STRUCTURAL STABILITY ANALYSIS AND BIFURCATION BEHAVIOR OF SOME PHASE SYNCHRONIZED SYSTEMS

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In this paper, we have attempted to give a general framework (from bifurcation theory point of view) for understanding the structural stability and bifurcation behavior in following phase synchronized systems: (a) coupled Poincare systems; (b) controlled linear oscillator and (c) 'predator-prey' system, on the base of a specific version of bifurcational theory (based on the computing first Lyapunov value (not exponent)). Our results suggest that for these three systems soft stability loss take place.

Key words: *structural stability, bifurcation analysis, first Lyapunov value*

1. Introduction

Synchronization is the manifestation of stability in interacting subsystems and it is an important phenomenon observed in nature and science [1–2]. Synchronization in a dynamical system is the phenomenon of the onset of balance between the phases of the subsystems state variables oscillations, which is caused by an onset of the energy balance. Usually that phenomenon is called phase synchronization (PS). Especially, PS is typical for many systems in biology, neuroscience and physics [1], [3]. Also, in [4] a rich variety of phenomena in the formulation of regular and chaotic phase synchronization in systems of coupled nonidentical circle maps was observed.

Bifurcation theory describes qualitative changes in phase portraits that occur as parameters are varied in the definition of a dynamical system [5]. In the modern theory of dynamical systems, their properties are mainly analyzed in a qualitative way in terms of their flow in phase space [6–8]. For the experimentalist, it is of great importance to know if any large deviation due to a change of parameter, occurs in his or her system. A better understanding of typical bifurcations is therefore required [9–10], [24].

It is well known that the stability conditions of limit cycles depend critically on the stability conditions of steady states. By knowing the sign of Lyapunov value at the bifurcation boundary (this is not a Lyapunov exponent – see for detailed discussion [11] or appendix in [12]), we are able indeed define not only the structural stability (roughness) of steady state, but – also the roughness of limit cycles or other types of trajectories. This is a very central point of the structural stability analysis (bifurcation theory) of steady states, periodic orbits and other types of trajectories. For details, see [2], [11], [13–14].

One dynamical system is structurally stable if its phase portrait is topologically equivalent to that of all points in a sufficiently small neighbourhood. If we consider a structurally stable

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