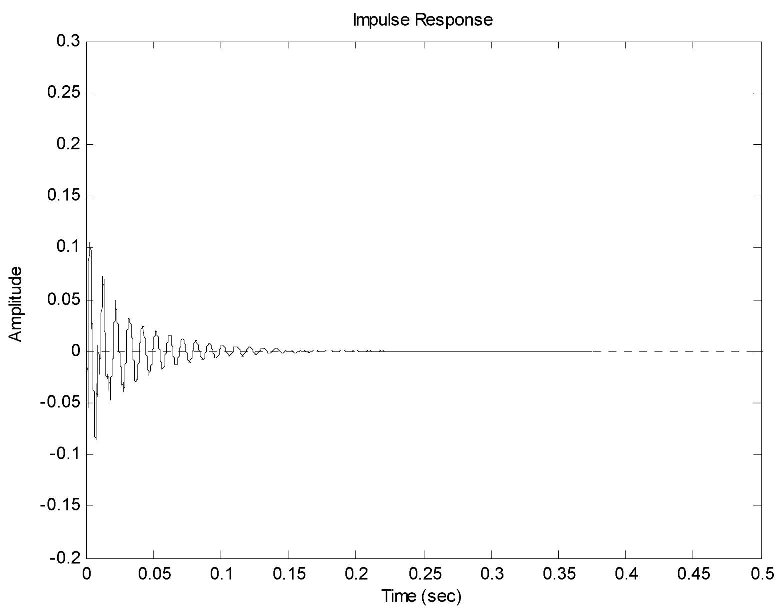

Fig.10: Impulse response of controlled system, state feedback controller designed by linear quadratic optimization method

In the case of self tuning controller design, both sensor and actuator are placed into the same location $[a/2, b/2]$ (see Fig. 12).

From Fig. 15 it follows that the self-tuning controller damps the first mode quite good but the damping of higher modes is insufficient.

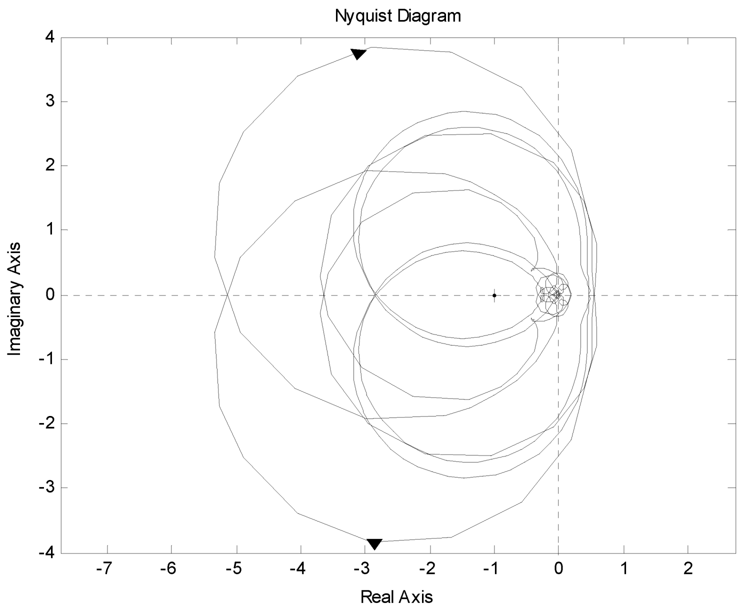

Fig.11: Nyquist diagram of open loop of the system with controller designed by linear quadratic optimization method

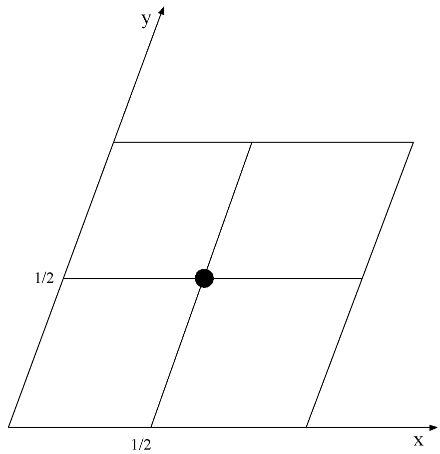

Fig.12: Positions of sensor and actuator

4. Conclusion

In this paper three possibilities of feedback control have been proposed as a means for the control of vibrations of plate structures. It was found that the system with the controller designed by incomplete pole assignment is more robust then the controller resulted form linear quadratic optimization. In this point, it is important to note that both the LQ and pole assignment state feedback were designed by Monte Carlo method which maximizes the measure of the robustness. All tested methods give rather good results when they are simulated in MATLAB (Simulink). However, only the self tuning controller with experimental identification can be simply implemented in real case.

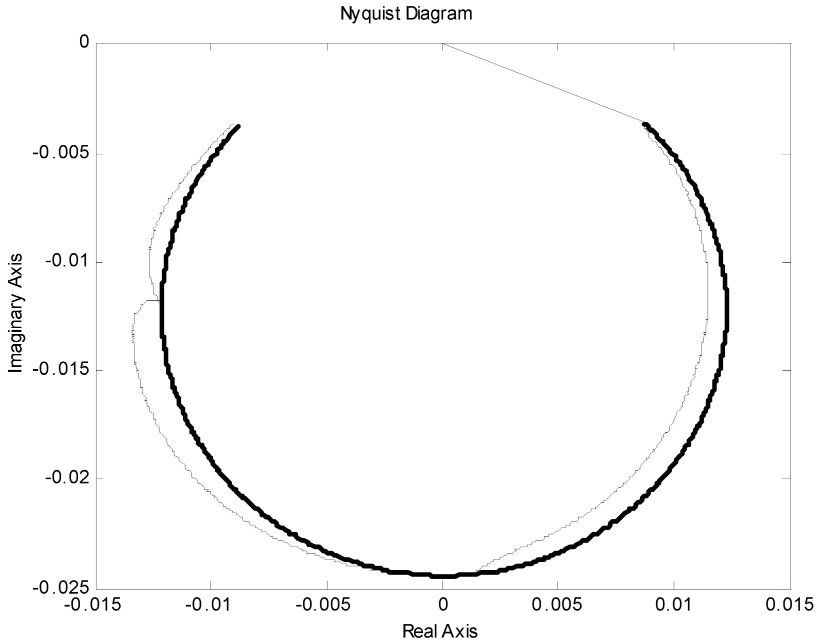

Fig.13: Approximation of the frequency response from SC2FA block by the second order model (heavy line) and identified points of response by SC2FA (thin line)

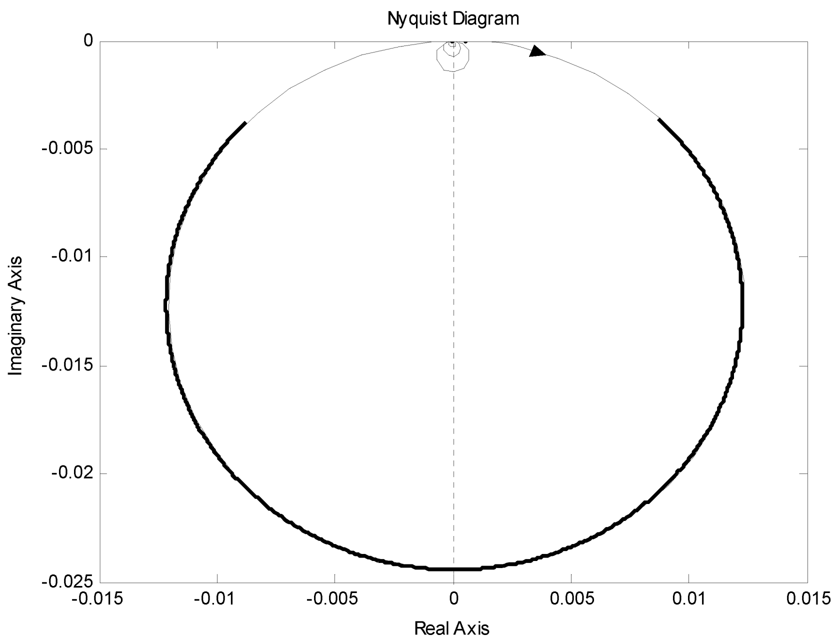

Fig.14: The responses of the second order model (heavy line) and the real system (thin line)

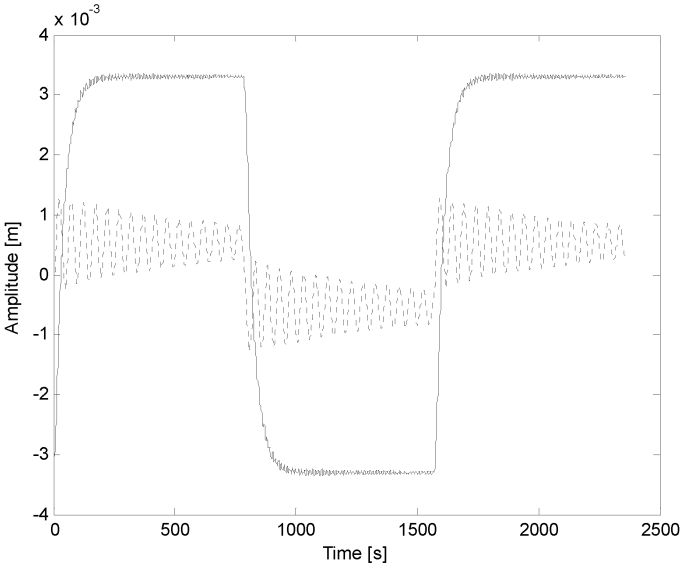

Fig.15: Open loop (dotted line) and closed loop (solid line) responses to the rectangular signal excitation for SC2FA controller

The purpose of this work is to compare the standard LQ control law with the controller designed by incomplete pole assignment and more practical self tuning controller. The results are preliminary, but the authors believe that they may be useful for robustness understanding of the problem considered.

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