COMPARISON OF CELL CENTERED AND VERTEX CENTERED FORMULATION OF FINITE VOLUME METHOD

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Finite volume methods for solving hyperbolic systems on unstructured meshes are known for a long time. There are two basic formulations of the method: cell centered and vertex centered. For the cell centered method, the (finite) volumes used to satisfy the integral form of the equation are the mesh elements itself. For the vertex centered approach, the finite volumes are elements of the mesh dual to the computational mesh. We present comparison of both formulations. The method is first evaluated on a scalar advection equation. Knowing the analytical solution of the problem, convergence studies are performed. More complex test cases involve the 3D transonic flow past an Onera M6 airfoil. Discussion includes influence of the reconstruction and limiters on the solution. The results of the parallel implementation for a Linux PC cluster both with explicit and implicit time integration method are presented.

Key words: *finite volume method, transonic flow, linear reconstruction, limiter, parallel method, implicit method*

1. Introduction and governing equations

This article deals with numerical methods for solving the hyperbolic problem

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \vec{\mathbf{f}} = 0, \qquad \forall \vec{x} \in \Omega \subset \mathbb{R}^d, \quad t \in I \tag{1}$$

where $\mathbf{u}(\vec{x},t) : \mathbb{R}^{d+1} \to \mathbb{R}^q$ is a set of q conserved variables and $\vec{\mathbf{f}}(\mathbf{u}) : \mathbb{R}^q \to \mathbb{R}^{q\times d}$ is a vector of flux functions. The problem is equipped with a proper set of initial and boundary conditions. In particular, equation (1) represents either a scalar advection equation or the set of Euler equations, i.e. conservation of mass, momentum and total energy.

The finite volume method will be considered, which is widely used as the current state of the art approach [4, 5, 3] for the given problem. Among the biggest advantages we can mention:

Accuracy: Second order of accuracy is routinely observed for the method with a linear reconstruction.

Shock capturing properties: The use of limiter [4] or WLSQR [11, 8, 9, 10] method gives a non-oscillatory solution even in the presence of strong shock waves and discontinuities.

Conservativity: The method can converge to the proper weak solution of the conservation law (the shock waves are located in the correct position).

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