A FUNDAMENTAL PROOF FOR THE EQUIVALENCE OF TRANSFORMATIONS WITH RESPECT TO FIXED AND MOVING COORDINATES IN REVERSE ORDER

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*In elementary robotics, it is very well known that the rotation of an object by the angles respectively $\Psi(x)$, $\Theta(y)$, $\Phi(z)$ wrt** a fixed coordinate system (RPY) results in the same angular position for the object as the position achieved by the rotation of that object by the angles respectively $\Phi(z)$, $\Theta(y)$, $\Psi(x)$ wrt a moving (with the object) coordinate system (euler angles). The proofs given up to now for such consequences are not general and for any such problem usually involve the calculation of the transformation matrix for both cases and observing the equivalence of the two matrices [1, 2, 3]. In this paper a fundamental and at the same time general proof is given for such results. It is shown that this equivalence in reverse order can be extended to the general class of transformations which keep the local relations constant (i.e., each transformation should keep the local relations constant). For example, rotation, translation and scaling are 3 types of transformations which can be located in this general class.*

Key words : *transformations, rigid body motion, Euler angles, fixed coordinate, moving coordinate, RPY*

1. Introduction

We first define what we mean by the transformation of a coordinate wrt another coordinate for 3 types of transformations (translation, rotation and scaling). This is done in part 2. Then we define what we mean by the constancy of local relations and define the class of transformations which keep the local relations constant and show that rotation & translation & scaling can be located in this class (part 3). Parts 4 to 8 represent the proof and the result. Part 9 illustrates the simplicity of the proof and part 10 models a special case of transformations noted in the proof in the matrix form.

Note that you can define and add to part 2, any other type of transformation which keeps the local relations constant Then, your defined transformation will also obey the law of equivalence in reverse order, too.

2. Transformation of a coordinate wrt another coordinate

In this part, we define 3 types of transformations which keep the local relations constant. You may define and add to this list, any other type of transformation which keeps the local relations constant. If you do that, our proof automatically covers your defined transformation too.

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** wrt = with respect to