NUMERICAL SIMULATION OF THREE-PHASE FLOW WITH A VIRTUAL MASS EFFECT

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We deal with the numerical simulation of a flow of solid-liquid-gas slurries with the virtual mass effect. The governing system of equations is strongly nonlinear hyperbolic with nonconservative terms. We propose a numerical scheme which belongs to the class of finite volume methods. In order to increase the order of convergence we apply a higher order reconstruction technique. Several numerical examples demonstrating the efficiency of the schemes are presented.

Key words : *solid-liquid-gas slurries, virtual mass effect, Rusanov scheme, MUSCL approach, finite volume methods, steady and unsteady flow simulation*

1. Introduction

We deal with the numerical simulation of an unsteady flow of solid-liquid-gas slurries, which can be employed, e.g., for a transport of a mixture of sand, oil and air flows in a pipe system. A flow of this mixture is described by a system of conservation laws with suitable constitutive relations. Since the liquid and gas phases of the mixture adhere to solid particles, we include in the model the so-called virtual mass effect, for description and determination see, e.g., [1], [9]. This model leads to a system of nonconservative hyperbolic equations with source terms.

There exist many papers dealing with the numerical simulation of multiphase flow, let us mention, e.g., [2], [5], [6], [8] and the references therein. However a numerical solution of a solid-liquid-gas mixture with the virtual mass effect can be found only in the original papers [1], [9] where a method of characteristics was employed.

In this paper we solve a three-phase flow with virtual mass effect by a finite volume scheme. At first we present a mathematical model introduced in [9]. Further we introduce the so-called Rusanov scheme and present its small modifications which is suitable for the solution of the considered problem. Moreover, a higher order scheme based on the MUSCL approach is introduced. Two numerical examples demonstrating the efficiency of the proposed methods are presented. Several concluding remarks are given at the end of the paper.

2. Governing equations

2.1. Original system of equations

We start with the model of a flow of solid-liquid-gas slurries in a pipe with a virtual mass effect, which was presented in [9]. Let the subscripts $_{\mathrm{s}}$, $_{\ell}$ and $_{\mathrm{g}}$ denote the solid, liquid and

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