NUMERICAL METHODS FOR MARKOV CHAIN MODELS

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We propose unified approach for analysis of finite discrete time Markov chains. We show some possibilities for computing stationary probabilty vectors even in the case of reducible transition matrix. Finaly we show one method for computing of Mean First Passage Times Matrices.

Key words: *Markov chains, numerical method*

1. Introduction

The basic motivation for the study of homogeneous reducible Discrete Time Markov Chain (DTMC) is a quantitative risk and reliability analysis for Railways signaling systems see [6, 4, 5]. A natural property of the risk model is a presence of several independent sets of persistent – absorbing states. The sets represent the fundamental classes of the system hazards. The probability characteristic of the transitions to these classes is the issue of the risk analysis. This is the reason why it is necessary for us to study the reducible homogeneous finite DTMC.

We show some possibilities for computing stationary probability vectors even in the case of reducible transition matrix. Finaly we show one method for computing of Mean First Passage Times Matrices.

Definition 1. *Let elements of* $\mathbf{T} \in \Re^{n\times n}$ *be non negative and* $\mathbf{T}\,\mathbf{e} = \mathbf{e}$*, where* $\mathbf{e} = (1,\dots,1)^{\mathrm{T}} \in \Re^n$*. Then we call* $\mathbf{T}$ *the stochastic matrix.*

Stochastic matrices are used for a description of Markov chains.

Definition 2. *A finite Markov chain is stochastic process, which moves through finite number of states, and for which the probability of entering a certain state depends only on the last state occupied.*

Suppose that $\{X_m \mid m = 0, 1, \dots\}$ is a finite homogeneous Markov chain on the states $S_1, \dots, S_n$. Let $\mathbf{T} \in \Re^{n\times n}$ be its corresponding transition matrix. More information on stochastic processes and Markov chains can be found in [2, 11]. From our point of view we are interested in the following characteristics: **long time behavior, probabilities of reaching some states from other states, and, if such transitions are possible, the time which is necessary to reach these transitions**.

Definition 3. *A transient state has a non-zero probability that the chain will never return to this state. A reccurent(persistent) state has a zero probability that the chain will never return to this state.*

The capital symbol $\mathbf{E}$ is used for the matrix of all ones and $\mathbf{e}$ is the vector with all items equal to 1. The dimensions of $\mathbf{E}$ and $\mathbf{e}$ will be always clear from the context.

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