ASYMPTOTIC DYNAMICS OF MODES IN SOLUTIONS TO THE HOMOGENEOUS NAVIER-STOKES EQUATIONS

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The main goal of the paper is the presentation of several new results on the asymptotic dynamics of modes in strong global solutions to the homogeneous Navier-Stokes equations. It is proved as the main result that if w is such a solution then there exists a unique eigenvalue od the Stokes operator such that its associated eigenfunctions prevail asymptotically in the solution w for $t \mapsto \infty$.

Key words: *Navier-Stokes equations, strong global solution, asymptotic dynamics*

1. Introduction

Let $\Omega \subset \mathbf{R}^3$ be a smooth bounded domain. We deal with the homogeneous Navier-Stokes initial-boundary value problem which is defined by the equations

$$\frac{\partial w}{\partial t} + (w \cdot \nabla)\, w = -\nabla p + \nu\, \Delta w \ , \tag{1}$$

$$\nabla \cdot w = 0 \tag{2}$$

in $\Omega \times (0, \infty)$, by the initial condition

$$w(x, 0) = w_0(x) \ , \quad \text{for every} \quad x \in \Omega \tag{3}$$

and by the homogeneous Dirichlet boundary conditions

$$w = 0 \quad \text{on} \quad \partial\Omega \times (0, \infty) \ . \tag{4}$$

The unknown $w = (w_1, w_2, w_3)$ stands for velocity, p denotes pressure and $\nu > 0$ is the kinematic viscosity. The equations (1) and (2) describe the flow of a Newtonian viscous incompressible fluid (water) and its evolution in time. The Navier-Stokes equations (1) express the conservation of momentum and the equation of continuity (2) expresses the conservation of mass. Although the mathematical theory of the Navier-Stokes equations is deeply elaborated, many important questions still remain open. The survey of main results and open problems can be found e.g. in [3]. The Navier-Stokes equations were originally derived in the 19th century and except others, one of the important assumptions was the a priori assumption on smoothness of velocity and pressure as functions of space and time. However, in spite of an enormous effort of many mathematicians, the question of the global in time existence of a smooth solution for arbitrarily large smooth initial data has not yet

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