FEM MODELING OF THERMO-MECHANICAL INTERACTION IN PRE-PRESSED RUBBER BLOCK

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The FEMLAB code based on weak formulation of PDE's problem was used at a solution of feedback thermo-mechanical interaction in pre-pressed rubber block used for resilient elements of the composed tram wheels. The structural motion and heat conduction equations are solved interactively as time dependent problems. The equality of heat energy density and dissipation energy density realizes the coupling between the equations. The dissipation energy density is computed according to the assumed proportional damping model. In the paper the results of thermo-mechanical processes in case of a plane strain deformation under static pre-pressed and cyclic dynamic loading are presented and analyzed.

Key words : *vibrodamping elements, thermo-mechanic interaction, rubber, dynamic loading*

1. Introduction

In frame of grant task GA CR 101/05/2669 'Dynamics and reliability of vibrodamping elements from thermo-visco-elastic-materials' we deal with the mathematical modeling of rubber segments that are used in passive damping for reduction of noise and vibration of composed railway wheels (see [1], [2]).

These segments that are pre-pressed between the rim and disk represent a transformation deformation element between both parts of the wheel. Then the total stress of the segments consists of static pre-press given by a mount and dynamic stress coming from rolling the wheel on the rail.

In case of the static pre-stress it is considered large deformation (cca 20 %) that is modeled in frame of the grant task by finite deformations and viscoelasticity theory [3]. The dynamic stress at relatively smaller deformations (up to 2 %) is herein modeled by linear theory of viscoelasticity concerning thermo-mechanical coupling since elastomers are generally characterized by high inner damping and also marked dependence on temperature [4]. The temperature field besides the mechanical straining also causes additional loading and can influence the lifetime of the elements.

The thermal processes in the elements are quite complicated since in addition to heat flow between a body and surroundings, heat is generated by transformation of dissipated mechanical energy, too. At stationary regime the temperature stabilizes after certain time so that the heat, which is generated by energy dissipated by the inner damping, is in equilibrium with the heat drained to environment. In addition the changes of temperature cause backwardly changes of elastic and damping behaviors and that consequently changes dynamic

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properties of a whole system. Since temperature changes inside the deformed elements due to thermo-elasticity were neglectable small with respect to dissipated energy temperature changes this effect was omitted.

The simple discrete mathematical model for analysis of thermo-mechanical processes at harmonic loading of the rubber resilient segments of the tram wheels is in [5]. Numerical modeling of temperature distribution inside the segments at mechanical harmonic excitation by finite element method (FEM) is presented in [6]. There is heat generation from mechanical energy absorption modeled by constant heat power density in the all volume.

The FEMLAB code based on weak formulation of PDE's problem was used at a solution of feedback thermo-mechanical interaction by FEM. In our approach structural motion and heat conduction equations are solved interactively as time dependent problems. The conservative energy law, i.e. equality of heat energy density and dissipation energy density, realizes the coupling between the equations. The dissipation energy density is numerically evaluated according to the assumed damping model.

In this paper the first results of numerical simulations are presented and analyzed under following considerations: a plane strain deformation under cyclic dynamic loading, proportional damping, heat generation by mechanical energy lost, heat transfer between rubber and outer air and steel parts and stiffness dependence on temperature. The material parameters of the model were identified from laboratory dynamic tests, such as temperature, force and displacement measurements obtained on the rubber samples of the composed tram wheels.

2. Mathematical model of thermo-mechanical interaction

Scheme of the mathematical model with a transfer of mechanical energy into heat is depicted on Fig. 1. It is concerned a time dependent plane deformation case of linear visco-elastic body including inertia forces. At vibration the energy Λ_{D} is changed to the heat Q_{prod} in the body (rubber). Part of the heat flows into surrounding (steel, air) by heat transfer $\dot{q}$. Part of the heat cumulates in the body and increases its temperature T. The boundary conditions of the body deformation and loading are defined so that the bottom part is fixed, upper part is free in deformation and is uniformly loaded either by stresses $p_{\mathrm{x}} = 0$, $p_{\mathrm{y}} \neq 0$ (press) or $p_{\mathrm{x}} \neq 0$, $p_{\mathrm{y}} = 0$ (shear) with harmonic time dependence. Heat conductivity coefficients and reference temperatures of surrounding (air – α_{a}; T_{a}, steel – α_{m}; T_{m}) give the boundary conditions of heat conduction. Initial conditions are described by vectors of displacements $\mathbf{u}_0$ and velocities $\dot{\mathbf{u}}_0$ and initial temperature T_0.

The equation of motion can be expressed in discretized form as

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{B}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{F}\ , \tag{1}$$

where mass $\mathbf{M}$, damping $\mathbf{B}$ and stiffness $\mathbf{K}$ matrices are generated from the Lame equation of motion by weak formulation of FEM. $\mathbf{F}$ is a vector of excitation forces, $\mathbf{u} = [u_1, v_1, \ldots, u_n, v_n]$ is displacement vector (n is number of nodes). A dot above the letter designates a time derivative. The damping matrix is defined according to the expression valid for proportional damping

$$\mathbf{B} = \alpha_{\mathrm{M}}\,\mathbf{M} + \beta_{\mathrm{K}}\,\mathbf{K}\ . \tag{2}$$

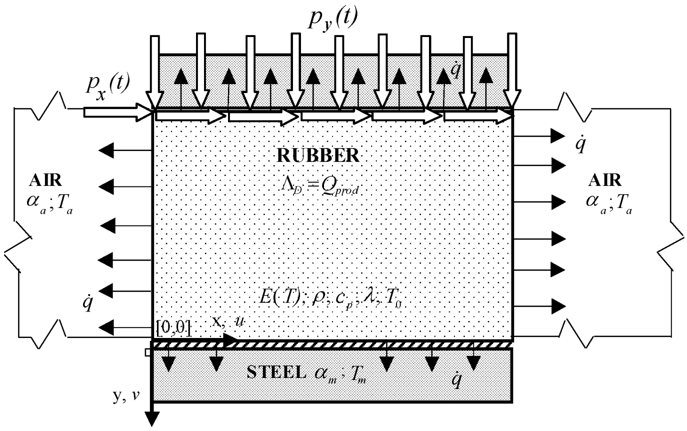


Fig.1: Scheme of mathematical model with thermo-mechanical interaction

The stiffness matrix changes with temperature since the non-linear dependence of Young modulus E on temperature T is introduced into the model:

$$E(T) = \frac{E_0}{1 + k_1\,(T - T_0)} \;, \tag{3}$$

where k_1 is parameter and T_0 is initial temperature of the body.

In continuum mechanics the dissipated energy $\Lambda_\mathrm{D} = \int_V \lambda_\mathrm{d}\,\mathrm{d}V$ of the volume V can be expressed for one loading period T in the form of density as

$$\lambda_\mathrm{d} = \oint \sigma_{\mathrm{d}\,ij}\,\mathrm{d}\varepsilon_{ij} = \int_T \sigma_{\mathrm{d}\,ij}\,\dot{\varepsilon}_{ij}\,\mathrm{d}t \;, \tag{4}$$

where σ_d is a tensor of the dissipated stress and ε, $\dot{\varepsilon}$ are tensors of strain and strain rate, respectively. Then the density of dissipated power can be written

$$\dot{\lambda}_\mathrm{d} = \sigma_{\mathrm{d}\,ij}\,\dot{\varepsilon}_{ij} \;. \tag{5}$$

For the case of plane deformation, the tensor notation can be transferred into vector form as

$$\dot{\lambda}_\mathrm{d} = \{\sigma_\mathrm{d}\}^\mathrm{T}\,\{\dot{\varepsilon}\} \;, \tag{6}$$

where components of the vectors are

$$\{\sigma_\mathrm{d}\} = [\sigma_{\mathrm{d\,xx}};\, \sigma_{\mathrm{d\,yy}};\, \tau_{\mathrm{d\,xy}}] = [\sigma_{\mathrm{d}\,11};\, \sigma_{\mathrm{d}\,22};\, \tau_{\mathrm{d}\,12}] \;, \quad \{\dot{\varepsilon}\} = [\dot{\varepsilon}_\mathrm{xx};\, \dot{\varepsilon}_\mathrm{yy};\, \dot{\gamma}_\mathrm{xy}] = [\dot{\varepsilon}_{11};\, \dot{\varepsilon}_{22};\, 2\,\dot{\varepsilon}_{12}] \;.$$

The components of strain rate for small deformations are defined

$$\dot{\varepsilon}_\mathrm{xx} = \frac{\partial \dot{u}}{\partial x} \;, \qquad \dot{\varepsilon}_\mathrm{yy} = \frac{\partial \dot{v}}{\partial y} \;, \qquad \dot{\gamma}_\mathrm{xy} = \frac{\partial \dot{u}}{\partial y} + \frac{\partial \dot{v}}{\partial x} \;. \tag{7}$$

The components of the dissipated stress for case of proportional damping (2) with coefficients $\alpha_\mathrm{M} = 0$, $\beta_\mathrm{K} \neq 0$ are

$$\begin{aligned}
\sigma_{\mathrm{d\,xx}} &= \beta_\mathrm{K}\,\frac{E}{1-\mu^2}\,(\dot{\varepsilon}_\mathrm{xx} + \mu\,\dot{\varepsilon}_\mathrm{yy}) \;,\\
\sigma_{\mathrm{d\,yy}} &= \beta_\mathrm{K}\,\frac{E}{1-\mu^2}\,(\dot{\varepsilon}_\mathrm{yy} + \mu\,\dot{\varepsilon}_\mathrm{xx}) \;,\\
\tau_{\mathrm{d\,xy}} &= \beta_\mathrm{K}\,\frac{E}{2\,(1+\mu)}\,\dot{\gamma}_\mathrm{xy} = \beta_\mathrm{K}\,G(\dot{\gamma}_\mathrm{xy}) \;.
\end{aligned} \tag{8}$$

After substituting Eq. (7), (8) into (6) we receive

$$\dot{\lambda}_{\mathrm{d}} = \beta_{\mathrm{K}} \frac{E}{1-\mu^2} \left\{ \left(\frac{\partial \dot{u}}{\partial x}\right)^2 + \left(\frac{\partial \dot{v}}{\partial y}\right)^2 + 2\,\mu \left(\frac{\partial \dot{u}}{\partial x}\right) \left(\frac{\partial \dot{v}}{\partial y}\right) + \right. \\ \left. + (1-\mu) \left[\left(\frac{\partial \dot{u}}{\partial y}\right)^2 + 2 \left(\frac{\partial \dot{u}}{\partial y}\right) \left(\frac{\partial \dot{v}}{\partial x}\right) + \left(\frac{\partial \dot{v}}{\partial x}\right)^2 \right] \right\} . \tag{9}$$

For case of proportional damping with $\alpha_{\mathrm{M}} \neq 0$ this expression gains an additional term $\dot{\lambda}_{\mathrm{d+}}$ due to energy lost from dissipation forces F_{d} that are from definition of proportional damping a function $\alpha_{\mathrm{M}}\,\varrho\,\{\dot{u}\}$. Then

$$\dot{\lambda}_{\mathrm{d+}} = \{F_{\mathrm{d}}\}^{\mathrm{T}} \{\dot{u}\} = \alpha_{\mathrm{M}}\,\varrho\,\{\dot{u}\}^2 , \tag{10}$$

where $\{F_{\mathrm{d}}\} = [F_{\mathrm{dx}}; F_{\mathrm{dy}}]$ and $\{\dot{u}\} = [\dot{u}, \dot{v}]$. Since the density of dissipated power is over the coefficient α_{M} dependent on absolute velocity (10) with respect to an inertial space this part of damping is called an absolute damping. In case of coefficient β_{K} there is a dependence of dissipated energy on strain rates (9) and therefore this part is denoted as internal (relative) damping (see [7]).

Hence the dissipated energy changes into the heat according to the equation $Q_{\mathrm{prod}} = \Lambda_{\mathrm{D}}$. The heat both cumulates in the body and passes to surrounding. Neglecting a thermal exchange by convection and concerning $\lambda, c_{\mathrm{p}}, \varrho = \mathrm{const.}$, this process can be mathematically described by Fourier equation of the linear heat conduction

$$\varrho\, c_{\mathrm{p}} \left(\frac{\partial T}{\partial t}\right) + \{L\}^{\mathrm{T}} \{\dot{q}\} = \dot{q}_{\mathrm{prod}} , \tag{11}$$

where L is divergence and gradient operator; $\dot{q}_{\mathrm{prod}} = \dot{\lambda}_{\mathrm{d}}$ is volume density of generated heat power $\dot{Q}_{\mathrm{prod}}$ and $\{\dot{q}\} = -\lambda\, \mathrm{grad}\, T$ surface density of heat flow. The heat transfer by surface B is described by the Newton cooling law

$$\dot{q} = -\alpha_B \left(T_{B\mathrm{e}} - T_{B\mathrm{i}}\right) , \tag{12}$$

where α_B ($B = \mathrm{a}, \mathrm{m}$ for air and metal, respectively) is a coefficient of heat transfer on boundary B, quantities $T_{B\mathrm{e}}$, $T_{B\mathrm{i}}$ are temperatures on outer and inner side, respectively, of the boundary B.

3. Numerical results of rubber segment model at dynamic loading with thermo-mechanic interaction

The above described mathematical model was used for solution of thermo-mechanical processes in rubber block (width 0.047 m, height 0.0258 m, thickness 0.05 m) as geometrically simplified rubber resilient segment used for the wheels under press and shear dynamic loadings. The material of the block was rubber on basis of synthetic isopren butadien elastomer with hardness Shore 80. The rectangular cross-section of the block was meshed by 516 triangle elements. The FEM model was developed in the program environment FEMLAB 3.1.

The mechanical and thermal parameters of the model were obtained from their identification based on previous measurements of loading force, displacement and inner temperature responses of the segments for case of harmonic loading at frequency 20 Hz, pre-press 6 kN and dynamic amplitude 2 kN (see [8]) : $\alpha_a = 30\,\mathrm{W/m^2/°C}$, $\alpha_m = 200\,\mathrm{W/m^2/°C}$ coefficients of heat transfer into air and steel, respectively, $T_{0a} = T_{0m} = 25.5\,°\mathrm{C}$ temperatures of air and steel, $c_p = 1000\,\mathrm{J/kg/°C}$ specific heat coefficient of rubber, $\lambda = 0.28\,\mathrm{W/m/°C}$ heat transfer coefficient of rubber, $\varrho = 1357\,\mathrm{kg/m^3}$ density of rubber, $E_0 = 52\,\mathrm{MPa}$ Young modulus for temperature $T_0 = 26\,°\mathrm{C}$, $k_1 = 0.06$ parameter of the Young modulus dependence on temperature, $\mu = 0.49$ Poisson constant. The initial temperature of rubber and surrounding was defined 26 °C.

Continuous uniformly distributed dynamic loading along the upper boundary of the block was for the press $p_y = p_0 \sin(2\,\pi\,f\,t)$, where amplitude $p_0 = 5.32{\times}10^5\,\mathrm{N\,m^{-2}}$, and for the shear $p_x = p_0 \sin(2\,\pi\,f\,t)$, where $p_0 = 2.13{\times}10^5\,\mathrm{N\,m^{-2}}$. For the press case, the amplitude of the total force on the whole segment corresponded to the test load amplitude 2 kN. Since the shear stiffness is about 3 times lower than press stiffness the amplitude of the shear was decreased so that the maximal shear deformation amplitudes were comparable with the press deformations. For both type of loading we chose different coefficients of proportional damping, i.e. $\beta_K = 12{\times}10^{-4}$, $\alpha_M = 0$ (cca 15 % damping ratio) for the press and $\beta_K = 8{\times}10^{-4}$, $\alpha_M = 0$ (cca 8 % damping ratio) for the shear.

Loading frequency f was 20 Hz for both cases. After loading time block in duration 200 s, the cooling period of 50 s without loading followed. In this period the generated heat was equal zero.

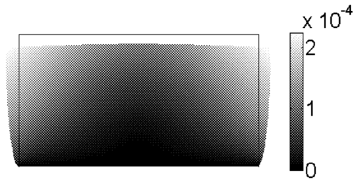


Fig.2: The total deformation field of the rubber block at dynamic loading (press – on the left, shear – on the right)

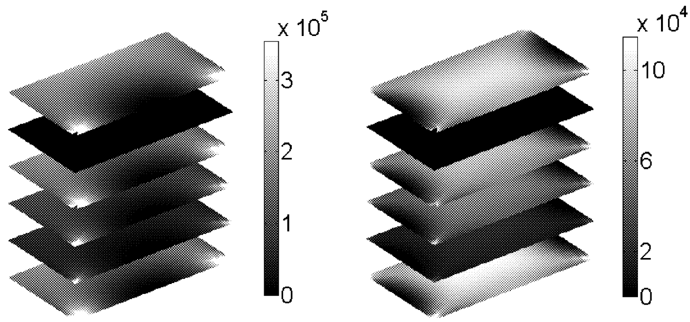


Fig.3: Fields of dissipated energy density of the block during one loading cycle (the press – on the left, the shear – on the right)

Time integration was performed in the FEMLAB, where after time discretization of differential equations, the linear solver UMFPACK that is based on multifrontal method and LU factorization of sparse coefficient matrices was used for solution of the equation system. The integration step was $1{\times}10^{-3}$ s and output time step for storage data was $1{\times}10^{-2}$ s.

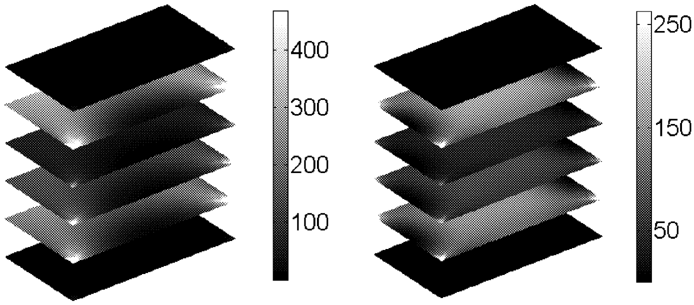
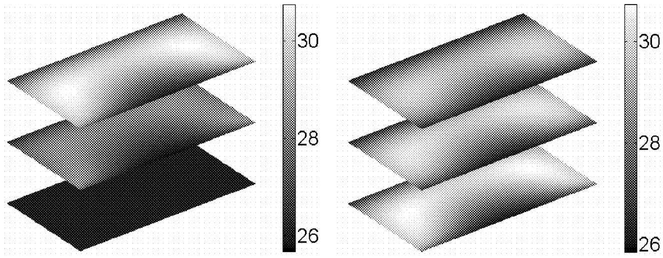


Fig.4: Fields of strain energy density of the block during one loading cycle (the press – on the left, the shear – on the right)

Figures 2 to 6 depict the results of numerical simulations. The total displacements, i.e. the Euclidean norm of displacement vector in single nodes, and deformed contours at a selected time are shown in Fig. 2. Regarding boundary condition of the upper side of the block for press loading, the maximum total displacements are in upper corners of the block where besides maximum vertical displacement also dilatation in horizontal direction occurs. In case of the shear loading the displacements are uniformly linearly distributed along height of the block.

On Fig. 3 and 4 there are shown fields of the density of dissipated energy and density of strain energy for both types of loading during one loading period $T = 0.05$ s. Each floor of these figures from bottom up corresponds to increasing sampled time ($\Delta T = 0.01$) of the cycle. By comparison of the time dependences from Fig. 3 and 4 it shows that there is shape affinity with phase lag between both quantities. This lag is caused by dependence of dissipated energy on strain rate besides of strain in the case strain energy. For harmonic loading it leads to the lag of quarter of a period. So where it is the peak amplitude of one quantity there is null amplitude of the other and vice verse. It can be seen at the bottom and upper floors of both figures. In the results of the energy and power distributions a side effect can be seen. There are spots of high concentration in the bottom and upper (in case of the shear) corners of the block due to geometric discontinuities and shear stresses in these points and insufficient dense space discretization. However since a large heat transfer into surrounding at these points, this side effect had not significant effect on a temperature distribution of the block as it is shown bellow.

The next two figures 5 and 6 depict temperature fields of the block for different times of loading ($t = 0$ s bottom, 100 s middle and 200 s upper floor) and cooling ($t = 200$ s, 225 s and 250 s) for the press and shear loadings, respectively. For both cases, it can be seen non-uniform distribution of temperature inside the block. The heat transfer disturbs a shape affinity of temperature fields with the fields of the densities (Fig. 3 and 4). Temperatures stabilize with time and the thermo-mechanical equilibrium settles. The distribution is dependent on the strain energy field to which a dissipated energy is directly dependent according to proportional damping. In addition the temperature field is dependent on the heat transfer into surrounding. So we can see lowest temperatures at the rubber-steel interface where is the largest heat transfer. The absolute values of temperatures also depend on the frequency of loading and size of damping coefficients. The generated heat is linearly dependent on both of them. The damping coefficients can be determined from the loss factor of given material or by parametric identification from measurement of temperature



Obr.5: Temperature field of the segment for different times of loading (0 s bottom, 100 s middle and 200 s upper view) (left side) and cooling (200 s, 225 s and 250 s) (right side) – case of press loading

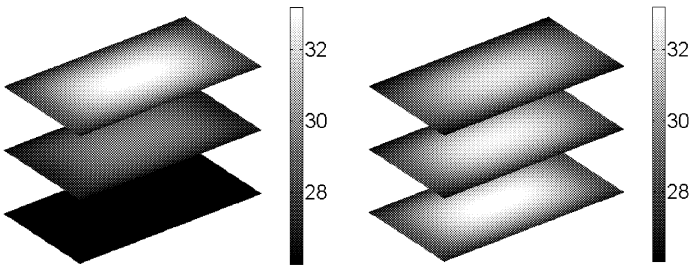


Fig.6: Temperature field of the segment for different times of loading (0 s, 100 s and 200 s) (left side) and cooling (200 s, 225 s and 250 s) (right side) – case of shear loading

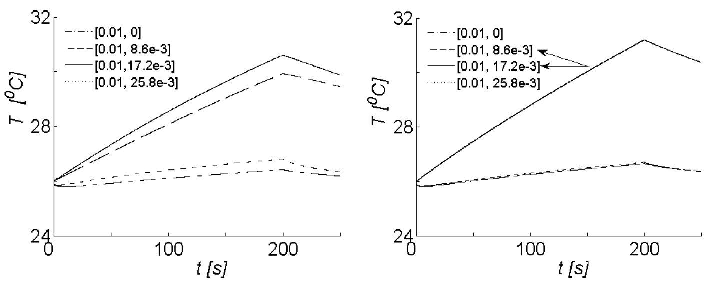


Fig.7: Temperature versus time characteristics of selected points along vertical section (1 cm aside from left edge) of the block (the press – on the left, the shear – on the right)

responses at dynamic tests. In the other case the drain of heat into surrounding has to be concerned.

The curve $[0.01, 0]$ belongs to the point on the bottom and $[0.01, 25.8\times10^{-3}]$ on the top of the block. For the press loading in contradiction to the shear loading it can be seen non-symmetric distribution in lower a upper halves of the block. The maximum temperature for the press is reached at the upper half of the segment – point $[0.01, 17.2\times10^{-3}]$. The lowest temperatures at the upper and bottom side due to marked heat transfer into steel parts. The increase of temperature up to 200 s corresponds to loading block and decrease after discontinuity in the characteristics belongs to the cooling period. In spite of a lower coefficient of proportional damping at the shear than at the press, the increase of temperature is comparable. It is cased partly by difference in strain distribution and partly by higher amplitudes of motions due to lower stiffness at the shear as it can be seen on hysteresis curves of Fig. 8. On this figure there are loops for the press (p_y versus v) and shear (p_x versus u) types of loading in the middle [0.0235, 0.0258] of the upper side of the block. The loops at the beginning of the loading are drawn by solid line and by dash line at its end. The tilting and prolonging of the loops during loading are cased by Young modulus dependence on temperature (Eq. 3) that increases.

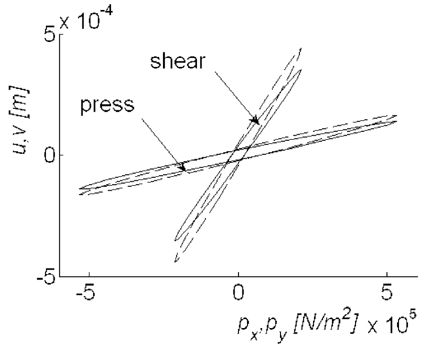


Fig.8: Hysteresis loops at the beginning (solid) and end (dash line) of the loading stage for the press and shear

4. Conclusion

The paper deals with the numerical modeling and solution of feedback thermo-mechanical interaction of a thermo-viscoelastic body at harmonic loading by finite element method. The proposed method was applied on the case of rubber resilient segment of tram wheels.

The aim of this investigation is to improve knowledge of thermo-rheological behavior of rubberlike materials. Therefore we designed mathematical model, whose coefficients can be evaluated by the identification process on the test pattern of a damping material under selected simple stress state, such as uni-axial tension or/and compression, pure shear. From loading point of view a harmonic type seems to be suitable, since changes of rheological parameters are rather slowly varying in time and there is longer time for their observation.

In addition from practical point of view an analysis of thermal processes in rubber resilient elements can lead to their better design with drain of generated heat into surrounding what can help to increase their lifetime at heavy service loading.

For identification of the model parameters we plan to perform next dynamic tests of rubber samples under different pre-presses, dynamic forces and temperature conditions using a hydraulic shaker and climat chamber in our laboratory in Plzen. For measurement of surface temperature fields will be used a thermo-camera.

This article is the extended paper [9].

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