GEOMETRICALLY NON-LINEAR TRUSS ELEMENT WITH VARYING STIFFNESS

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In this paper a new truss element with variation of the cross-sectional area and/or Young modulus along its axis is presented which can be used for geometrically non-linear analysis of uniaxially graded bars. The non-linear stiffness matrices of this element contain the unknown nodal displacements. The non-incremental Lagrangian approach, without any linearisation, has been used for deriving the stiffness matrices. The new shape functions of the beam element have been established which describe the stiffness variation along the element length very accurately. Numerical experiments have been performed – the large displacement of Von Mises two truss structure with varying stiffness, which results confirm the effectiveness and accuracy of the new truss element. In addition, this element fulfils the equilibrium equations in both the global and local sense.

Key words: *geometrical non-linearity, truss element, varying stiffness, functionally graded material, Von Mises truss structure*

1. Motivation

There are many architectural and economical reasons for the use of truss structures with varying stiffness along their length. New technologies enable to develop materials with non-constant material properties. Such materials are called functionally graded materials (FGM). Structures, which are created from FGM materials, have to be modeled by finite elements, that enable accurate inclusion such types of materials.

At the present time the static analysis of such structures (linear or non-linear) can be performed using the truss or beam elements with constant ‘average’ cross-sectional area and Young modulus (although several new beam finite elements with FGM material has been developed for linear static analysis [1, 2, 3]). Sufficient accuracy can be achieved by increasing the number of integration points in the assembled stiffness matrix, by refining the mesh of elements, and by choosing of higher order interpolation polynomials as shape functions. It is known, that for geometrically non-linear or buckling analyses a coarse mesh can lead to inaccurate numerical results. Numerical inaccuracy is caused by usage of the classical shape functions (Hermitean or isoparametric), and by using a linearised form of the incremental non-linear FEM equations.

The main goal of this paper is to present new more effective and accurate truss element with continuous variation of the stiffness along its axis suitable for the solution of geometric non-linear problems. The non-incremental non-linearised Lagrangian formulation of the non-linear FEM-equations will be used to avoid inaccuracy caused by the linearisation of the Green-Lagrange strain tensor increment.

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