DYNAMIC BEHAVIOR OF INVERTED PENDULUM WITH A CYCLOIDAL OSCILLATING SUSPENSION POINT

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The study of inverted pendulum with oscillating suspension point is of interest to several fields of physics, mechanics and engineering. In this work we consider a parametrically forced inverted pendulum with a cycloidal (ordinary cycloid, cardioid and astroid) oscillating suspension point. Regarding the three cases outlined above, the stability of the pendulum inverse state substantially depends on the initial conditions. Also, we make detailed analytical and numerical investigation of its dynamic behavior.

Key words: *inverted pendulum, dynamic behavior, stabilization, oscillating suspension point*

1. Introduction

The dynamic stabilization of the inverted pendulum with a vertically and horizontally oscillating suspension point (see Fig. 1) possesses an interesting behavior. It is well know that, the inverted pendulum becomes stabilized after its instability, destabilizes again, and so forth ad infinitum [1]. Here, we could note that the first example of this dynamic stabilization is the inverted pendulum with a vertically oscillating suspension point [2]. It is well known also that, when the verticaly oscillations are increased, the inverted pendulum becomes stabilized one. On the other hand, when the horizontaly oscillations are increased it becomes unstabilized [3]. Such inverted pendulums with a vertically and horizontally suspension point can be used to model many physical [3] and biological (biped locomotion) systems [9].

Several investigators have sought to develop nonlinear dynamical models of the pendulum and the inverted pendulum when the suspension points oscillating one [3–6], [9]. In the recent years, the nonlinear chaotic dynamics of the inverted pendulums has attracted considerable attention [1], [7–8]. In [1] a parametrically forced pendulum with the vertically oscillating suspension point is considered. It is found that the inverted state stabilizes via alternating 'reverse' subcritical pitchfork and period-doubling bifurcations, while it destabilizes via 'normal' supercritical period-doubling and pitch-fork bifurcations.

Here we are interested in the stability of the inverted state when the suspension point makes verticaly and horizontaly oscillations, i.e. ordinary cycloidal oscillations; cardioidal oscillations and astroidal oscillations. In this case (following [3]), the differential equation of motion of the pendulum has the form

$$m\,l^2\,\ddot{\varphi} + c\,\dot{\varphi} - m\,l\,\left[g + \ddot{s}_1(t)\right]\,\sin\varphi = m\,l\,\ddot{s}(t)\,\cos\varphi\ , \tag{1}$$

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